Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 1 Hour) For 4 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. The long-run growth rate of per capita consumption is independent of individual preferences.

2. Individuals who are risk averse should put all of their savings into assets without risk.

3. Consumption is procyclical in a real business cycle model.

4. Permanent tax increases generate larger changes in consumption and labor than temporary ones.

5. In the basic Ramsey model, taxes, depreciation rates and population growth do not affect per-capita income.

6. There are many empirical studies showing that output is positively correlated with anticipated changes in the money supply. Taken collectively these studies are evidence against the rational expectations hypothesis.
Section 2. (Suggested Time: 2 Hours) Answer any 3 of the following 4 questions.

7. Consider a finite-horizon endowment economy dated in discrete time by the index $t = 1, 2, \ldots, T$. The economy has a deterministic business cycle periods of the following form: in odd periods $Y_t = Y_{odd}$ while in even periods $Y_t = Y_{even}$ where $Y_{odd} > Y_{even}$. There are $J$ agents (indexed by $j$) who have identical homothetic preferences given by:

$$\sum_{t=1}^{T} \beta^{t-1} u(c_t^j)$$

where $0 < \beta < 1$ and the $u$ is increasing and concave. Each agent $j$ has an endowment $\{y_t^j\}_{t=1}^{T}$ of a perishable consumption good where $y_t^j > 0$ for all $t$ and $\sum_{j=1}^{J} y_t^j = Y_t$. In each period, the agents trade a one-period bond that is in zero net supply. Their period-$T$ bondholdings must be nonnegative.

(a) Are one-period interest rates countercyclical or procyclical in this environment? (i.e. is $r_{odd} > r_{even}$ or vice-versa?) Explain your answer.

(b) Now consider this economy with a government that, prior to trading, announces a policy in which it taxes each individual $\tau_{odd}$ units of consumption in odd periods and $\tau_{even}$ units of consumption in even periods. The tax is procyclical: $\tau_{odd} > \tau_{even}$. The government throws the proceeds of the tax into the ocean: What happens to the fluctuations in the interest rate in the economy relative to part (a)? What about fluctuations in aggregate consumption? Explain your answers.

(c) Now suppose that the government is able to borrow and lend in an international market at a constant interest rate $r = 1/\beta - 1$. The government is small enough that it treats $r$ as given. Before the beginning of trading, the government commits to a stream of lump-sum taxes and transfers for each agent which satisfies the government's intertemporal budget constraint:

$$\sum_{t=1}^{T} \frac{\tau_t}{(1 + r)^t} = 0.$$ 

The government chooses a Pareto optimal sequence of taxes and transfers $\{\tau_t\}_{t=1}^{T}$. The agents take this sequence of taxes and transfers as given when they trade with one another in the one-period bond market. What happens to the fluctuations in the interest rate in the economy relative to part (a)? What about fluctuations in aggregate consumption? Explain your answers.
8. Consider the following simplified version of a stochastic growth model. There are a fixed number of price-taking producers that solve

\[
\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,
\]

\[Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad \text{(PRF)}\]

where: \(\Pi_t\) is profit; \(L_t\) is labor; and \(W_t\) is the real wage. There is no population growth, and the population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The government’s flow budget constraint is given by

\[H_t = G_t,\]

where \(H_t\) is the total value of lump-sum taxes, and \(G_t\) denotes government purchases of goods and services. Note that lump-sum taxes are driven by government spending, which follows an AR(1) process around the log of its steady state value:

\[
\hat{\epsilon}_t = \ln(G_t/G_{ss}) = \phi \hat{\epsilon}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1, \quad \text{(TS)}
\]

where \(\{\varepsilon_t\}\) is an exogenous i.i.d. zero-mean process, and \(G_{ss}\) is steady-state government spending. To simplify the analysis, assume that government spending has no effect on the utility households receive from private consumption, and no effect on the private production function. (You can also assume that \(0 < G_t < Y_t, \forall t\).)

The preferences of the representative household over consumption \((C_t)\) and labor \((L_t)\) are given by:

\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_t) - \chi \frac{1}{1 + \gamma} L_t^{1+\gamma} \right] \right),
\]

\[0 < \beta < 1, \quad \gamma \geq 0, \quad \chi > 0.\]

Households receive labor income and profits from firms. Households earn a gross return of \((1 + \tau)K_t\) on their assets, with \(\beta (1 + \tau) = 1.\) As usual, assume that assets held at the beginning of period \(t + 1, K_{t+1},\) are chosen in period \(t.\) Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption, investment in capital, and lump-sum taxes. Households also face the usual non-negativity and No-Ponzi-Game conditions.

(a) Find the first order conditions for profit and utility maximization. Imposing equilibrium, find: the labor allocation condition; the Euler equation; and the capital accumulation equation.

(b) Let lower-case letters with carats "~" denote deviations of logged variables around their steady state values. Show that log-linearized expressions for labor and output are

\[
\tilde{e}_t = -\theta \tilde{c}_t, \quad \tilde{y}_t = -\lambda \tilde{c}_t, \quad \theta > \lambda > 0.
\]
(c) Suppose that the steady state consumption-to-capital ratio, $C/K$, is $\psi > 0$, and that the steady state government spending-to-capital ratio is $\zeta > 0$, with $(\psi + \zeta) > r$. It is easy to show that the steady state output-to-capital ratio, $Y/K$, is $\psi + \zeta - r$. Using these results, log-linearize the capital accumulation equation, and then apply your answer from part (b) to show that
\[ \hat{k}_{t+1} = (1 + r) \hat{k}_t - \zeta \hat{g}_t - \omega \hat{c}_t, \]
\[ \omega > 0. \]

(d) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA')) to express consumption as a function of capital and government spending.
\[ \hat{c}_t = \eta \hat{k}_t - \mu \hat{g}_t, \]  
\[ \eta = r/\omega > 0, \]
\[ \mu = \frac{r \zeta}{(1 - \phi + r) \omega} > 0. \]

1. Using (CF) and your answer from part (b), express $\hat{g}_t$ and $\hat{c}_t$ as functions of capital and government spending. Is output increasing or decreasing in government spending? Why?

2. Consider the log of average labor productivity, $\omega p_t = \hat{g}_t - \hat{c}_t$. Suppose that the economy experiences an unexpected increase in government spending ($\hat{g}$ raises). In what direction will output, labor and average productivity initially respond? Extrapolating from the initial response, would you expect average labor productivity to be pro- or counter-cyclical in this model? Explain intuitively. Graphs are nice, but a short answer is fine.
9. Consider the following economy populated by a dynastic household. The preferences of the household are represented by the utility function

\[
\sum_{t=0}^{\infty} \beta^t N_t \frac{c_t^{1-\sigma}}{1-\sigma}, \quad 0 < \beta < 1, \ \sigma > 1,
\]

where \( N_t \) is population and \( c_t \) is per capita consumption. Population grows at rate \( n \) so that the period-\( t \) population is given by \( N_t = (1 + n)^t \) with initial population \( N_0 = 1 \). Each individual has one unit of time in each period which can be allocated between production of the consumption good or re-organizational activities that increase the stock of organization capital. Organizational capital can be thought of as "blueprints" that outline general management practices within firms. Per capita output of the consumption good is given by the production function

\[
c_t = \alpha u_t x_t
\]

where \( \alpha > 0 \), \( x_t \) is the beginning-of-period-\( t \) stock of organization capital per capita and \( u_t \) is the fraction of time allocated to goods production. Finally, organization capital is accumulated by combining existing organization capital with the time allocated to re-organizational activities according to the expression

\[
x_{t+1} = z (1 - u_t) (1 - \delta) x_t
\]

where \( z > 0 \), \( 0 < \delta < 1 \), and \( x_0 > 0 \) is given.

(a) Set up the social planner’s problem as a dynamic programming problem and derive the corresponding Euler equation.

(b) Show that per capita consumption, organization capital, and output grow at the same rate and determine this growth rate.

(c) Determine the fraction of time devoted to production activities in each period. What is the economic intuition behind this expression?

(d) Derive a condition on the model’s preference and technology parameters sufficient to guarantee positive long-run growth in per-capita consumption. Do you think that this model captures an important element of long-run (endogenous!) growth? Explain why or why not. (Hint: What is the “engine of growth” in this economy?)
10. Consider the following simplified version of a stochastic growth model. The preferences of the representative household over consumption ($\bar{C}_t$) and labor ($\bar{L}_t$) are given by:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ a\bar{C}_t - \frac{1}{2} \bar{C}_t^2 + b\bar{L}_t - \frac{d}{2} \bar{L}_t^2 \right] \right),$$

$$0 < \bar{\beta} < 1, \quad a, b, d > 0.$$  

The usefulness of the tilde ("\(\sim\)) notation will soon be apparent. Production is given by

$$\bar{Y}_t = r\bar{K}_t + f\bar{I}_t + u_t,$$

$$= \bar{C}_t + \bar{I}_t,$$

$$f, r > 0, \quad \bar{\beta} (1 + r) > 1,$$

where $\bar{K}_t$ denotes capital, $\bar{I}_t$ denotes investment, and $u_t$ is a zero-mean exogenous technology shock. $r$ is the constant marginal product of capital. Finally, the law of motion for the capital stock is

$$\bar{K}_{t+1} = \bar{K}_t + \bar{I}_t.$$  

You can assume that all quantities are intensive. This implies that $r$ implicitly includes population and technology growth, as well as depreciation.

(a) Write down the social planner’s problem associated with this model. Assuming interiority, find the first order conditions that characterize an equilibrium. These are: the Euler equation; the labor allocation condition; and the capital accumulation equation.

(b) Set $u_t = 0$ and consider the associated steady state. Using non-tilde notation to denote deviations from this steady-steady, define

$$\beta = \bar{\beta} (1 + r),$$

$$C_t = \bar{C}_t - a,$$

$$L_t = \bar{L}_t - \frac{b}{d},$$

$$K_t = \bar{K}_t - \frac{1}{r} \left[ a - \frac{fb}{d} \right].$$

Assume that $C_t$, etc. will in fact equal 0 in the steady state (this can be shown). Show that the first order conditions can be written as

$$K_{t+1} = (1 + r) K_t - F C_t + u_t, \quad F > 1,$$

$$C_t = \beta E_t (C_{t+1}).$$

(c) Derive the economy’s expected present value budget constraint (EPVBC), then express consumption as a function of permanent income. To simplify your algebra, define $R = 1 + r$. Be sure to list all of the assumptions behind your derivation.
(d) Note that the output gap, $Y_t$, is given by

$$Y_t = rK_t + fL_t + u_t.$$  \hspace{1cm} \text{(PRF2)}

Holding $K_t$ at its steady state value of 0, consider how $Y_t$ responds to $u_t$.

1. Modify equation (PRF2) to express $Y_t$ as a function of $K_t$, $u_t$, and permanent income.

2. Consider the response of $Y_t$ to an increase in $u_t$. It is not essential, but you can assume that $u_t$ follows a stationary ARMA process, with i.i.d. innovations. Is the coefficient on this response bigger or smaller than 1? Explain. Does this result make sense to you? Explain.