**Instructions**: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

**Section 1.** (Suggested Time: 1 Hour) *For 3 of the following 6 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations typically appeal to specific macroeconomic models.*

1. Persistent changes in productivity generate larger changes in consumption and labor than temporary ones.

2. Oil price increases cause inflation.

3. Government policies aimed at increasing saving have little effect on the economy’s long-run growth rate.

4. An economic downturn (recession) should lead to lower real interest rates.

5. An increase in government spending cannot stimulate the economy, as it crowds out private expenditures one for one.

6. If agents have rational expectations, changes in the nominal money supply have no real effects.
Section 2. (Suggested Time: 2 Hours) Answer any 3 of the following 4 questions.

7. (Inspired by Den Haan and Kaltenbrunner, 2004.) Consider the following simplified version of a stochastic growth model. There are a fixed number of price-taking producers that solve

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = Z_t L_t^{\eta}, \quad 0 < \eta < 1,$$

(PRF)

where: $Z_t$ is exogenous total factor productivity; $\Pi_t$ is profit; $L_t$ is labor; $W_t$ is the real wage; and $Y_t$ is output. The population, number of firms and steady state technology level are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption ($C_t$) and labor ($L_t$) are given by:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) - \frac{\nu}{1+\chi} L_t^{1+\chi} \right] \right),$$

$$0 < \beta < 1, \quad \chi \geq 0, \quad \nu > 0.$$

Households receive labor income and profits from firms. Households earn a gross return of $(1 + r) K_t$ on their assets, $K_t$, with $\beta (1 + r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, $K_{t+1}$, are chosen in period $t$. Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households also face the usual non-negativity and No-Ponzi-Game conditions.

The log of the productivity parameter $Z_t$ follows an AR(1) process

$$\hat{z}_t \equiv \ln \left( \frac{Z_t}{Z} \right) = \phi \hat{z}_{t-1} + \epsilon_t, \quad Z > 0, \quad 0 \leq \phi < 1,$$

(TS)

where $\{\epsilon_t\}$ is an exogenous stationary martingale difference sequence.

In addition to observing the current technology shock, $Z_t$, at time $t$ agents observe next period’s shock, $Z_{t+1}$.

(a) Write down the social planner’s problem for this economy as a dynamic programming problem. Find the equilibrium conditions for this economy.

(b) Briefly discuss how you might calibrate $\eta$ and $\phi$.

(c) Let lower-case letters with carats “~” denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

$$\hat{\ell}_t = \theta (\hat{z}_t - \hat{c}_t); \quad \hat{y}_t = (1 + \alpha) \hat{z}_t - \alpha \hat{c}_t.$$

(d) Suppose that the steady state consumption-to-capital ratio, $C_{ss}/K_{ss}$, is $\psi$, with $\psi > r$, so that the steady state output-to-capital ratio, $Y_{ss}/K_{ss}$, is $\psi - r$. Using these results, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1 + r) \hat{k}_t + \omega_1 \hat{z}_t - \omega_2 \hat{c}_t.$$
(e) One can log-linearize the Euler equation and solve the resulting system (which also includes (TS) and (CA')) to express consumption as a function of capital and productivity

\[ \hat{c}_t = \lambda \hat{k}_t + \mu \hat{z}_t + \gamma \hat{z}_{t+1} = \lambda \hat{k}_t + (\mu + \gamma \phi) \hat{z}_t + \gamma \hat{z}_{t+1}, \]  

(CF)

with

\[ \lambda = \frac{r}{\omega_2}; \quad \mu = r \frac{\omega_1}{\omega_1 + r \omega_2}; \quad \gamma = \frac{\mu}{1 - \phi + r}. \]

(Take this as given.)

1. One can show that \( \gamma \) is positive. Does this make sense to you? Briefly explain.

2. Beaudry and Portier (2004) find that an increase in expected future productivity leads to higher labor hours today. Is this model consistent with such a result? Explain.

8. Consider an economy populated by a representative consumer who is endowed with \( y_t \) units of a perishable consumption good in each period. There is a government that taxes the consumer’s endowment at the constant rate \( \tau \in (0, 1) \) in each period and transforms the proceeds into the consumption good \( g_t \) unit for unit, i.e. \( g_t = \tau y_t \) in each period \( t \). The consumer has preferences over private consumption and government consumption given by the utility function

\[ \sum_{t=0}^{\infty} \beta^t \ln (c_t + \alpha g_t) \]

where \( 0 < \alpha < 1 \) and \( 0 < \beta < 1 \). There is a market in one-period bonds; a bond purchased in period-\( t \) at price \( p_t \) pays off one unit of the consumption good in period-\( (t + 1) \). Let the quantity of bonds held by the consumer at the beginning of period-\( (t + 1) \) be denoted \( b_{t+1} \). The gross return on the asset purchased in period \( t \) is defined as \( R_t = 1/p_t \). Note that the government is assumed to balance its budget in each period.

(a) Write down the agent’s utility maximization problem in recursive form. That is, write down the Bellman equation and any constraints that the consumer’s optimal consumption problem must satisfy in a competitive equilibrium.

(b) Derive the Euler equation that the consumer’s optimal consumption sequence must satisfy.

(c) Suppose that the endowment is constant across time: \( y_t = y \) for all \( t \). Derive an expression for the period-\( t \) equilibrium gross return on the asset. What is the time-series pattern of returns? Explain your results.

(d) Suppose that in period-\( t \) the government announces a once-and-for-all increase in the tax rate to take effect in the following period. That is, \( \tau_t = \tau \) for \( t = 0, 1, \ldots t \), and \( \tau_t = \tau^* \) for \( t = t + 1, t + 2, \ldots, \tau^* \). Compare the time-series pattern of gross returns with that derived in part (c) and explain the economic intuition behind any differences. (Problem continues on next page.)
(e) Consider an economy identical to the set forth above, with the single exception that it has a higher value for the preference parameter $\alpha$. Redo your analysis of part (c) and (d) and explain the economic intuition behind any differences.

9. Consider an economy with many identical agents, each with preferences given by the utility function
\[ \sum_{t=0}^{\infty} \beta^t \ln c_t \]
where $0 < \beta < 1$ and $c_t$ is period-$t$ consumption. Each consumer is endowed with one unit of time per period and the same initial amount of capital $k_0$. In each period $t$, consumers sell off $n_t$ units of their time as labor services to a representative firm, receiving the real wage $w_t$. They also rent capital $k_t$ to the firm, receiving the real rental rate $r_t$. Consumers receive the profits generated by the firm and have the ability to transform current output into future capital unit-for-unit. The depreciation rate on capital is 100% per period. The period-$t$ budget constraint of consumers is therefore
\[ c_t + k_{t+1} = w_t n_t + r_t k_t + \pi_t \]
where $\pi_t$ is a consumer’s share of period-$t$ profits. The representative firm has access to a technology represented by the production function
\[ y_t = A k_t^\alpha n_t^{1-\alpha} K_t^\theta \]
where $K_t$ denotes average capital holdings across all agents in the economy. Note the following parametric restrictions: $A > 0$, $\theta > 0$, $0 < \alpha < 1$, and $\alpha + \theta = 1$.

(a) Devise a centralized planning problem that, when solved, will yield the competitive equilibrium paths for labor effort, consumption, and capital accumulation. Write this problem in recursive form, identifying the period-$t$ state variables and choice variables.

(b) Assuming that the value function is differentiable, derive the corresponding first-order condition, envelope condition, and Euler equation.

(c) Characterize the competitive equilibrium balanced growth path of capital and consumption.

(d) Determine the Pareto optimal growth rate. Is the competitive equilibrium growth rate faster or slower than the optimal growth rate?

(e) Does your result in part (d) help you to understand the existence of investment tax credits and research and development subsidies? Explain.
10. Consider an economy with a single representative consumer. The consumer’s preferences over consumption and leisure are:

\[
Et \left( \sum_{j=0}^{\infty} \beta^j \frac{1}{1-\gamma} \ell_{t+j}^{1-\gamma} (1-\ell_{t+j}) \right), \quad 0 < \beta < 1, \quad 0 \leq \gamma < 1.
\]

The sole source of the single non-storable good is a representative farm that produces the good using labor and an everlasting tree. The farm’s objective is to maximize each period’s profits:

\[
\max_{t \geq 0} \pi_t = y_t - w_t \ell_t,
\]

\[
y_t = c_t = d_t \ell_t^{1-\alpha}, \quad 0 < \alpha < 1,
\]

where: \( \pi_t \) is profit; \( y_t \) is the tree’s “fruit” or output; \( \ell_t \) is the labor input; and \( w_t \) is the real wage. Output depends on the exogenous shifter \( d_t \), where \( d_t \) is a non-negative random variable governed by a Markov process with the stationary transition density \( f(d', d) \).

Consumers receive labor income and profits from farms. At the beginning of time 0, each consumer owns one farm. Let \( p_t \) be the price at time \( t \) of a title to all future profits from a farm, and let \( q(d', d_t) \) be the kernel used to price the one-step-ahead contingent claim that delivers one unit of the consumption good when \( d_{t+1} = d' \).

(a) Write down the consumer’s problem in recursive form and find the first order conditions.

(b) Continuing...

1. Solve the farm’s problem and find the equilibrium wage.
2. Imposing clearing in the markets for goods and labor, find the equilibrium labor allocation, \( \ell_t = \ell \).
3. Find equilibrium output and profits, \( y_t = y(d_t) \) and \( \pi_t = \pi(d_t) \).

(c) Derive the equilibrium pricing function for stocks, \( p_t = p(d_t) \). (This function should not include any expected future prices.)

(d) Derive the equilibrium kernel used to price one-step-ahead contingent claims, \( q(d_{t+1}, d_t) \). Let \( R_t^{-1} = R^{-1}(d_t) \) be the time-t price of a risk-free discount bond that pays one unit of consumption at time \( t+1 \) under any state. Use the pricing kernel to find \( R_t^{-1}(d_t) \).