Partial Answer Key

Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer.

1. A persistent current account deficit, perhaps one that is everlasting, is not socially optimal and should be corrected by the government.

2. A country’s long-run growth rate does not depend on its income tax structure.

3. A deficit-financed expansion of government spending has no effect on aggregate output.

4. Changes in the structure of income tax can affect the distribution of (pre-tax) incomes.

5. Consumers with linear-quadratic preferences will not engage in precautionary saving.

6. Temporary increases in the government budget deficit designed to increase economic activity, i.e. “fiscal stimulus” packages, are welfare-enhancing.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider a world with $T$ periods and $J$ agents. There is a single perishable consumption good. All agents have the same utility function

$$u(c_j^1, c_j^2, \ldots, c_j^T) = \sum_{t=1}^{T} \beta^{t-1} \frac{c_t^{1-\alpha}}{1-\alpha}, \quad j = 1, 2, \ldots, J.$$ 

Each agent is given a lifetime endowment of the consumption good $(y_j^1, y_j^2, \ldots, y_j^T)$.

(a) Solving the social planner’s problem or the Pareto problem yields first-order conditions that equate marginal rates of substitution across agents,

$$\left(\frac{c_s^1}{c_t^j}\right)^{-\alpha} = \left(\frac{c_s^j}{c_t^j}\right)^{-\alpha} \Rightarrow c_s^j = c_s^j \frac{c_t^j}{c_t^j}.$$ 

Thus, for some constant $\lambda^j$,

$$c_t^j = \lambda^j c_t^1,$$

for all $j$. Summing both sides over all agents gives

$$Y_t \equiv \sum_j y_t^j = \left(\sum_j \lambda^j\right) c_t^1,$$

or

$$c_t^1 = \frac{1}{\sum_j \lambda^j} Y_t.$$ 

In other words, agent 1’s period-$t$ consumption is a constant fraction of the period-$t$ aggregate endowment. Multiplying both sides of this equation by $\lambda^j$ gives the expression

$$c_t^j = \lambda^j c_t^1 = \lambda^j \frac{\lambda^j}{\sum_j \lambda^j} Y_t,$$

for all $j \neq 1$. That is, the set of interior Pareto optima is

$$(k_1 Y_t, k_2 Y_t, \ldots, k_J Y_t)_{t=1}^T,$$

where

$$k_j = \frac{\lambda^j}{\sum_j \lambda^j}$$

is a time-invariant fraction of the aggregate endowment, with $0 < k_j < 1$ and $\sum_j k_j = 1$.

(b) A competitive equilibrium in an Arrow-Debreu economy consists of a consumption allocation $(c_j^1, c_j^2, \ldots, c_j^T)_{j=1}^J$ and prices $(q_1, q_2, \ldots, q_T)$ that satisfy

$$\max_{\{c_j^1\}} u(c_1^1, c_2^1, \ldots, c_T^1)$$

s.t. $\sum_{t=1}^{T} q_t c_t^j \leq \sum_{t=1}^{T} q_t y_t^j,$

$$c_t^j \geq 0, \quad \forall t,$$
for each agent $j$, and

$$\sum_{j=1}^{J} c_j^t \leq \sum_{j=1}^{J} y_j^t \equiv Y_t, \quad \forall t. \quad \blacksquare$$

The first-order conditions to agent $j$’s utility maximization problem imply that

$$\frac{\beta(t-1) (c_j^t)^{-\alpha}}{(c_j^t)^{-\alpha}} = \frac{q_t}{q_1}, \quad t = 2, \ldots, T. \quad \blacksquare$$

Identical homothetic preferences imply strong aggregation, i.e., prices can be computed as if there is a single agent that receives the aggregate endowment. Market clearing implies

$$\frac{q_t}{q_1} = \beta(t-1) \left( \frac{Y_t}{Y_1} \right)^{-\alpha}, \quad t = 2, \ldots, T. \quad \blacksquare$$

(c) A competitive allocation in a Radner economy consists of a consumption-discount bond allocation $\left\{ (c_{it}^j, b_{it}^j) \right\}_{j=1}^{J}$ and bond prices $\left\{ p_t^* \right\}_{t=1}^{T}$ that satisfy

$$\max \left\{ c_{it}^j, c_{i2}^j, \ldots, c_{iT}^j \right\}$$

$$s.t. \quad c_{it}^j + p_t b_{it}^j \leq y_{it}^j + b_{i(t-1)}^j, \quad c_{it}^j \geq 0, \quad b_{iT}^j \geq 0, \quad \forall t, j,$$

and market clearing conditions

$$\sum_{j=1}^{J} c_{it}^j \leq \sum_{j=1}^{J} y_{jt}^j \equiv Y_t, \quad \forall t,$$

$$\sum_{j=1}^{J} b_{it}^j = 0, \quad \forall t,$$

given $b_{it}^j = 0$ for all $j$. \quad \blacksquare$

The first-order conditions to agent $j$’s utility maximization problem imply that

$$\frac{\beta (c_{it+1}^j)^{-\alpha}}{(c_{it}^j)^{-\alpha}} = p_t, \quad t = 2, \ldots, T. \quad \blacksquare$$

Applying strong aggregation and market clearing implies:

$$p_t = \beta \left( \frac{Y_{t+1}^j}{Y_t^j} \right)^{-\alpha}, \quad t = 2, \ldots, T,$$

with the (gross) real interest rate $R_t = p_t^{-1}$.\[3\]
(d) **First welfare theorem:** Any competitive equilibrium allocation is Pareto optimal if \( u^j \) is increasing for all \( j \).

**Second welfare theorem:** If all \( u^j \)'s are quasiconcave, then for every Pareto optimum, there exists initial endowments that make it a competitive equilibrium. Both theorems are satisfied. Markets are complete and the conditions of the theorems are clearly satisfied. Also, solving for equilibrium consumptions in parts (b) and (c) will yield an allocation on the contract curve computed in part (a).

8. We are considering a variant of the Lucas tree model where there are two types of trees: apple trees ("A"), which have a constant yield; and banana trees ("B"), whose yield varies with the intensity of the "banana blight". The amount of fruit produced by each type of tree is given by

\[
d_t^A = \frac{1}{2}d, \quad d > 0,
\]

\[
d_t^B = d \left( \frac{1}{2} + e_t \right),
\]

where \( e_t \) follows a symmetric two-state Markov chain with the values \( \{-\varepsilon, \varepsilon\} \). The transition density \( f(\epsilon', e) \) is given by

\[
f(-\varepsilon, -\varepsilon) = \Pr(e_{t+1} = -\varepsilon | e_t = -\varepsilon) = \pi = f(\varepsilon, \varepsilon),
\]

\[
f(\varepsilon, -\varepsilon) = \Pr(e_{t+1} = \varepsilon | e_t = -\varepsilon) = 1 - \pi = f(-\varepsilon, \varepsilon).
\]

The preferences of the representative consumer are

\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \sigma} \left[ c_t^{1-\sigma} - 1 \right] \right), \quad 0 < \beta < 1, \quad \sigma > 0.
\]

where \( c_t \) denotes total consumption of apples and bananas; agents are indifferent between the two. The economy starts off with each agent owning one apple and one banana tree apiece.

(a) Let \( q(\epsilon', e) \) denote the price of the one-step-ahead contingent claim that pays off when \( e_{t+1} = \epsilon' \). Writing the consumer’s problem as a Lagrangean, we get

\[
V(x_t, e_t) =
\]

\[
\min_{\lambda_t \geq 0} \max_{c_t \geq 0, \ s^A_{t+1}, \ s^B_{t+1}, \ z(e')} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \lambda_t \left( x_t - c_t - p^A_t s^A_{t+1} - p^B_t s^B_{t+1} - \sum_{\epsilon' \in \{-\varepsilon, \varepsilon\}} q(\epsilon', e_t) z(\epsilon') \right)
\]

\[
+ \beta \sum_{e_{t+1} \in \{-\varepsilon, \varepsilon\}} f(e_{t+1}, e_t) \times V \left( \left( z(e_{t+1}) + \left[ p^A_{t+1} (e_{t+1}) + \frac{d}{2} \right] s^A_{t+1} + \left[ p^B_{t+1} (e_{t+1}) + d \left( \frac{1}{2} + e_{t+1} \right) \right] s^B_{t+1} \right), \ e_{t+1} \right),
\]

The FOC for an interior solution are:

\[
c_t^{1-\sigma} = \lambda_t,
\]

\[
\lambda_0 p_i^t = \beta \sum_{e_{t+1} \in \{-\varepsilon, \varepsilon\}} f(e_{t+1}, e_t) \times \frac{\partial V [t+1]}{\partial x_{t+1}} \left[ p^i_{t+1} (e_{t+1}) + d^i_{t+1} (e_{t+1}) \right], \quad i \in \{A, B\},
\]

\[
\lambda_t q(\epsilon', e_t) = \beta \frac{\partial V (x_{t+1}(\epsilon'), e')}{\partial x_{t+1}} f(\epsilon', e_t), \quad \epsilon' \in \{-\varepsilon, \varepsilon\}.
\]
Since (following Benveniste-Scheinkman),
\[
\frac{\partial V}{\partial x_t} = \lambda_t,
\]
the Euler equations are
\[
\begin{align*}
\pi^i t^{-\sigma} = \beta E_t \left( c_{t+1} (e_{t+1})^{-\sigma} \left[ p_{t+1}^i (e_{t+1}) + d_{t+1}^i (e_{t+1}) \right] \right), & \quad i \in \{A, B\}, \\
q(e', e_t) = \beta \left( \frac{c_t}{c_{t+1}(e')} \right)^{\sigma} f(e', e_t), & \quad e' \in \{-\varepsilon, \varepsilon\}.
\end{align*}
\]

(b) To achieve equilibrium, we impose \( s_{t+1}^A = s_{t+1}^B = 1, z(e_{t+1}) = 0, \forall e_{t+1} \) so that \( c_t = d_t^A + d_t^B = d(1 + e_t) \). Then the equilibrium prices for contingent claims are
\[
\begin{align*}
q(-\varepsilon, -\varepsilon) &= \beta \frac{[d(1 - \varepsilon)]^\sigma}{[d(1 - \varepsilon)]^\sigma} f(-\varepsilon, -\varepsilon) = \beta \pi, \\
q(\varepsilon, -\varepsilon) &= \beta \frac{[d(1 - \varepsilon)]^\sigma}{[d(1 + \varepsilon)]^\sigma} f(\varepsilon, -\varepsilon) = \beta (1 - \pi) \left( \frac{1 - \varepsilon}{1 + \varepsilon} \right)^\sigma, \\
q(\varepsilon, \varepsilon) &= \beta \frac{[d(1 + \varepsilon)]^\sigma}{[d(1 + \varepsilon)]^\sigma} f(\varepsilon, \varepsilon) = \beta \pi, \\
q(-\varepsilon, \varepsilon) &= \beta \frac{[d(1 + \varepsilon)]^\sigma}{[d(1 - \varepsilon)]^\sigma} f(-\varepsilon, \varepsilon) = \beta (1 - \pi) \left( \frac{1 + \varepsilon}{1 - \varepsilon} \right)^\sigma.
\end{align*}
\]

When \( \pi = 1 - \pi \), it follows from \( \sigma > 0 \) that \( q(-\varepsilon, \varepsilon) \) has the largest value. This implies that the most valuable contingent claim is a claim for low future output that is purchased when current output is high. In this configuration, the marginal utility of current consumption is low—as output/consumption is high—while the marginal utility of future consumption is high. Consumption-smoothing agents are thus very eager to trade current consumption for consumption in this future state, leading to a high price.

(c) Let’s derive closed-form expressions for \( p^A^- = p^A(-\varepsilon) \) and \( p^A^+ = p^A(\varepsilon) \).

1. In equilibrium, the Euler equation for \( p^A(e_t) \) is
\[
p^A(e_t) [d(1 + e_t)]^{-\sigma} = \beta E_t \left( [d(1 + e_{t+1})]^{-\sigma} \left[ p^A(e_{t+1}) + \frac{1}{2} d \right] \right)
\]

2. Evaluating this Euler equation at \( e_t = \varepsilon \) and \( e_t = -\varepsilon \) yields
\[
\begin{align*}
p^A^- &= [d(1 - \varepsilon)]^\sigma \beta \left( \pi [d(1 - \varepsilon)]^{-\sigma} \left[ p^A^- + d/2 \right] + (1 - \pi) [d(1 + \varepsilon)]^{-\sigma} \left[ p^A^+ + d/2 \right] \right) \\
&= \beta \left( \pi \left[ p^A^- + d/2 \right] + (1 - \pi) \left( \frac{1 - \varepsilon}{1 + \varepsilon} \right)^\sigma \left[ p^A^+ + d/2 \right] \right) \\
&= \beta \left( \pi p^A^- + (1 - \pi) \left( \frac{1 - \varepsilon}{1 + \varepsilon} \right)^\sigma p^A^+ + \left[ \pi + (1 - \pi) \left( \frac{1 - \varepsilon}{1 + \varepsilon} \right)^\sigma \right] d/2 \right), \\
p^A^+ &= [d(1 + \varepsilon)]^\sigma \beta \left( (1 - \pi) [d(1 - \varepsilon)]^{-\sigma} \left[ p^A^- + d \right] + \pi [d(1 + \varepsilon)]^{-\sigma} \left[ p^A^+ + d/2 \right] \right) \\
&= \beta \left( \pi p^A^+ + (1 - \pi) \left( \frac{1 + \varepsilon}{1 - \varepsilon} \right)^\sigma p^A^- + \left[ \pi + (1 - \pi) \left( \frac{1 + \varepsilon}{1 - \varepsilon} \right)^\sigma \right] d/2 \right).
\]
3. The expressions just derived comprise a system of two linear equations in two unknowns, which can be solved for $p^{A-}$ and $p^{A+}$. To see this more clearly, note that we can rewrite the equations as

$$
\begin{pmatrix}
1 & -\lambda (1 - \pi) \chi \\
-\lambda (1 - \pi) \chi^{-1} & 1
\end{pmatrix}
\begin{pmatrix}
p^{A-} \\
p^{A+}
\end{pmatrix}
= \left( \begin{array}{c}
\lambda \pi + \lambda (1 - \pi) \chi \\
\lambda \pi + \lambda (1 - \pi) \chi^{-1}
\end{array} \right) d/2,
$$

with $\lambda \equiv \beta (1 - \beta \pi)^{-1}$, $\chi \equiv \left( (1 - \varepsilon) / (1 + \varepsilon) \right)^{\sigma}$.

4. The prices just derived are not the only ones that solve the Euler equation. In particular, we can append to these prices any bubble term $b(e_t)$ that satisfies $E_t (c_{t+1}^\sigma b_{t+1}) = \beta c_t^\sigma b_t$: if the function $p_A^A (e_t)$ solves the Euler equation, so does the function $p_A^A (e_t) + b(e_t)$. We rule out bubbles with the transversality condition:

$$
\lim_{J \to \infty} E_t \left( \beta^J p_A^A (c_{t+J}^{\sigma-\sigma}) \right).
$$

(Note that $\lim_{J \to \infty} E_t (\beta^J b_{t+J} c_{t+J}^{\sigma}) = b_t$.) If the transversality condition fails to hold, the consumer’s budget set is unbounded, and she will want to consume more than the equilibrium quantity of $d(1 + e_t)$.

9. The production function for this economy is given by

$$
Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1. \quad \text{(PRF)}
$$

The preferences of the representative household are

$$
E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \gamma} C_t^{1-\gamma} (1 - L_t),
$$

$$
0 < \beta < 1, \quad \gamma > 0. \quad \text{(TS)}
$$

The capital accumulation equation is

$$
K_{t+1} = (1 + r) K_t + L_t^{1-\alpha} - C_t - G_t, \quad \text{(CA)}
$$

with government spending following an AR(1) process around the log of its steady state value:

$$
\tilde{\gamma}_t \equiv \ln \left( G_t / G_{ss} \right) = \phi \tilde{\gamma}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad \text{(TS)}
$$

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process.

(a) In recursive form, the social planner’s problem is

$$
V(K_t, G_t) = \max_{\{C_t, L_t\}} \frac{1}{1 - \gamma} C_t^{1-\gamma} (1 - L_t) + \beta E_t \left( V \left( (1 + r) K_t + L_t^{1-\alpha} - C_t - G_t, G_{t+1} \right) \right).
$$

The first order conditions are

$$
C_t^{1-\gamma} (1 - L_t) = \beta E_t \left( \frac{\partial V (K_{t+1}, G_{t+1})}{\partial K_{t+1}} \right), \quad \text{(FOC1)}
$$

$$
\frac{1}{1 - \gamma} C_t^{1-\gamma} = \beta E_t \left( \frac{\partial V (K_{t+1}, G_{t+1})}{\partial K_{t+1}} \right) (1 - \alpha) L_t^{-\alpha}. \quad \text{(FOC2)}
$$
Using Benveniste and Scheinkman’s results, we find that
\[
\frac{\partial V (K_t, G_t)}{\partial K_t} = \beta E_t \left( \frac{\partial V (K_{t+1}, G_{t+1})}{\partial K_{t+1}} \right) (1 + r).
\]
Inserting equation (FOC1), this reduces to
\[
\frac{\partial V (K_t, G_t)}{\partial K_t} = C_t^{-\gamma} (1 - L_t) (1 + r),
\]
and (FOC1) becomes
\[
C_t^{-\gamma} (1 - L_t) = \beta (1 + r) E_t \left( C_{t+1}^{-\gamma} (1 - L_{t+1}) \right) = E_t \left( C_{t+1}^{-\gamma} (1 - L_{t+1}) \right). \tag{EE}
\]
Combining equations (FOC1) and (FOC2) yields
\[
\frac{1}{1 - \gamma} C_t^{1-\gamma} = C_t^{-\gamma} (1 - L_t) (1 - \alpha) L_t^{-\alpha},
\]
which reduces to
\[
\frac{1}{1 - L_t} L_t^\alpha = (1 - \alpha) (1 - \gamma) \frac{1}{C_t}. \tag{LL}
\]
The capital accumulation equation (CA) was derived above when formulating the social planner’s problem.

(b) Let’s consider how to calibrate $\alpha$ and $\gamma$.

1. Under perfect competition $(1 - \alpha)$ equals labor’s share of national income, which in the U.S. is around 65 percent.

2. Note that equation (LL) can be written as
\[
\frac{Y_t}{C_t} = L_t^{1-\alpha} \frac{1}{C_t} = \frac{L_t}{1 - L_t} (1 - \alpha)^{-1} (1 - \gamma)^{-1}
\equiv F (L_t; \alpha, \gamma).
\]
The data give us the average values of $Y_t/C_t$ (around 1.5 for the U.S.) and $L_t$ (around 0.2 to 0.35, depending on how labor is defined). Plugging in these values, along with the value of $\alpha$ in part 1, we can solve for $\gamma$.

(c) Log both sides of equation (LL):
\[
-\ln (1 - \exp (\ln L_t)) + \alpha \ln (L_t) = \ln ((1 - \alpha) (1 - \gamma)) - \ln (C_t).
\]
Implicitly differentiating this expression yields
\[
\frac{\alpha d \ln L_t - \exp (\ln L_t)}{1 - \exp (\ln L_t)} d \ln L_t = -d \ln C_t.
\]
Letting lower-case letters with carats “^” denote deviations of logged variables around their steady state values, this implies
\[
\hat{\ell}_t \approx - \left[ \alpha + \frac{L_{ss}}{1 - L_{ss}} \right]^{-1} \hat{c}_t
\equiv -\theta \hat{c}_t, \quad \theta > 0, \tag{LL’}
\]
with the last line following from $\alpha, L_{ss} \in (0, 1)$. Proceeding similarly, it follows from equation (PRF) that

$$
\hat{y}_t = (1 - \alpha) \hat{\ell}_t \\
\approx - (1 - \alpha) \theta \hat{c}_t, \\
\equiv -\lambda \hat{c}_t, \quad 0 < \lambda < \theta. \tag{PRF'}
$$

(d) Logging both sides of Equation (EE) yields:

$$
-\gamma \ln C_t + \ln (1 - \exp(\ln L_t)) = \ln \left( E_t\left( C_{t+1}^{-\gamma} (1 - L_{t+1}) \right) \right) \\
\approx E_t \left( -\gamma \ln C_{t+1} + \ln (1 - \exp(\ln L_{t+1})) \right),
$$

with the second line using the approximation $\ln \left( E_t(\ln X_t) \right) \approx E_t(\ln(\ln X_t))$. Implicitly differentiating, we get

$$
-\gamma d \ln C_t + \frac{-\exp(\ln L_t)}{1 - \exp(\ln L_t)} d \ln L_t \approx E_t \left( -\gamma d \ln C_{t+1} + \frac{-\exp(\ln L_t)}{1 - \exp(\ln L_t)} d \ln L_{t+1} \right),
$$

or

$$
\gamma \hat{c}_t + \frac{L_{ss}}{1 - L_{ss}} \hat{\ell}_t \approx E_t \left( \gamma \hat{c}_{t+1} + \frac{L_{ss}}{1 - L_{ss}} \hat{\ell}_{t+1} \right).
$$

Inserting equation (LL'), we get

$$
\gamma \hat{c}_t - \frac{L_{ss}}{1 - L_{ss}} \theta \hat{c}_t \approx E_t \left( \gamma \hat{c}_{t+1} - \frac{L_{ss}}{1 - L_{ss}} \theta \hat{c}_{t+1} \right),
$$

or

$$
\hat{c}_t \approx E_t \left( \hat{c}_{t+1} \right). \tag{EE'}
$$

(e) Suppose that $C_{ss}/K_{ss} = \psi$, while $G_{ss}/K_{ss} = \chi$, with $\psi + \chi > r$ which implies that $Y_{ss}/K_{ss}$ is $\psi + \chi - r$. One can then log-linearize equation (CA) as

$$
\hat{k}_{t+1} = (1 + r) \hat{k}_t - \chi \hat{g}_t - \omega \hat{c}_t, \tag{CA'}
$$

$$
\omega = \psi + (\psi + \chi - r) \lambda.
$$

One can solve the resulting system (which also includes (TS) and (EE')) to express consumption as a function of capital and government spending:

$$
\hat{c}_t = \eta \hat{k}_t - \mu \hat{g}_t, \tag{CF}
$$

$$
\eta = \frac{r}{\omega} > 0,
$$

$$
\mu = \frac{r \chi}{(1 - \phi + r) \omega} > 0.
$$

1. Combining equation (CF) with equations (LL') and (PRF') yields

$$
\hat{\ell}_t = -\theta \eta \hat{k}_t + \theta \mu \hat{g}_t,
$$

$$
\hat{y}_t = -\lambda \eta \hat{k}_t + \lambda \mu \hat{g}_t.
$$
2. As the persistence parameter $\phi \to 1$, $\mu$ and thus $\lambda \mu$ grow larger, so that increases in government spending lead to increasingly large increases in output. This is because government spending operates solely through wealth effects: government spending diverts resources and makes consumers poorer. As $\phi$ grows, shocks to government spending get more persistent, leading to larger changes in lifetime wealth, and thus larger wealth effects.

10. Consider a deterministic version of a Lucas tree economy inhabited by a representative agent. There is a single tree that bears a constant amount of fruit equal to $y_t = y$ units in each period. There is a market in equity shares entitling the owner to a fraction of the tree’s fruit yield. For convenience, the number of shares is normalized to one. Let $a_t$ denote the number of shares owned by the agent at the end of period $t$; let $p_t$ denote the period $t$ price of the tree. In equilibrium, $p_t$ adjusts so that the agent is willing to hold all shares: $a_t = 1$. The gross return on the asset purchased in period $t$ is defined as $R_t = (p_{t+1} + y_{t+1}) / p_t$. The agent has preferences given by the utility function:

$$\sum_{t=1}^{\infty} \beta^{t-1} \gamma_t u(c_t)$$

where $c_t$ is period-$t$ consumption of the agent and $u(\cdot)$ is increasing and strictly concave. The agent’s preferences are peculiar in that he receives more utility from a given quantity of consumption in periods that are multiples of four. In these periods, momentary utility is given by $\gamma u(\cdot)$. Thus, $\gamma_t = \gamma > 1$ for $t = 4, 8, \ldots$ and $\gamma_t = 1$ otherwise. The fruit cannot be stored.

(a) The agent solves

$$\max_{\{c_t, a_t\}} \sum_{t=1}^{\infty} \beta^{t-1} \gamma_t u(c_t)$$

s.t. $c_t + p_t a_t \leq (p_t + y_t) a_{t-1}, \quad \forall t,$

$c_t \geq 0, \quad a_t \geq 0, \quad \forall t,$

given $a_0 = 1$. The first-order conditions imply the following set of Euler equations

$$p_t u'(c_t) = \beta (p_{t+1} + y_{t+1}) u'(c_{t+1}), \quad t = 1, 2, 5, 6, 9, 10, \ldots$$

$$p_t u'(c_t) = \beta (p_{t+1} + y_{t+1}) \gamma u'(c_{t+1}), \quad t = 3, 7, 11, \ldots$$

$$p_t \gamma u'(c_t) = \beta (p_{t+1} + y_{t+1}) u'(c_{t+1}), \quad t = 4, 8, 12, \ldots$$

Substituting for the rate of return and applying market clearing gives:

$$R_t = \frac{p_{t+1} + y_{t+1}}{p_t}$$

$$= \frac{1}{\beta}, \quad t = 1, 2, 5, 6, 9, 10, \ldots$$

$$= \frac{1}{\gamma \beta} < \frac{1}{\beta}, \quad t = 3, 7, 11, \ldots$$

$$= \frac{\gamma}{\beta} > \frac{1}{\beta}, \quad t = 4, 8, 12, \ldots$$
(b) Period 7: The asset return is low as the agent attempts to save in anticipation of the subsequent period in which consumption is especially valuable. The interest rate falls so that the agent is content consuming the endowment.

Period 8: The asset return is high in this period as the agent attempts to increase consumption by borrowing. The equilibrium interest rate increases so that the agent is content consuming the endowment.

Periods 9 and 10: The return is flat over these two periods because the agent’s intertemporal marginal rate of substitution is stable.

(c) Note: Intuition goes most of the way in answering this part. In the competitive equilibrium, each agent’s marginal rate of substitution must equal the gross asset return. For the agent with linear momentary utility, the marginal rate of substitution is constant (regardless of the endowment) so that his first order conditions reduce to

\[ R_t = \frac{1}{\beta}, \]

for all \( t \). The new agent’s (constant) marginal rate of substitution therefore establishes the equilibrium rate of return in each period. The first agent will borrow from the second (new) agent in period 7 and pay him back in period 8, allowing the first agent the jagged consumption profile that he desires. The new agent essentially functions as an entrepreneur who arbitrages away predictable asset return fluctuations by providing credit to the first agent.

(d) See part (c).