Section 1. (Suggested Time: 45 Minutes) For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.

1. Monetary neutrality implies that monetary policy has no effect on output.

2. If recessions were an efficient response to changes in productivity, we should observe an increase in beach ball sales (i.e. leisure activities) during downturns in the economy.

3. Permanent decreases in income taxes are better at stimulating consumption and labor supply than temporary decreases.

4. Procyclical average productivity of labor means that business cycles cannot be caused by demand shocks.

5. The labor supply of high-wealth workers is more elastic than the labor supply of low-wealth workers.

6. An increase in government spending reduces output.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Sidrauski model with seigniorage

**Time**: Discrete, infinite horizon

**Demography**: A continuum, mass normalized to 1, of (representative) infinite lived consumer/worker households. There is a large number, mass $N$, of firms owned jointly and equally by the households.

**Preferences**: The instantaneous household utility function over consumption, $c_t$, and real money balances, $m_t$, is $u(c_t, m_t)$. The function $u(\cdot, \cdot)$ is twice differentiable, strictly increasing in both arguments and strictly concave. The discount factor is $\beta \in (0, 1)$.

**Technology**: Aggregate output, $Y_t = F(K_t, L_t)$ where $K_t$ is the aggregate capital stock and $L_t = 1$ is the aggregate labor supply. The function $F(\cdot, \cdot)$ is twice differentiable, strictly increasing in both arguments, concave and exhibits constant returns to scale. It will be helpful to use $f(k_t)$ as output per worker where $k_t$ is the capital stock per worker. Capital depreciates by the fraction $\delta$ for each period it is in use.

**Endowments**: Each household has one unit of labor and an initial endowment of capital $k_0$. Each also has an initial nominal money holding $H_0$.

**Information**: Complete, perfect foresight.

**Institutions**: Competitive markets in each period for capital, the consumption good, labor and money. There is a government that has to meet the exogenous sequence of per capita expenditures, $g_t$. To do so, it issues new money and buys output from the market. Its policy instrument is the nominal money growth rate, $\sigma$, such that $H_t = (1 + \sigma)H_{t-1}$.

(a) Using $M_{t+1}$ as period $t$ nominal money demand, $P_t$ as the period $t$ price of the consumption good, $r_t$ as the rental rate on capital and $w_t$ as the wage paid per effective unit of labor, write down and solve the household’s problem.

(b) Solve the problem faced by the firms and, write down the market clearing conditions and the government budget constraint.

(c) Define a monetary equilibrium and solve for the equations that characterize the equilibrium.

(d) Consider now $g_t = g$ for all $t$. In steady state, for a given value of $\sigma$, what are the maximum sustainable values of $g$ and $m^*$ (steady state real money balances). To what extent do these values depend on $\sigma$? Explain your answer.
8. Consider an economy with a single representative consumer. The consumer’s preferences over consumption and labor (leisure) are:

\[ E_t \left( \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1-\sigma}c_{t+j}^{1-\sigma} - \theta_t \ell_{t+j} \right] \right), \]

\[ 0 < \beta < 1, \quad \sigma > 0. \]

The preference shifter \( \theta_t \) is a non-negative exogenous random variable governed by a Markov process with the stationary transition density \( f(\theta', \theta) \).

The sole source of the single non-storable good is a representative farm that produces the good using labor and an everlasting tree. The farm’s objective is to maximize each period’s profits:

\[ \max_{\ell_t \geq 0} \pi_t = y_t - w_t \ell_t, \]

\[ y_t = \ell_t^{1-\alpha}, \quad 0 < \alpha < 1, \]

where: \( \pi_t \) is profit; \( y_t \) is the tree’s “fruit” or output; \( \ell_t \) is the labor input; and \( w_t \) is the real wage.

Consumers receive labor income and profits from farms. Let \( q(\theta', \theta_t) \) be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when \( \theta_{t+1} = \theta' \). At the beginning of time 0, the consumer owns contingent claims equivalent to the ownership of 1 farm. The consumer allocates her financial resources between these contingent claims and consumption.

(a) Write down the consumer’s problem in recursive form and find the first order conditions for an interior solution.

(b) Solving the social planner’s problem, find the equilibrium expressions for labor, \( \ell_t = \ell(\theta_t) \), and consumption, \( c_t = c(\theta_t) \). Using this result, find the equilibrium price for contingent claims.

(c) Ignoring any terms related to the transition density \( f(\theta', \theta) \), explain intuitively how \( q(\theta', \theta_t) \) depends on the current and future values of the preference shifter \( \theta_t \), and how that dependence in turn depends on \( \sigma \) and \( \alpha \).

(d) Let \( R_t^{-1} = R^{-1}(\theta_t) \) be the price of a risk-free discount bond that pays one unit of consumption at time \( t + 1 \). Use your answer to part (b) to find \( R_t^{-1}(\theta_t) \).
9. One-sided search with recall

**Time:** Discrete, infinite horizon.

**Demography:** Single worker who lives for ever.

**Preferences:** The worker is risk-neutral (i.e. \( u(x) = x \)). He discounts the future at the rate \( r \).

**Endowments:** When unemployed, the worker receives income \( b > 0 \) per period. Also with probability \( \alpha \) he gets an offer of employment at a wage \( w \sim F(w) \). The distribution function \( F \) has support on \([0, \bar{w}]\) where \( \bar{w} > b \).

When the worker accepts a wage, \( w \), he becomes employed and earns \( w \) until he is laid-off, which occurs with probability \( \lambda \).

The only change from the standard one-sided model is that the unemployed worker can now recall any offer received during the current spell of unemployment. Clearly, he will only potentially recall the highest offer to date. Call this \( \hat{w} \). (If no offer has been received then \( \hat{w} = 0 \).)

(a) Write down the Bellman-type asset equations for each of the states the worker can be in and briefly explain each one. (**Hint:** The value to unemployment can now potentially depend on \( \hat{w} \).)

(b) Obtain the flow value equations from the Bellman equations.

(c) Define (algebraically) the reservation wage \( w^*(\hat{w}) \) that a worker with current highest offer \( \hat{w} \) will accept.

(d) Solve for the appropriate reservation wage equation.

(e) Comment on the value of permitting the worker to recall offers.

10. **Time-saving public goods.** Consider the following version of a stochastic growth model. There are a fixed number of price-taking producers that solve

\[
\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t, \\
Y_t = L_t^{1-\alpha}, \quad 0 < \alpha < 1,
\]

where: \( \Pi_t \) is profit; \( Y_t \) is output \( L_t \) is labor; and \( W_t \) is the real wage. The population and number of firms are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption, \( C_t \), government spending, \( G_t \), and labor are given by

\[
E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) - \chi \frac{1}{1+\gamma} (L_t - \sigma G_t)^{1+\gamma} \right] \right),
\]

\[
0 < \beta < 1, \quad \gamma \geq 0, \quad \chi > 0, \quad \sigma > 0.
\]

Households receive labor income and profits from firms. They pay lump-sum taxes, \( T_t \), to the government. Households earn a gross return of \((1 + r)\) on their assets, \( K_t \),
with $\beta (1 + r) = 1$. As usual, assume that assets held at the beginning of period $t + 1$, $K_{t+1}$, are chosen in period $t$. Note that capital is used only as a storage device, and not as a factor of production. Households face the usual initial, non-negativity and No-Ponzi-Game conditions.

The government’s flow budget constraint is given by

$$
T_t = G_t,
$$

where $T_t$ is the total value of lump-sum taxes. Lump-sum taxes are driven by government spending, which follows an AR(1) process around the log of its steady state value: $\sigma$

$$
\hat{y}_t \equiv \ln \left( \frac{G_t}{G_{ss}} \right) = \phi \hat{y}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1,
$$

(TS)

where $\{\varepsilon_t\}$ is an exogenous i.i.d. process, and $G_{ss}$ is steady state government spending. You can also assume that $G_t < Y_t$ and $\sigma G_t < L_t$, $\forall t$.

(a) Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.

(b) Let lower-case letters with carats “$^\wedge$” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$
\hat{\ell}_t = \theta_1 \hat{g}_t - \theta_2 \hat{c}_t, \quad \theta_1 \geq 0, \theta_2 > 0,
$$

$$
\hat{y}_t = \lambda_1 \hat{g}_t - \lambda_2 \hat{c}_t.
$$

(c) Suppose that the steady state consumption-to-capital ratio, $C_{ss}/K_{ss}$, is $\psi$, and that the steady state government spending-to-capital ratio, $G_{ss}/K_{ss}$, is $\zeta$, with $(\psi + \zeta) > r$. It is then straightforward to show that the steady state output-to-capital ratio, $Y_{ss}/K_{ss}$, is $\psi + \zeta - r$. (Take this as given). Using this result, log-linearize the capital accumulation equation to show that

$$
\hat{k}_{t+1} = (1 + r) \hat{k}_t + \omega_1 \hat{g}_t - \omega_2 \hat{c}_t.
$$

(CA)

(d) What are the signs of $\omega_1$ and $\omega_2$? What happens to these signs as $\gamma \to 0$? Provide a brief intuitive explanation. (Hint: You should look at the sign of $\theta_1$.)