Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. An increase in the real interest rate signals an increase in the likelihood of a recession in the near future.

2. If two countries have the same preferences and technologies, they must experience the same rate of per-capita output growth.

3. Monopolistic competition eliminates monetary neutrality in equilibrium.

4. The more a country saves, the faster it grows.

5. The contemporaneous correlation between consumption and real interest rates is negative.

6. A positive contemporaneous correlation between money and output proves that money is not neutral.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider the following two-person, two-period economy. The economy is populated by two agents, Art and Bart.

Art’s preferences are given by:

\[ E_0 \{ \ln \left( c_0^A \right) + \ln \left( c_1^A \right) \}; \quad 0 < \beta < 1, \]

where \( c_t^A \) denotes Art’s consumption at time \( t \).

Each period, Art is endowed with 1 unit of labor, which he supplies inelastically. Art’s labor income is \( \pi_t^A w_t \), where \( \pi_t^A \) is Art’s idiosyncratic labor productivity, and \( w_t \) is the market wage for effective labor, \( \pi_t^A \). In period 0, Art’s labor productivity is 1. In period 1, however, Art’s labor productivity is stochastic: with probability \( \frac{1}{2} \), his labor productivity is \( 1+\gamma \), \( 0 < \gamma < 1 \), and with probability \( \frac{1}{2} \), his labor productivity is \( 1-\gamma \).

Art uses his period-0 income to purchase 2 assets: contingent claims that pay when Art’s period-1 productivity is high, selling at the price \( p_H \) per unit of consumption delivered; and contingent claims that pay when Art’s period-1 productivity is low, selling at the price \( p_L \). It follows that in period 0, Art’s flow budget constraint is

\[ w_0 = c_0^A + p_H x_H^A + p_L x_L^A, \]

where \( x \) denotes quantities of contingent claims. Art’s period 1 flow budget constraint is

\[ \pi_t^A w_1 + x_H^A \times 1 \{ \pi_t^A = 1 + \gamma \} + x_L^A \times 1 \{ \pi_t^A = 1 - \gamma \} = c_1^A, \]

where \( 1 \{ A \} \) is the indicator function that returns 1 if event \( A \) occurs and 0 otherwise.

(a) Find the Euler equations for Art’s utility maximization problem. You can assume that \( w_1 \) is independent of \( \pi_t^A \).

(b) Bart’s preferences and endowments are completely analogous to Art’s—you can replace A’s with B’s—except that when Art’s productivity is \( 1+\gamma \), Bart’s is \( 1-\gamma \), and when Art’s productivity is \( 1-\gamma \), Bart’s is \( 1+\gamma \).

Output is produced by a price-taking firm with a production function given by

\[ Y_t = \theta \left[ \pi_t^A \ell_t^A + \pi_t^B \ell_t^B \right]; \quad \theta > 0, \]

where \( \ell \) denotes labor supply. Output is non-storable.

1. What is aggregate output in each period? What is \( w_t \)?
2. What are the aggregate supplies of \( x_H \) and \( x_L \)?
3. Find the equilibrium values of \( p_H \), \( p_L \), \( x_H^A \), \( x_L^A \), \( x_H^B \) and \( x_L^B \). (Hint: It is probably quickest to make an intuitive guess and then verify it.)
4. What would be the equilibrium price of a 1-period risk-free bond?
8. Consider an overlapping generations model in which agents live two periods. Each agent inelastically supplies one unit of labor services when young. For an agent born in period $t$, utility is given by:

$$U_t = \frac{c_{1t}^{1-\theta} - 1}{1 - \theta} + \left( \frac{1}{1 + \rho} \right) \frac{c_{2t+1}^{1-\theta} - 1}{1 - \theta},$$

where $c_{1t}$ is consumption when young, $c_{2t+1}$ is consumption when old, $0 < \rho < 1$ is the rate of time preference, and $\theta > 0$ is a parameter. The agent’s budget constraints in each period of life are given by:

$$s_t + c_{1t} = w_t (1 - \tau_t),$$
$$c_{2t+1} = (1 + r_{t+1}) s_t,$$

where $s_t$ is savings when young, $w_t$ is the wage rate, $\tau_t$ is the tax rate on wages, and $r_{t+1}$ is the interest rate on savings. Aggregate output ($Y_t$) is given by:

$$Y_t = AK_t^\alpha L_t^{1-\alpha},$$

where $A$ is a fixed parameter representing technology, $K_t$ is the aggregate capital stock, and $L_t$ is the number of workers, equivalently the population of the young generation. To simplify, assume that population is fixed over time. The government spends all of its tax revenue each period such that:

$$G_t = L_t w_t \tau_t.$$

(a) Derive the Euler equation relating consumption when old to consumption when young as a function of the interest rate. Interpret $\theta^{-1}$.

(b) Now, set $\theta = 1$, and solve for consumption in both periods and for savings as a function of the wage rate and the interest rate. Does saving depend on the interest rate? Explain in terms of income and substitution effects.

(c) Assuming perfectly competitive markets, write expressions for the real wage rate and the interest rate. Use these results to solve for the reduced form for the steady-state stock of capital.

(d) How does an increase in government spending affect the capital stock? Compare the effects in the OLG model with those of representative agent Ramsey model. Explain.

(e) Is this economy dynamically efficient for all feasible tax rates? Explain.
9. Consider a representative agent, money-in-the-utility-function model. Utility is given by:

\[ U_t = \int_{t=0}^{\infty} e^{-\rho t} (\gamma \ln c_t + (1 - \gamma) \ln m_t) \, dt, \]

where: \( c_t \) denotes real consumption; \( m_t = \frac{M_t}{P_t} \) denotes real money balances; \( M_t \) is nominal money; and \( P_t \) is the price level. The rate of time preference is given by \( \rho \) and \( 0 < \gamma < 1 \). The representative agent inelastically supplies one unit of labor services. To simplify, assume that there is a single agent. Output \( (Y_t) \) is given by:

\[ Y_t = AK_t^\alpha \]

where \( K \) is the capital stock, and \( 0 < \alpha < 1 \). Production is assumed to be perfectly competitive. The agent’s budget constraint is given by:

\[ \dot{K}_t + \left( \frac{1}{P_t} \right) \dot{M}_t = r_t K_t + w_t - c_t + x_t, \]

where \( r_t \) is the rate of return on capital \( w_t \) is the real wage rate, \( c_t \) is real consumption, and \( x_t \) is government transfers. A dot denotes a time derivative. Note that capital does not depreciate. The government’s budget constraint is given by:

\[ \left( \frac{1}{P_t} \right) \dot{M}_t = \theta m_t = x_t, \]

where \( \theta = \dot{M}/M \) and is constant.

(a) Justify and criticize the assumption that real money balances yield utility.
(b) Define \( a_t = K_t + \frac{M_t}{P_t} \), and write the agent’s budget constraint in terms of \( a_t \) and \( m_t \). (Eliminate \( K \)).
(c) Set up the Hamiltonian and derive the first order necessary conditions.
(d) Use the first order conditions to write expressions for the consumption Euler equation and for money demand as a function of consumption and the nominal interest rate. (You’ll need to derive an expression for the nominal interest rate.)
(e) Combine agent and government budget constraints and write an equation for the evolution of the capital stock over time after eliminating money. Use this, together with the Euler equation to draw a phase diagram for the system in \( c, K \) space. Evaluate stability properties.
(f) Solve for the steady-state capital stock. Is money neutral? Is money superneutral?
(g) What is the optimal rate of growth of money? Explain.
Consider the following simplified version of a stochastic growth model. There are a fixed number of price-taking producers that solve
\[
\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t, \\
Y_t = L_t^\eta, \quad 0 < \eta < 1,
\]
where: \( \Pi_t \) is profit; \( L_t \) is labor; and \( W_t \) is the real wage. There is no population or technology growth, and the population, number of firms and steady state technology level are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption \( (C_t) \) and labor \( (L_t) \) are given by:
\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) - \Gamma L_t \right] \right), \\
0 < \beta < 1, \quad \Gamma > 0.
\]

Households receive labor income and profits from firms. They pay taxes on this income and receive lump-sum transfers from the government. Households earn a gross return of \((1 + r)K_t\) on their assets, \(K_t\), with \(\beta (1 + r) = 1\). As usual, assume that assets held at the beginning of period \(t+1\), \(K_{t+1}\), are chosen in period \(t\). Note that in this economy, capital is used only as a storage device. Households also face the usual non-negativity and No-Ponzi-Game conditions.

Each period, the government gives households the lump-sum transfer \(S_t\), and levies income taxes on the household’s labor and profit income. (Asset income is not taxed.) The income tax rate, \(\tau_t\), obeys a balanced budget rule:
\[
\tau_t Y_t = S_t. \quad \text{(BB)}
\]

The log of transfers follows an AR(1) process
\[
\hat{s}_t \equiv \ln \left( S_t / S \right) = \phi \hat{s}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad \text{(TS)}
\]
where \(S\) is steady-state government spending and \(\{\varepsilon_t\}\) is an exogenous stationary martingale difference sequence.

(a) Find the equilibrium allocation:
1. Find the first order conditions for profit and utility maximization.
2. Imposing equilibrium, eliminate wages and the tax rate \(\tau_t\) to find: the labor allocation condition; the Euler equation; and the aggregate resource constraint.

(b) Let’s examine the labor allocation condition more closely.
1. Let lower-case letters with carats “\(\hat{\phantom{a}}\)” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:
\[
\hat{l}_t \approx -\theta \left[ \lambda \hat{s}_t + \hat{c}_t \right]; \quad \hat{y}_t \approx -\alpha \left[ \lambda \hat{s}_t + \hat{c}_t \right].
\]
2. As $P \equiv S/Y_{ss}$ goes from 0 to 1, what happens to the signs and magnitudes of $\theta$ and $\lambda$? (Be sure to describe the entire interval.) Why?

(c) Suppose that the steady state consumption-to-capital ratio, $C_{ss}/K_{ss}$, is $\psi$, with $\psi > r$. It is then straightforward to show that the steady state output-to-capital ratio, $Y_{ss}/K_{ss}$, is $\psi - r$. Using these results, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1 + r) \hat{k}_t - \omega_1 \hat{s}_t - \omega_2 \hat{c}_t.$$ \hspace{1cm} (CA')

(d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA')) to express consumption as a function of capital and transfers

$$\hat{c}_t = \chi \hat{k}_t - \mu \hat{s}_t,$$ \hspace{1cm} (CF)

with

$$\chi = \frac{r}{\omega_2}; \quad \mu = \frac{r \omega_1}{\omega_2 (1 - \phi + r)}.$$ 

As $P$ goes from 0 to 1, what happens to the sign of $\chi$?