Instructions: Read the questions carefully and make sure to show your work. You have three hours to complete this examination. Good luck!

Section 1. (Suggested Time: 45 Minutes) For 3 of the following 5 statements, state whether the statement is true, false, or uncertain, and give a complete and convincing explanation of your answer. Note: Such explanations often appeal to specific macroeconomic models.

1. In the real business cycle model, government purchases of goods and services always reduce welfare.

2. If the data showed that most consumers were not borrowing-constrained, we should conclude that borrowing constraints have few real-world effects.

3. Information imperfections in credit markets implies a valid role for governments to act as financial intermediaries.

4. A procyclical money supply is evidence that money is not neutral.

5. A decrease in the real interest rate signals an increase in future economic activity.

6. Decades of stagnant economic growth in sub-Saharan Africa provides direct evidence against the neoclassical growth model.
Section 2. (Suggested Time: 2 Hours, 15 minutes) Answer any 3 of the following 4 questions.

7. Consider the following economy dated in discrete time by the index $t = 1, 2, \ldots, T$. The economy is populated by a single representative agent endowed with a stream of a single, physically distinct consumption good $\{y_t\}_{t=1}^T$. The agent has preferences over date-contingent consumption represented by the utility function

$$\sum_{t=1}^{T} \beta^{t-1} u(c_{t-1}, c_t), \quad 0 < \beta < 1,$$

where $c_t$ is the period-$t$ consumption of agent $j$. Let $c_0 = 0$. The period-utility function $u$ is twice continuously differentiable with the following derivative properties: $u_1 > 0$, $u_2 > 0$, and $u_{12} = u_{21} > 0$.

(a) Letting $q_t$ denote the price of period-$t$ consumption, define the Arrow-Debreu competitive equilibrium for this economy. Is the competitive equilibrium allocation Pareto optimal? Explain.

(b) Derive an expression for the Arrow-Debreu competitive equilibrium prices $\hat{q}_t$, $t = 2, \ldots, T$ and the corresponding real interest rates $r_t$, $t = 1, \ldots, T$ in the equivalent sequence-of-markets competitive equilibrium.

(c) Suppose that the aggregate endowment increases in some period $t = s > 2$, and that this increase is known before the beginning of trading in period-1. Characterize as completely as you can, the effect on the period-$t$ Arrow-Debreu price $\hat{q}_t$ and corresponding real interest rate $r_t$.

(d) Consider the same experiment as in part (c), except with the assumption that $u_{21} = 0$.

(e) Compare your results in part (c) and (d) and explain the intuition behind the differences.
8. Consider the following variant of the Lucas tree model. There are two types of trees: wet-weather trees, which will be indexed by \(W\); and arid- (dry) weather trees, indexed by \(A\). Each tree gives off \(d_i\), \(i \in \{W, A\}\), units of non-storable fruit (dividends). The amount of fruit produced by each type of tree depends on the amount of moisture (rainfall), \(m_t\), with

\[
\begin{align*}
    d^W_t &= d + m_t, \\
    d^A_t &= d - m_t.
\end{align*}
\]

As their names suggest, wet-weather trees yield better in wet years, and arid-weather trees yield better in dry years. The amount of moisture, \(m_t\), is a zero-mean exogenous random variable with support \((-d, d)\). \(m_t\) is governed by a Markov process with the stationary one-step transition density \(f(m', m)\). This density is also symmetric, in that \(f(m', m) = f(-m', -m)\).

The preferences of the representative consumer are

\[
E_0 \left( \sum_{i=0}^{\infty} \beta^i \frac{1}{1 - \sigma} \left[ c^i_{-\sigma} - 1 \right] \right), \quad 0 < \beta < 1, \quad \sigma > 0.
\]

where \(c_t\) denotes total consumption of wet- and arid-weather fruit; agents are indifferent between the two. The economy starts off with half of the agents owning one wet-weather tree apiece, and the other half owning one arid-weather tree. In addition, initial dividends just happen to be equivalent: \(m_0 = 0\).

Let \(p_t^i\) be the price at time \(t\) of a title to all future dividends from a tree of type \(i\). Let \(R_t^{-1}\) be the time-\(t\) price of a risk-free discount bond that pays one unit of consumption at time \(t+1\) under any state. Finally, let \(x_t\) denote the consumer’s Chancial resources, which she allocates between bonds, stocks and consumption.

(a) Write down the consumer’s problem in recursive form and find the first order conditions.

(b) Solve the social planner’s problem for this economy. You can assume that the social planner cares equally about each consumer. What is the optimal allocation of consumption? What distribution of trees would achieve this allocation in a competitive economy?

(c) Derive the equilibrium pricing function for wet-weather stocks, \(p_t^W = p^W(m_t)\). (This function should not include any expected future prices.) Be sure to point out any restrictions that you impose in deriving this function.

(d) Find the equilibrium pricing function for bonds, \(R_t^{-1} = R^{-1}(m_t)\). Next, define the return on wet-weather stocks, \(R_t^W\), by

\[
R_t^W = \frac{p_{t+1}^W + d_{t+1}^W}{p_t^W},
\]

and define the equity premium, \(e_t^W\), by \(e_t^W = E_t (R_t^W) - R_t\). Find \(e_t^W\). (Hint: You may skip the calculations if you can provide a convincing economic argument.)
9. Consider an economy with many identical agents each with preferences given by the utility function
\[
\sum_{t=0}^{\infty} \beta^t \ln c_t
\]
where \(0 < \beta < 1\) and \(c_t\) is period-\(t\) consumption. Each consumer is endowed with one unit of time per period and the same initial amount of capital \(k_0\). In each period \(t\), consumers sell off \(n_t\) units of their time as labor services to a representative firm, receiving the real wage \(w_t\). They also rent capital \(k_t\) to the firm, receiving real rental rate \(r_t\). Consumers receive the profits generated by the firm and have the ability to transform current output into future capital unit-for-unit. The depreciation rate on capital is 100\% per period. The period-\(t\) budget constraint of consumers is therefore
\[
c + k_{t+1} = w_t n_t + r_t k_t + \pi_t
\]
where \(\pi_t\) is a consumer’s share of period-\(t\) profits. The representative firm has access to a technology represented by the production function
\[
y_t = Ak_t^\alpha n_t^{1-\alpha} K_t^\theta
\]
where \(A > 0, \theta > 0, 0 < \alpha < 1\), and \(K_t\) denotes average capital holdings across all agents in the economy.

(a) Explain the intuition behind the production technology in this model.
(b) Define a competitive equilibrium in this economy and derive expressions for the equilibrium real wage and real rental rate in each period \(t\).
(c) Devise a centralized planning problem that, when solved, will yield the competitive equilibrium paths for labor effort, consumption, and capital accumulation. Write this problem in recursive form and identify the period-\(t\) state variables and choice variables.
(d) Assuming that the value function is differentiable, derive the corresponding first-order condition, envelope condition, and Euler equation.
(e) Assuming that \(\alpha + \theta \geq 1\), characterize the competitive equilibrium growth path of capital and consumption as completely as you can.
(f) Is the competitive equilibrium growth rate faster or slower than the optimal growth rate? Explain the intuition behind your answer.
10. Consider the following simplified version of a stochastic growth model. There are a fixed number of price-taking producers that solve

\[ \max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t, \]

\[ Y_t = L_t^\eta, \quad 0 < \eta < 1, \]  

(PRF)

where: \( \Pi_t \) is profit; \( L_t \) is labor; \( W_t \) is the real wage; and \( Y_t \) is output. There is no population or technology growth, and the population, number of firms and steady state technology level are normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

The preferences of the representative household over consumption \( (C_t) \) and labor \( (L_t) \) are given by:

\[ E_0 \left( \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) - \frac{\Gamma}{1 + \chi} L_t^{1+\chi} \right] \right), \]

\[ 0 < \beta < 1, \quad \chi \geq 0, \quad \Gamma > 0. \]

Households receive labor income and profits from firms. Households earn a gross return of \( R_t K_t \) on their assets, \( K_t \). As usual, assume that assets held at the beginning of period \( t + 1 \), \( K_{t+1} \), are chosen in period \( t \). Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption and investment in assets. Households also face the usual non-negativity and No-Ponzi-Game conditions.

Finally, the gross rate of return on assets, \( R_t \), follows an AR(1) process:

\[ \hat{r}_t \equiv \ln \left( \frac{R_t}{1 + r} \right) = \phi \hat{r}_{t-1} + \epsilon_t, \quad \beta (1 + r) = 1, \quad 0 \leq \phi < 1, \]

(TS)

where \( \{ \epsilon_t \} \) is an exogenous stationary martingale difference sequence.

(a) Write down the social planner's problem for this economy as a dynamic programming problem. Find the equilibrium conditions for this economy, namely: the labor allocation condition; the Euler equation; and the capital accumulation equation.

(b) Let lower-case letters with carats "\( \hat{\} \)" denote deviations of logged variables around their steady state values. Show that the log-linearized expressions for labor hours and output are:

\[ \hat{\ell}_t = -\theta \hat{\alpha}_t; \quad \hat{y}_t = -\alpha \hat{\alpha}_t. \]

Using a similar approach, show that the log-linearized Euler equation can be written as:

\[ E_t (\hat{\alpha}_{t+1} - \hat{\alpha}_{t+1}) = \hat{\alpha}_t. \]

(EE')

Hint: at low levels of variance,

\[ \ln (E_t (Z_t)) \approx E_t (\ln (Z_t)). \]
(c) Suppose that the steady state consumption-to-capital ratio, $C_{ss}/K_{ss}$, is $\psi$, with $\psi > r$. It is then straightforward to show that the steady state output-to-capital ratio, $Y_{ss}/K_{ss}$, is $\psi - r$. Using these results, log-linearize the capital accumulation equation to show that

$$\hat{k}_{t+1} = (1 + r)\hat{k}_t + \omega_1 \hat{r}_t - \omega_2 \hat{c}_t.$$  \hspace{1cm} (CA')

(d) One can solve the system given by equations (TS), (CA') and (EE') to express consumption as a function of capital and the rate of return:

$$\hat{c}_t = \lambda \hat{k}_t + \mu \hat{r}_t,$$  \hspace{1cm} (CF)

with

$$\lambda = \frac{r}{\omega_2} \in (0, 1); \quad \mu = \frac{1}{1 - \phi + r} \left( \frac{\omega_1}{\omega_2} - \phi \right).$$

(Take this as given.)

1. Consider the coefficient $\mu$, which gives the response of consumption to a rate of return shock. When $\phi = 0$, so that return shocks are completely transitory, will consumption be increasing or decreasing in $\hat{r}_t$? Interpret. As $\phi$ increases, what ultimately happens to $\mu$? (You can assume that $\omega_2 > r\omega_1$, which will be the case for almost all plausible parameter values.) Interpret.

2. Next, combine equation (CF) with your answer to part (b) to express $\hat{\ell}_t$ as a function of $\hat{r}_t$. Are there parameter values where $\hat{\ell}_t$ is increasing in $\hat{r}_t$? Provide an economic interpretation.