Duration Dependent UI Payments and Equilibrium Unemployment

Melvyn Coles
University of Essex

Adrian Masters*
University of Essex

November 2001

Abstract

This paper develops a dynamically consistent model of equilibrium unemployment when unemployment insurance (UI) payments are duration dependent. Assuming that the government does not observe job offers, there is a moral hazard problem as the option of receiving further UI payments while unemployed distorts job seeker reservation wages. Extending the duration of UI payments, while holding the generosity of the scheme constant, increases the distortion on reservation wages at all durations. Simulations suggest that the aggregate distortion by moving from a 6 month UI scheme to a one-year scheme is relatively small. However, an indefinite payment scheme has a large impact on equilibrium unemployment.

Key Words: Equilibrium Unemployment, Matching, Bargaining

JEL Classification: J64, J65, J41

*Corresponding author: Department of Economics University of Essex, Wivenhoe Park, Colchester, CO4 3SQ England. amaste@essex.ac.uk

1 Introduction

This paper develops a dynamically consistent model of equilibrium unemployment when unemployment insurance [UI] payments are duration dependent. Assuming that the government does not observe job offers, there is a moral hazard problem as the option of receiving further UI payments while unemployed distorts job seeker reservation wages. Although such reservation wage distortions are well known in the sequential search literature [e.g. Mortensen (1977), van den Berg (1990)], it has not been possible to embed that structure with duration dependent UI payments into an equilibrium framework. By instead considering an ex-post wage bargaining model, this paper not only captures the impact of changing the duration of UI payments on worker reservation wages, it is the first to compute its impact on equilibrium unemployment. The finding is that for natural parameter values, extending the duration of UI payments while holding the generosity of the scheme constant, increases the distortion on reservation wages at all durations. Simulations suggest that the aggregate distortion by moving from a 6 month scheme to a one-year scheme is relatively small. However, an indefinite payment scheme has a large impact on equilibrium unemployment levels. Such a result appears consistent with the empirical findings of Nickell (1997).

The insights offered are a significant advance on previous work on this issue. The usual argument for reducing the duration of UI payments is that the government needs to encourage greater search effort by individual job seekers. But Atkinson and Mickelwright (1991) argue that such effects are not large in the U.K. and cannot explain the large rise in U.K. unemployment since the 1970's. Nevertheless, the cross country evidence [e.g. Nickell (1997)] establishes that differences in UI systems have a large macro-impact on unemployment. This suggests that the central channel by which a UI system distorts a market economy is more likely to be through distorting equilibrium wages than through reducing job search effort. The argument here is not that the government wishes to encourage greater search effort, rather it needs to minimise the wage distortions inherent with a public unemployment insurance program when job offers are unobserved. Indeed, this perspective suggests a potentially useful re-interpretation of the policy impact of the U.K. Job Restart scheme. This scheme perhaps encourages greater search effort by the longer-term unemployed, but it also generates verifiable job offers so that the government can credibly threaten to withdraw further benefits should a job seeker reject a suitable employment offer.

The moral hazard problem when the government does not observe job offers is that a job seeker, on receiving an employment offer, has the option of rejecting that offer and continuing to receive further UI payments. This option increases the job seeker's reservation wage [relative to the case of no UI coverage] and so raises equilibrium wages. Embedding this wage distortion into a standard matching framework [e.g. Diamond (1981), Pissarides (1990)], this paper identifies how the duration of UI payments affects equilibrium unemployment. Simulations find that moving from a 6 month UI scheme to an indefinite payment scheme, while reducing the level of payments to keep generosity constant, can result in a 35% increase in equilibrium unemployment, corresponding to an overall increase of around 2 percentage points. The reason for such a large employment effect is best understood using the notion of 'credible threats'. In equilibrium, the negotiated wage reflects the value of the worker's option of remaining unemployed. Although the worker does not expect to remain unemployed indefinitely, the option or 'threat' of receiving UI payments for an indefinite period significantly affects the worker's reservation wage. The resulting wage distortion then has a significant impact on equilibrium unemployment.

There are two closely related literatures. The classic search approach assumes firms make take-it-or-leave-it wage offers and workers search sequentially for job offers. Taking the wage offer distribution as given, Mortensen (1977) and van den Berg (1990) describe the job seeker's optimal search strategy given duration dependent UI payments. The sequential search assumption implies a reservation wage strategy is optimal, and higher UI payments imply job seekers have a higher reservation wage. Further, if UI payments decrease with unemployment duration, then a job seeker's reservation wage also falls with duration. Exactly the same effect occurs in our framework as both structures assume the same moral hazard problem - that the government does not observe job offers and so job seekers retain the option of remaining unemployed and continuing to receive UI [until their UI payments expire]. Unfortunately given a duration dependent UI payment scheme and the corresponding non-stationary search strategies of workers, determining the wage offer distribution endogenously [consistent with optimal wage posting by firms and optimal job search by workers] is a formidable problem. Albrecht and Vroman (2001) is the only paper which explicitly considers this problem, and does so by assuming UI payments expire according to an exogenous Poisson process. A more tractable structure is obtained here by instead replacing the take-it-or-leave-it wage offer structure with wage bargaining.

A second literature instead assumes that job search effort is unobserved. Shavell and Weiss (1979) establish that optimal insurance against layoff risk implies UI payments should decrease with duration. In a recent paper, Frederiksson and Holmlund (2001) extend those results using an equilibrium matching model where wages are endogenously determined. However as has been typically done in the literature, they determine wages using the Nash bargaining approach assuming the worker's threatpoint is the value of being unemployed with full UI coverage [also see Blanchard and Diamond (1994), Millard and Mortensen (1997), Davidson and Woodbury (1997)]. This latter assumption has no formal microeconomic justification and seems inconsistent with the fact that workers who quit are not entitled to receive UI. In contrast, here we describe a formal wage bargaining game where, given job offers are not observed, the worker has the option of remaining unemployed and continuing to receive further benefits during wage negotiations. Assuming firms and workers can contract on wages, the negotiated wage then depends on the unemployed worker's entitlement to further UI. Our paper therefore captures within a dynamically consistent framework how a duration dependent UI payment scheme affects equilibrium wages and unemployment.

2 The Model

The aim of the paper is to understand the macroeconomic interaction between a UI program which offers a duration dependent payment scheme and equilibrium wage formation in a non-competitive labour market. As the assumed degree of risk aversion of workers is not central to this issue, we simplify by assuming workers are risk neutral.[1] As we omit the insurance value of UI, we do not claim to identify an 'optimal' UI program, though we shall discuss this issue further in the Conclusion. Here we demonstrate that the wage distortion under consideration can generate large macroeconomic distortions.

The infinite horizon, continuous time economy comprises of a continuum of workers and a continuum of firms, where all are risk neutral and have the same discount rate $r > 0$. The number [measure] of workers is normalized to unity. Although the number of workers in the economy does not change over time, there is turnover. In particular, each worker leaves the market according to a Poisson process with parameter $\delta > 0$. Of course for a steady state, δ also describes the inflow of new unmatched workers.

[1] This also avoids the savings problem, where workers might self-insure using a savings strategy.

Throughout we only consider steady state equilibria. In any such equilibrium, each worker is either employed or unemployed, where U denotes the steady state measure of unemployed workers. Following Pissarides (1990), the steady state measure of vacancies, V, will be determined by a free entry condition.

Because there are matching frictions, it takes time for the U job seekers to contact the V vacancies. We describe that process using an aggregate matching function $M = M(U, V)$ where $M(U, 0) = M(0, V) = 0$, and M is a continuous, increasing and concave function which is homogeneous of degree 1; i.e. there are constant returns to matching. If α_w denotes the rate at which a job seeker contacts a vacancy, and α_f is the arrival rate of job seekers to any vacancy, symmetry implies

$$\alpha_w = M(U, V)/U \quad \text{and} \quad \alpha_f = M(U, V)/V. \tag{1}$$

Of course if we let $\phi = V/U$ [which is referred to as labour market tightness] and define $m(\phi) = M(1, \phi)$, then constant returns to matching implies these arrival rates reduce to $\alpha_w = m(\phi)$ and $\alpha_f = m(\phi)/\phi$.

Let $H(\tau)$ denote the steady-state distribution of unemployment spell lengths τ across the unemployed. Search is random in that given a vacancy has been contacted by an unemployed worker, that worker's duration τ is considered as a random draw from H.

As is standard in continuous time analysis, we introduce the concept of the infinitesimal time period $dt > 0$. The eventual modelling equations will be those which emerge in the limit as dt approaches 0. While unemployed, then, a worker with unemployment duration τ receives benefit payment $b(\tau)dt \geq 0$ in the next instant. An unemployed worker also obtains flow payoff $u \geq 0$ through leisure while unemployed. As workers are risk neutral and there is positive discounting, the worker immediately consumes the benefit payment and so obtains total flow payoff $[u + b(\tau)]dt$ while unemployed.

As described in Pissarides (1990), a firm must pay a fixed cost $a > 0$ per unit time to advertise a vacancy. If the firm does not pay this fixed cost, it is not contacted by job seekers. With free entry, the number of vacancies adjusts so that the expected discounted profit by advertising is zero.

Wages are determined by bargaining. Of course in stationary environments there is a close relationship between the Nash bargaining approach and the Rubinstein model of bargaining [e.g. Binmore (1987), Binmore et al. (1986)]. We reconsider that relationship in detail below. For now, note that given the worker's duration $\tau \geq 0$ and that a gain to trade exists, the bargaining game will imply a [reduced form] equilibrium wage agreement $w^*(\tau)$. Given that agreement, a contract is then written which specifies that wage in all future periods. The corresponding equilibrium payoffs to the firm and worker [given contact and τ] are assumed to be

$$\pi_f(\tau) = \frac{k - w^*(\tau)}{r + \delta} \text{ and } \pi_w(\tau) = \frac{w^*(\tau)}{r + \delta},$$

where $k > 0$ is match productivity.

Note, the above payoff functions implicitly assume no on-the-job search, no quits and no layoff risk - the job is for life [until the worker quits the labour market]. Assuming no on-the-job search is a standard simplifying assumption.[2] Ruling out quits is straightforward if we assume a worker who quits is not entitled to receive UI payments [which is usually the case]. As we shall see, receiving UI payments puts a worker in a stronger bargaining position which raises the negotiated wage. Quitting to search for another [identical] employer with no entitlement to UI payments is a retrograde step which no worker would take in equilibrium.

Assuming no layoff risk is also a simplifying assumption; in essence we consider an

[2]See Burdett and Mortensen (1998) for the complications that might arise when there is on-the-job search.

inflow-outflow model where school-leavers replace retiring workers. Introducing layoff risk, say by assuming firm specific productivity shocks as described in Mortensen and Pissarides (1994), would introduce re-entitlement effects. In particular, a laid-off worker whose UI payments have expired would find that getting re-employed re-entitles him/her to future UI payments [perhaps after a certain qualification period]. In contrast, the value of this re-entitlement effect for a worker who has just been laid off is zero as he/she is already fully entitled to receive UI. Coles and Masters (2001) establish within a stochastic [non-steady-state] matching framework that such re-entitlement effects are employment stabilising over the cycle. However here we focus on steady state, and so restricting attention to an inflow-outflow model of turnover conveniently abstracts from those effects.[3]

2.1 The Strategic Bargaining Game.

Typically one thinks of the Nash bargaining solution as a reduced form approximation for the strategic bargaining approach. Although the relationship between the two frameworks has been clearly described in Binmore (1987), Binmore, Rubinstein and Wolinsky (1986), there is some confusion on the role of Nash bargaining 'threatpoints' in the equilibrium matching literature. In fact as long as bargaining is jointly efficient and while bargaining power remains a free parameter, then typically the choice of 'threatpoint' makes little qualitative difference in a matching equilibrium.

To see this consider the extreme case where firms have all the bargaining power,

[3]If we had assumed risk averse workers, omitting layoff risk would be a more important defect of the model - risk averse workers and layoff risk together imply the UI system increases the value of any match and so is potentially employment enhancing. Burdett and Mortensen (1980) explore this feature of a UI system in detail. But with risk neutral workers, the UI system generates no such effect and so including layoff risk would complicate the framework without producing any useful insight.

$\theta = 1$. This implies that workers can never obtain any surplus from a match. Hence, for **any** chosen threatpoints consistent with privately efficient bargaining [e.g. those consistent with some breakdown rate of the underlying strategic bargaining game], a matching equilibrium and the choice of $\theta = 1$ implies the equilibrium wage outcome is an employment contract whose value to the worker is equal to the value of being unemployed forever [as a worker can never extract any surplus by becoming employed]. This is also the lowest wage which is consistent with individual worker rationality.

The same insight holds when workers have all the bargaining power $\theta = 0$. As firms cannot obtain any surplus through bargaining, then regardless of the chosen threatpoint [consistent with some breakdown rate of the underlying strategic bargaining game], a matching equilibrium would imply a wage equal to the value of output. An intermediate choice of $\theta \in (0, 1)$ therefore parameterises how much [reduced form] surplus workers obtain in a matching equilibrium consistent with individual rationality. The point being that while bargaining power remains a free parameter, the choice of threatpoint - whether it is the value of continued search [an infinite breakdown rate] or the value of bargaining forever [a zero breakdown rate] or some weighted average [a finite, potentially endogenous, breakdown rate] - typically plays little role.[4]

Of course there are exceptions. One is when the focus is on the limiting frictionless case where $r \to 0$ [e.g. Rubinstein and Wolinsky (1984), Gale (1987), Mortensen and Wright (2000)].[5] A second is when payoffs evolve dynamically. For example using

[4]Not surprisingly some specifications are mathematically more convenient than others. For example, when the threatpoints are the value of continued search, Hosios (1990) identifies the Nash bargaining power which implements the socially efficient outcome. More specifically, constant returns to matching implies a wage exists which internalises the relevant externalities, and social optimality requires that the assumed bargaining power and threatpoint implement that price. A different threatpoint would imply a different bargaining power. Of course the algebra is most elegant for the Hosios (1990) case.

[5]In this case, the value functions take arbitrarily large values, while the surplus to any match

a monetary model of search, Coles and Wright (1998) show that equilibrium trading cycles are possible when the terms of trade are determined by [forward looking] strategic bargaining, but cannot exist if a static Nash bargaining rule is imposed. As agents ought to be forward looking in the bargaining problem [as they are in all other aspects of the model], dynamic consistency requires that the terms of trade are determined by strategic bargaining when payoffs evolve dynamically.

The bargaining problem here corresponds to this second case - UI payments change with duration - and so dynamic consistency requires determining the terms of trade using the strategic bargaining approach. However consistent with the above arguments, we leave bargaining power as a free parameter but note that the description of the bargaining game will imply particular strategic 'threatpoints'.

When a job seeker and a firm with an unfilled vacancy meet, assume they both observe the job seeker's current duration of unemployment τ. The agreed wage is determined by strategic bargaining with random alternating offers with period $dt > 0$ between counteroffers. The crucial assumption is that the government does not observe offers of employment, and so the worker continues to receive UI benefits $b(\tau)$ after rejecting an offer.

At the beginning of time interval $[t, t + dt)$, the firm and the worker first decide whether to continue bargaining. If either chooses to separate then the worker's payoff is the expected value of continued search, while the firm's payoff is zero in a free entry equilibrium. If both agree to bargain, Nature with probability $\theta \in [0, 1]$ chooses the firm to make a contract offer w, and with probability $1 - \theta$ chooses the worker to make an offer. If the contract offer is accepted, the match is consummated and the payoffs are $\pi_f = [k - w]/(r + \delta), \pi_w = w/(r + \delta)$. If the contract offered by one party is rejected by the other, there is a one-period delay during which there is no

becomes arbitrarily small as agents become arbitrarily patient. Small modelling differences can potentially have large effects in this case.

production. During this delay, the firm receives a zero flow payoff, while the worker obtains $[u+b(\tau)]dt$. The sequence is then repeated over time interval $[t+dt, t+2dt)$, given the increased unemployment duration $\tau+dt$. Of course equilibrium will reflect the worker's worsening bargaining position - his UI payments may soon expire.

We make the standard tie-breaking assumptions; that an agent accepts a contract offer if indifferent to doing so, and an agent who is indifferent between continued bargaining and separation chooses to continue bargaining. The first tie-break assumption is standard and is necessary for existence of an equilibrium, the second plays no important role.

To simplify matters, we assume throughout that unemployment benefits are non-increasing with duration and that $u + b(0) < k$, which imply that a gain to trade always exists between a firm and an unemployed worker.

As is standard in the matching literature, the worker's outside option - which is to separate and continue search - is never binding on the bargaining equilibrium [identical firms and a positive discount rate imply it is always better to keep bargaining than be unmatched]. Further, positive discounting suggests that the most natural bargaining equilibrium is one where the firm and worker always reach immediate agreement. We refer to such an outcome as an Immediate Trade Equilibrium [ITE].

Claim 1. For any duration $\tau \geq 0$, and in the limit as $dt \to 0$, an ITE exists, is unique and implies equilibrium wage agreement

$$w^*(\tau) = (1-\theta)k + \theta \int_\tau^\infty e^{-(r+\delta)(t-\tau)}(r+\delta)[u+b(t)]dt. \tag{2}$$

Before proving Claim 1, we first show that it in fact corresponds to a particular Nash bargaining decomposition. Define the worker's disagreement payoff $d_w(\tau)$ as

$$d_w(\tau) = \int_\tau^\infty e^{-(r+\delta)(t-\tau)}[u+b(t)]dt,$$

which at duration τ is the worker's value of remaining unemployed forever. Also define 'surplus' $\psi(\tau)$ as

$$\psi(\tau) = k/(r+\delta) - d_w(\tau) \equiv \int_\tau^\infty e^{-(r+\delta)(t-\tau)}[k - u - b(t)]dt, \tag{3}$$

where the assumption $u + b(t) < k$ for all t guarantees $\psi(\tau) > 0$ for all τ. The above wage equation is in fact equivalent to

$$w^*(\tau)/(r+\delta) = d_w(\tau) + (1-\theta)\psi(\tau); \tag{4}$$

i.e. the worker negotiates a contract with value equal to his disagreement payoff $d_w(\tau)$ - the value of remaining unemployed forever - plus a share of the surplus $\psi(\tau) > 0$. Also note that as UI payments fall with duration, then both the worker's disagreement payoff d_w and the equilibrium wage agreement w^* fall with duration.

The firm's corresponding payoff $\pi_f(\tau) = [k - w^*(\tau)]/(r+\delta)$ is

$$\pi_f(\tau) = \theta\psi(\tau);$$

i.e. the firm obtains a constant share of the surplus. As previously explained, note that $\theta = 1$ implies the worker negotiates the value of being unemployed forever, while $\theta = 0$ implies the wage equals the value of output [the firm makes zero profit]. Hence $\theta \in [0,1]$ parameterises how much [reduced form] surplus workers obtain [consistent with individual rationality] while the strategic bargaining approach implies these prices are dynamically consistent - both agents prefer immediate agreement at wage $w^*(\tau)$ than defer trade.

2.2 The derivation of Claim 1.

We quickly sketch the relevant argument and refer the reader to Coles and Wright (1998) for a more formal approach. Fix $dt > 0$ and let $w^*(t)$ denote the expected wage agreement at duration t in an ITE. Now consider any duration $\tau \geq 0$ and suppose Nature chooses the firm to make a wage offer, w. By accepting that offer, the worker obtains expected payoff $w/(r+\delta)$. However, should the worker reject that offer, the worker will receive UI payment $b(\tau)$ for the subsequent period [the government does not observe job offers] and, in an ITE, expects agreement next period at expected wage $w^*(\tau+dt)$. The worker's expected payoff by rejecting the job offer is therefore

$$\frac{1-\delta dt}{1+rdt}\left[[u+b(\tau)]dt+\frac{w^*(\tau+dt)}{r+\delta}\right].$$

The worker's optimal bargaining strategy has the *reservation wage property:* the worker rejects any wage offer which has less value than the continuation value of bargaining. That reservation wage, denoted $R_w(\tau)$, is therefore

$$\frac{R_w(\tau)}{r+\delta}=\frac{1-\delta dt}{1+rdt}\left[[u+b(\tau)]dt+\frac{w^*(\tau+dt)}{r+\delta}\right],$$

and the worker is just indifferent to accepting wage offer $w=R_w$. Most importantly, note that given $w^*(\tau+dt)$, the worker's reservation wage at duration τ is strictly increasing in the UI payment $b(\tau)$.

Of course the same argument applies should Nature choose the worker to make a wage offer. The firm believes that if it rejects that offer, then they will reach agreement next period at expected wage $w^*(\tau+dt)$. The firm's reservation wage, denoted $R_f(\tau)$, is therefore

$$\frac{k-R_f(\tau)}{r+\delta}=\frac{1-\delta dt}{1+rdt}\left[\frac{k-w^*(\tau+dt)}{r+\delta}\right].$$

A nice insight is that the difference between these two reservation wages is the surplus that is lost should a one-period delay occur. To see this, add up the above two equations and re-arrange to obtain

$$\frac{R_f(\tau)-R_w(\tau)}{r+\delta}=\frac{1-\delta dt}{1+rdt}\left[k-u-b(\tau)\right]dt, \tag{5}$$

where $u+b(\tau)<k$ guarantees that the right hand side of (5) is strictly positive. As the reservation wage of the firm is strictly greater than the reservation wage of the worker, equilibrium always results in immediate agreement. If the firm makes the wage offer, it extracts that one-period surplus by offering the worker's reservation wage, which the worker accepts. Similarly if the worker makes the offer, he extracts that surplus by offering the firm's reservation wage, which the firm accepts.

Hence the expected wage agreement at duration τ is

$$w^*(\tau)=\theta R_w(\tau)+(1-\theta)R_f(\tau).$$

Using the above to substitute out $R_w(\tau), R_f(\tau)$ and rearranging implies:

$$w^*(\tau)-\frac{1-\delta dt}{1+rdt}w^*(\tau+dt)=\frac{1-\delta dt}{1+rdt}(r+\delta)dt\left[\theta[u+b(\tau)]+\frac{(1-\theta)k}{1-\delta dt}\right] \quad (6)$$

This recursive equation describes today's expected wage agreement given the player's expectations on the expected wage agreement tomorrow should either turn down the other's offer. Solving this [unstable] forward looking difference equation, noting that an ITE requires a bounded solution $w^*(t)\in[0,k]$ for all t, implies the unique solution

$$w^*(\tau)=(1-\theta)k+\theta\sum_{i=0}^{\infty}\left[\frac{1-\delta dt}{1+rdt}\right]^{i+1}(r+\delta)[u+b(\tau+idt)]dt. \quad (7)$$

Letting $dt\to 0$,equation (5) implies $R_f(\tau), R_w(\tau)\to w^*$, and so w^* describes the actual wage agreement in this limit. (7) then implies the Claim.

Note that as $dt\to 0$, then $w^*(t)-R_w(t)$ becomes arbitrarily small. (6) therefore describes the worker's reservation wage in the bargaining game as dt becomes small. It is worth contrasting this reservation wage outcome with the sequential search approach where instead firms post wages and F describes the distribution of wage offers. The usual recursive arguments [see van den Berg (1990) for example] imply the reservation wage of a worker at duration τ, denoted $R(\tau)$, satisfies the difference equation:

$$R(\tau)-\frac{1-\delta dt}{1+rdt}R(\tau+dt)=\frac{1-\delta dt}{1+rdt}(r+\delta)dt\left[[u+b(\tau)]+\frac{\alpha_w}{r+\delta}\int_{R(\tau+dt)}^{\infty}[1-F(w)]dw\right]$$

which has a very similar structure to (6). Indeed, this reservation wage is also a discounted sum of future $[u + b(t)]$. However the additional non-linear term implies a closed form solution does not exist, which is why the above bargaining approach is more tractable for characterising equilibrium. Nevertheless the underlying moral hazard problem is identical - future UI payments distort current reservation wages.

3 The Matching Equilibrium.

We take as given the government's UI program b, where $b(\tau)$ is a positive and decreasing function of τ, with $u + b(0) < k$. Given equilibrium wages w^* defined in Claim 1, a Matching Equilibrium is a steady state triple $\{U, V, H\}$ where unemployment U and the distribution of uncompleted unemployment spells $H(\tau)$ are consistent with steady state turnover, while vacancies V are consistent with free entry [see Pissarides (1990) for example]. We characterise each component in turn.

3.1 Steady-State Vacancy Creation

In a steady state and over any brief interval dt, the value Π_0 to holding an unfilled vacancy is given by,

$$\Pi_0 = \frac{1}{1 + rdt} \left[-adt + \alpha_f dt \mathrm{E} \left[\pi_f(\tau) \right] + (1 - \alpha_f dt) \Pi_0 \right].$$

Note, adt is the flow cost to holding an unfilled vacancy. With probability $\alpha_f dt$, the firm contacts a worker with some unemployment duration τ and obtains expected profit $\mathrm{E}[\pi_f(\tau)]$, where a Matching Equilibrium implies τ has distribution H. If the firm is not contacted by a job seeker, the vacancy remains unfilled with expected payoff Π_0. Free entry of vacancies implies $\Pi_0 = 0$, and so the above implies

$$a = \alpha_f \mathrm{E} \left[\pi_f(\tau) \right] \tag{8}$$

describes the appropriate free entry condition.

3.2 Steady-State Unemployment

Steady-state unemployment implies

$$\delta = M(U, V) + \delta U,$$

where the left hand side describes the inflow of market entrants into the pool of unemployment, and the right hand side describes the flow of workers out of the pool of unemployment. As $\alpha_w = M(U, V)/U$, steady state unemployment implies

$$U = \frac{\delta}{\alpha_w + \delta}. \tag{9}$$

3.3 Steady-State $H(\tau)$

Note that the exit rate of unemployed workers is not duration dependent - each exits at rate $[\alpha_w + \delta]$. If $U(\tau)$ denotes the measure of unemployed worker who have been unemployed no longer than duration τ, then given unemployment inflow $\delta > 0$,

$$U(\tau) = \int_0^\tau \delta \exp[-(\alpha_w + \delta)t]dt,$$

where $\exp[-(\alpha_w + \delta)t]$ is the probability a worker does not find work before duration t. Integration implies

$$U(\tau) = \frac{\delta}{\alpha_w + \delta}[1 - \exp\{-(\alpha_w + \delta)\tau\}],$$

and as $H(\tau) \equiv U(\tau)/U$, we obtain

$$H(\tau) = 1 - \exp\{-(\alpha_w + \delta)\tau\}. \tag{10}$$

3.4 Equilibrium

In any Matching Equilibrium $\{U, V, H\}$, we can define equilibrium labour market tightness $\phi^* = V/U$. Note, the matching function implies $\alpha_w = m(\phi^*)$ and (9) implies $U = \delta/[\alpha_w + \delta]$. Further, (10) implies $H \equiv \widehat{H}(\tau \mid \phi^*)$ where

$$\widehat{H}(\tau \mid \phi) = 1 - \exp\{-(m(\phi) + \delta)\tau\}. \tag{11}$$

ϕ^* is tied down by the free entry condition (8). As equilibrium wage formation implies $\pi_f(\tau) = \theta\psi(\tau)$, where ψ is the surplus function defined earlier, and as the matching function implies $\alpha_f = m(\phi^*)/\phi^*$, then the free entry condition (8) requires finding a ϕ^* which satisfies

$$a = \frac{m(\phi)}{\phi} \int_0^\infty \theta\psi(\tau) d\widehat{H}(\tau \mid \phi). \tag{12}$$

If a solution ϕ^* exists, we have identified a Market Equilibrium where U, H are as described above and $V = \phi^* U$.

Assuming $\theta > 0$ and that the Matching Function has standard properties [that for $\phi > 0$, $\alpha_f = m(\phi)/\phi$ is continuous and strictly decreasing in ϕ, with $\lim_{\phi \to 0} m(\phi)/\phi) = \infty, \lim_{\phi \to \infty} m(\phi)/\phi) = 0$] implies a non-degenerate Matching Equilibrium exists and is unique.[6]

4 Discussion and Simulations

Our focus here is on the effect of duration dependent unemployment benefits on equilibrium unemployment. In particular, we now restrict attention to unemployment

[6]Note that $\phi_1 < \phi_2$ implies $\widehat{H}(\tau \mid \phi_1)$ first order stochastically dominates $\widehat{H}(\tau \mid \phi_2)$. As ψ is strictly positive and increasing for $\tau \geq 0$, then for $\theta \in (0, 1]$, the integral term on the right hand side (RHS) of (12) is strictly positive and decreases with ϕ. The properties of the matching function then imply the RHS is a continuous and strictly decreasing function of ϕ for $\phi > 0$, where the RHS becomes arbitrarily large as $\phi \to 0$, and converges to zero as $\phi \to \infty$. Hence for any $a > 0$, a Matching equilibrium exists and is unique.

benefit schemes of the form

$$b(\tau) = \begin{cases} b_0 \text{ for } \tau \leq T \\ 0 \text{ otherwise.} \end{cases}$$

The policy parameters are the level of benefit payments b_0, and the duration of the scheme T.

Of course holding b_0 constant while increasing T implies a more generous benefit scheme, and implies all wages for $\tau < T$ increase. To identify a pure duration effect, we consider a compensated change where b_0 is reduced as T increases so that the expected cost of the UI scheme is held constant.

Given α_w, the expected cost of UI payments per job seeking entrant, denoted C_b, is

$$C_b = \int_0^\infty e^{-(\alpha_w+\delta)\tau} b(t)dt,$$

as each unemployed worker exits the state of unemployment at rate $\delta + \alpha_w$. Hence given policy parameters (b_0, T), generosity

$$C_b = \frac{b_0[1 - e^{-(\alpha_w+\delta)T}]}{\alpha_w + \delta}. \tag{13}$$

Now fix $C_b > 0$ and consider how variations in (b_0, T) satisfying (13) affect equilibrium wages. Claim 1 implies

$$w^*(\tau) = (1-\theta)k + \theta \int_\tau^\infty e^{-(r+\delta)(t-\tau)}(r+\delta)[u + b(t)]dt.$$

Obviously if $b = 0$, the equilibrium wage reduces to $w = (1-\theta)k + \theta u$. To focus on the wage distortion generated by the UI scheme, define $\Delta w = w^*(\tau) - [(1-\theta)k + \theta u]$, which is the wage increase at duration τ due to the UI scheme. The above gives

$$\begin{aligned} \Delta w &= \theta \int_\tau^\infty e^{-(r+\delta)(t-\tau)}(r+\delta)b(t)dt \\ &= \begin{cases} \theta b_0[1 - e^{-(r+\delta)[T-\tau]}] & \text{for} \quad \tau < T \\ 0 \quad \text{for} \quad \tau \geq T. \end{cases} \end{aligned}$$

A critical insight is that although the worker anticipates becoming re-employed at rate α_w, the wage distortion Δw does not depend on α_w. The worker's reservation wage instead depends on the value of remaining unemployed indefinitely, and it is the **option** of continuing to receive UI payments in the future which distorts w^*. Further, the value of that option reflects the worker's rate of time preference $r > 0$.

Also note that the time form of this wage deviation is closely related to that identified in the sequential search literature with duration dependent UI – the [reservation] wage decreases as $\tau \to T$, but the change is small for $T - \tau$ large, and large as τ approaches T [see Mortensen (1977), van den Berg (1990)].

For given C_b, we can use (13) to substitute out b_0 which gives

$$\Delta w(\tau, T) = \frac{1 - e^{-(r+\delta)[T-\tau]}}{1 - e^{-(\alpha_w+\delta)T}} \theta(\alpha_w + \delta) C_b$$

for $\tau < T$. This equation describes the wage distortion at any duration $\tau < T$ by varying (b_0, T) while holding generosity, C_b, constant.

Claim 2. Given α_w and for $r < \alpha_w$, $\Delta w(\tau, T)$ is strictly increasing in T for all $0 \leq \tau < T$; i.e. ceteris paribus, an increase in the duration of the UI scheme [while holding generosity C_b constant], increases wages at all durations $\tau < T$.

Proof: Differentiating the equation for Δw with respect to T gives:

$$\begin{aligned}
&\frac{\partial}{\partial T}[\Delta w(\tau, T)] \\
= \quad & \frac{(1 - e^{-(\alpha_w+\delta)T})(r+\delta)e^{-(r+\delta)[T-\tau]} - (1 - e^{-(r+\delta)[T-\tau]})(\alpha_w+\delta)e^{-(\alpha_w+\delta)T}}{[1 - e^{-(\alpha_w+\delta)T}]^2} \theta(\alpha_w+\delta) C_b \\
\geq \quad & \frac{(1 - e^{-(\alpha_w+\delta)T})(r+\delta)e^{-(r+\delta)T} - (1 - e^{-(r+\delta)T})(\alpha_w+\delta)e^{-(\alpha_w+\delta)T}}{[1 - e^{-(\alpha_w+\delta)T}]^2} \theta(\alpha_w+\delta) C_b \\
= \quad & \frac{e^{-(r+\delta)T}\theta(\alpha_w+\delta) C_b}{[1 - e^{-(\alpha_w+\delta)T}]^2} \left[(r+\delta) + (\alpha_w - r)e^{-(\alpha_w+\delta)T} - (\alpha_w+\delta)e^{-(\alpha_w-r)T}\right]
\end{aligned}$$

Inspection establishes that the bracketed term in the last equation above is strictly increasing in T. As the bracketed term is zero at $T = 0$, the bracketed term is strictly positive for all $T > 0$, which establishes the Claim.

Ignoring any equilibrium feedback effects - that the wage distortion also changes equilibrium α_w - Claim 2 establishes that increasing the duration of the UI scheme, while reducing b_0 to hold generosity constant, pushes up equilibrium wages. [Note that $r < \alpha_w$ is the natural case; e.g. in the UK the average duration of unemployment is around 26 weeks, which implies an annual matching rate $\alpha_w = 2$, while annual discount rates are typically assumed to be around 5%.] The underlying insight is that when job offers are unobservable, each worker's reservation wage depends on the worker's option value of remaining unemployed and continuing to receive further UI payments. As the worker's rate of time preference $r \ll \alpha_w$, extending the duration of UI payments while holding generosity constant implies the option of remaining unemployed increases in value.

To illustrate the potential magnitude of this wage distortion, the following uses a numerical simulation based on the parameter values chosen by Millard and Mortensen (1997) [henceforth MM] in their calibrations of the U.S. economy. Assuming a Cobb-Douglas matching function $M = U^{1-\eta}V^{\eta}$, Table 1 reports the MM values for u, r, η, θ, k where the assumed time unit is one quarter.

To focus on the moral hazard implications of non-observable job offers, the theory section was simplified by assuming no job destruction. As a result, steady state job turnover equals δ. The purpose of the simulations that follow is to demonstrate how large the resulting wage distortion on unemployment can be using a baseline economy in which the unemployment figures correspond roughly to actual economic activity in the US. The Current Population Survey [CPS] indicates that between 1985 and 1995, the average unemployment rate and the average duration of unemployment were 6.4% and 15 weeks respectively. To fit these averages, steady state requires that the inflow of job seekers $\delta = 0.0555$ per quarter [by comparison, Cole and Rogerson (1999) use 0.055 when describing aggregate job destruction rates].

k	u	r	η	θ	δ	a	b_0
1	0.285	0.01	0.6	0.7	.0555	8.56	0.25

Table 1: Simulation Parameter Values

The unemployment insurance system in the U.S. is considered to be $T_b = 2$ quarters with a replacement rate of around 0.5. The remaining parameter values a and b_0 are chosen to approximate this replacement rate while matching the unemployment figures from the CPS. Table1 describes those parameter values, and row 2 in Table 2 describes the corresponding market equilibrium.[7]

Given these parameter values, Table 2 describes the effect of changing the duration of payments in a UI scheme while holding generosity constant. For each scheme, newly unemployed workers expect to receive 3.1 weeks of match output in total UI payments [about 6 weeks of wages].[8] The second and third columns describe the scheme. $\bar{w}$ is the average wage paid in the economy while $\overline{\tau}$ is the average duration of unemployment.

As demonstrated by Claim 2, increasing T [while holding C_b constant] increases the option value of remaining unemployed, and so increases wages for all $\tau < T$. Table 2 demonstrates these effects are potentially large. Going to an indefinite payment scheme [from row 2 to row 4] implies a 19% increase in average market wages. The corresponding increase in unemployment is around 35% [just over two percentage points] and the average duration of unemployment increases from 15 to 20 weeks. However, most of this change occurs when we move from the one year to the indefinite

[7] The value for a may look excessively large under its usual interpretation as an advertising cost. In actuality, it has to capture all of the capital costs associated with job creation.

[8] with $T = 2$ and $b_0 = 0.25$, the computed steady state cost of the UI scheme is 0.0131. Note that as the inflow δ is the same across schemes, the steady state cost of the UI scheme per entrant is also constant.

row #	T weeks	b_0	$\bar{w}$	U %	$\bar{\tau}$ weeks
0	0	-	0.499	6.17	14.5
1	13	0.365	0.505	6.27	14.7
2	26	0.250	0.511	6.38	15.0
3	52	0.205	0.524	6.62	15.5
4	∞	0.152	0.606	8.62	20.2

Table 2: Simulation Results

payment scheme - the distortion is relatively small for a shift from 6 months to one year.

Figure 1 plots the wage functions $w^*(.)$ for each of the policy parameters (b_0, T). As implied by claim 2, these wage functions shift up as T increases. Apart from the case $T = \infty$, these wage functions decrease with duration, where a worker's bargaining position deteriorates the closer the expiry date of his/her UI entitlement.

Note that equilibrium unemployment levels change with these policy parameters. On each wage function described in figure 1, we have also marked the average duration of unemployment $\overline{\tau}$ [as described in Table 2]. For these parameter values, the distortion on $\overline{\tau}$ [and U] is small for T small [no more than one year]. The implied employment elasticity with respect to average wages is only 1% in this region [going from row 1 to row 3 in Table 2]. However the employment distortion going to the indefinite payment case is relatively large.

As all are risk neutral, we cannot describe the insurance value of each UI scheme. Nevertheless it is well known when all are risk neutral, that the market outcome is not efficient unless the Hosios condition holds [Hosios (1990)]. For the above parameter values [and $b = 0$] workers have 'too little' bargaining power [as $1 - \theta = 0.3 < 1 - \eta =$

0.4], and a weak threatpoint [which is the value of being unemployed forever, rather than the equilibrium value of being unemployed]. The negotiated wage in row 0, for the case $b = 0$, is therefore below the Hosios optimum. Equilibrium there implies 'too much' vacancy creation.

As firms make zero expected profit, the Utilitarian Social Welfare measure USW corresponds to the steady state flow utility of all workers, which by the above is

$$\begin{aligned} USW &= [1-U]\overline{w} + \delta \int_0^\infty e^{-(\alpha_w+\delta)t}[u+b(t)]dt \\ &= [1-U]\overline{w} + \delta d_w(0). \end{aligned}$$

Given the above parameter values and Table 2, this welfare measure implies that row 4, the indefinite UI scheme, is in fact the preferred UI system. By improving the bargaining position of workers, this UI scheme raises equilibrium wages closer to the Hosios wage level [at which point all congestion externalities are fully internalised]. Of course for different parameters, say workers have too much bargaining power, the opposite insight would apply - row zero in Table 2, which can alternatively be interpreted as making a lump sum payment of C_b at $\tau = 0$, would then be preferred - all other UI schemes distort wages further away from the Hosios optimum.

5 Conclusion

This paper has developed a dynamically consistent model of equilibrium unemployment when UI payments are duration dependent. The finding is that for natural parameter values (that imply $r < \alpha_w$), extending the duration of UI payments while holding the generosity of the scheme constant, increases the distortion on reservation wages at all durations. Simulations suggest that an indefinite payment scheme has a particularly large impact on equilibrium wages and unemployment levels.

To illustrate clearly the nature of the moral hazard problem, this paper has assumed that workers are risk neutral. But as there is no insurance role for UI in this case, we have minimised any welfare discussions. An important direction for future research is to allow strictly risk averse workers. If workers also have 'too much' bargaining power so that market wages are potentially too high [in the Hosios sense], the government then has a trade-off between providing efficient insurance against unemployment risk, and distorting market wages. Unfortunately such an extension is complex. The central difficulty is that given duration dependent UI payments, the equilibrium wage bargaining outcome with a strictly risk averse worker is described by a non-linear differential equation which has no closed form solution [see Coles and Wright (1998) for example]. Although it should be possible to compute numerical equilibria, the clean theoretical insights obtained above are necessarily lost. Nevertheless, intuition strongly suggests a Shavell-Weiss type program will be optimal. In Shavell and Weiss (1979), the government reduces UI payments with duration to reduce the value of remaining unemployed and so encourage greater job search effort. The same insight applies here should workers have too much bargaining power. By reducing UI payments with duration, the government reduces the value of remaining unemployed which, this time, reduces the distortion on worker reservation wages. As in Shavell and Weiss (1979), the government can then offer relatively generous UI payments for short durations, and so provide reasonable insurance for employed workers against layoff risk, while minimising the moral hazard distortion on equilibrium wage formation. Computing such equilibria and establishing formally this intuition is left for future research.

References

Albrecht, J. and S. Vroman (2001), "Equilibrium Search with Time Varying Unemployment Benefits" mimeo, Georgetown University.

Atkinson, A. and J. Micklewright (1991) "Unemployment Compensation and Labour Market Transitions: a Critical Review" *Journal of Economic Literature,* **29,** 1679-1727.

Binmore, K. (1987) "Perfect Equilibria in Bargaining Models", in *The Economics of Bargaining,* Ken Binmore and Partha Dasgupta, eds. Oxford, U.K.: Basil Blackwell.

Binmore, K., A. Rubinstein, and A. Wolinsky, (1986) "The Nash Bargaining Solution in Economic Modelling" *Rand Journal of Economics,* **17,** 176-88.

Blanchard, O. and P. Diamond, (1994) "Ranking, Unemployment Duration and Wages", *Review of Economic Studies*, **61**, 417-34.

Burdett, K. and D. Mortensen (1980) "Search, Lay-offs and Labor Market Equilibrium", *Journal of Political Economy,* **88,** 652-72.

Cole, H. and R. Rogerson, (1999) "Can the Mortensen-Pissarides Matching Model Match the Business Cycle Facts?" *International Economic Review,* **40,** 933-60.

Coles, M. and A. Masters (2001) "Unemployment Insurance and the Cyclical Pattern of Unemployment", mimeo University of Essex, UK.

Coles, M. and R. Wright, (1998) "A Dynamic Equilibrium Model of Search, Bargaining and Money" *Journal of Economic Theory*, **83**, 32-54.

Davidson, C. and S. Woodbury, (1997) "Optimal Unemployment Insurance", *Journal of Public Economics",* **64,** 359-87.

Diamond, P. (1981) "Wage Determination and Efficiency in Search Equilibrium", *Review of Economic Studies*, **49**, 217-28.

Fredriksson, P. and B. Holmund, (2001) "Optimal Unemployment Insurance in Search Equilibrium", *Journal of Labor Economics,* **19,** 370-99.

Gale, D. (1987) "Limit Theorems for Markets with Sequential Bargaining", *Journal of Economic Theory,* **43,** 20-54.

Hosios, A. (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment", *Review of Economic Studies*, **57,** 279-98.

Millard, S. and D. Mortensen, (1997) "The Unemployment and Welfare Effects of Labour Market Policy: A Comparison of the USA and the UK" in *Unemployment Policy,* Snower, D. and de la Dehesa, G. eds. Cambridge: Cambridge University Press (for CEPR).

Mortensen, D. (1977) "Unemployment Insurance and Job Search Decisions", *Industrial and labour Relations Review,* **30,** 505-17.

Mortensen, D. and C. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies,* **61,** 397-415.

Mortensen, D. and R. Wright (2000) "Competitive Pricing and Efficiency in Search Equilibrium", mimeo, University of Pennsylvania.

Nickell, S. (1997) "Unemployment and Labour Market Rigidities: Europe versus North America" *Journal of Economic Perspectives,* **11.3,** 55-74.

Pissarides, C. (1990). *Equilibrium Unemployment Theory.* Oxford: Basil Blackwell.

Rubinstein, A. and A. Wolinsky "Equilibrium in a Market with Sequential Bargaining", *Econometrica,* **53,** 1133-50.

Shavell, S. and L. Weiss (1979) "The Optimal Payment of Unemployment Insurance Benefits over Time" *Journal of Political Economy*, **87**, 1347-62.

van den Berg, G. (1990) "Nonstationarity in Job Search Theory", *Review of Economic Studies,* **57,** 255-77.

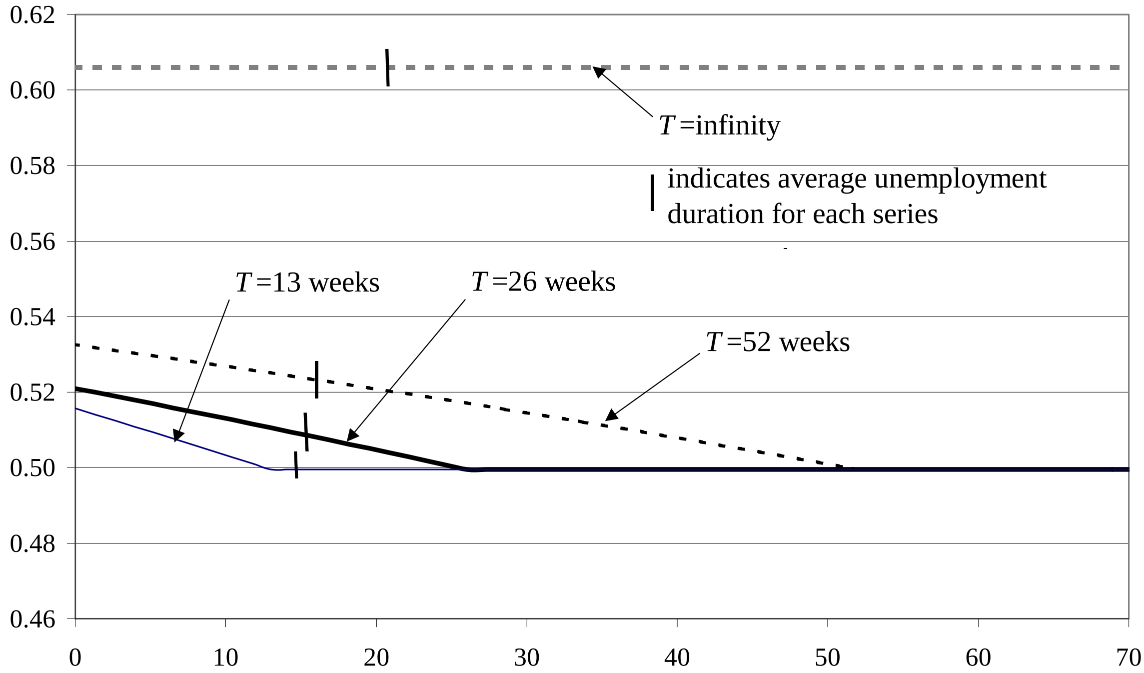

Figure 1: Wage *versus* Duration of Unemployment in Weeks