

Middlemen in Search Equilibrium

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Abstract

This paper shows how including divisibility of goods and productive heterogeneity leads to the emergence of middlemen in an equilibrium search environment. In the baseline model, middlemen are welfare reducing and their number increases as market frictions are reduced. When the model is extended to allow for time taken in production and increasing returns-to-scale in the market meeting technology, middlemen can be beneficial to society by speeding up the meeting process.

Keywords: middlemen, search, matching, commodity money

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1 Introduction

This paper provides a simple modification of Diamond's [1982] (coconut) search model to investigate the emergence of middlemen in markets characterized by matching frictions. Individuals in the model are not born with, nor are they able to acquire, any special skills that might enhance their ability to act as a market intermediary. Still, the finding is that as long as goods are divisible, and there is sufficient heterogeneity in productive ability, some people will become middlemen simply to avoid the cost of producing. Moreover, in the baseline model intermediation is detrimental to society in that a ban on such activities would improve economic efficiency.

In the baseline model, individuals cycle between trading and production which occur in separate locations. Trading is driven by the unpalatability of one's own output and takes place in an anonymous market with matching frictions. Production is instantaneous but costly. To become a middleman an individual simply has to forego consumption once, and remain in the market. Holding an edible good strengthens a middleman's bargaining position with future trading partners - he acquires a trading partner's whole good for part of the one he is carrying. He consumes the remainder of the divided good and leaves with the partner's good intact; ready to trade again. Middlemen, therefore, survive by 'buying low' and 'selling high'. They never need to produce, so intermediation is more attractive to those with higher production costs.

In this environment, middlemen simply clog up the system. They have no effect on the frequency of trading opportunities but when they are around some trades only involve the consumption of one unit of output rather than two. Producers (non-middleman traders) do business with middlemen to

avoid the time cost of looking for another producer (from whom they would get a whole good). Output is necessarily lower when individuals are permitted to act as middlemen than when they are banned from the practice. As the production cost they avoid is less than the utility received from consumption, a *laissez-faire* economy with middlemen is necessarily inefficient. The source of inefficiency is an entry/exit type externality (see Hosios [1990]); in their decision to become middlemen, individuals do not take account of the effect of their choice on future trading partners.

This paper goes on to consider an extension of the model in which production takes time. In this environment, a snap shot of the economy finds some proportion of the population involved in production. As middlemen remain in the market there are more potential trading opportunities when they are around than when they are forced to produce. How this affects the social contribution of intermediation depends on how the aggregate number of meetings changes with the number of people in the market. It is common in the labor literature, for instance (see Pissarides [2000]), to assume constant returns to scale in the meeting technology. In the current context this would mean individual meeting rates are unaffected by the number of people in the market. So, with constant returns to scale, middlemen do not improve the trading opportunities of those who do produce. Only if individual meeting rates increase with the population of people in the market does the presence of middlemen act to reduce the time producers take to trade. It will be shown then, that if the meeting technology exhibits sufficiently strong increasing returns to scale, *laissez-faire* equilibrium with middlemen can be preferred to the allocation that results from a ban on such intermediation. In this context, the individual incentives to become a middleman are the same as in the baseline model. The increasing returns in the meeting technology

simply redirects the effect of the entry/exit externality.

The literature on the role of intermediaries in matching environments with search frictions started with Rubinstein and Wolinsky [1987]. Their model has 3 types of agents: buyers, sellers and middlemen. While buyers and sellers are free to trade with each other, middlemen are able to match more quickly which gives them a natural advantage in the market. Buyers and sellers simply flow through the market while middlemen remain there. The terms of trade faced by each type of agent are determined endogenously by Nash bargaining. They show that equilibria in which middlemen make positive profits are possible. While they show that the existence of middlemen has an ambiguous effect on the welfare of buyers and sellers, the profits of middlemen are necessarily ignored in these calculations. This is because the middlemen are infinite lived and the others survive only as long as it takes them to trade. The current paper avoids such non-comparability because the middlemen emerge from the same population as the buyers and sellers.

The work on middlemen subsequent to Rubinstein and Wolinsky [1987] falls into two strands. One strand has introduced private information over the quality of goods. If buyers cannot tell lemons from peaches, producers face a moral hazard problem. Biglaiser [1993] shows that there is a role for middlemen when the technology for quality assessment has increasing returns to scale. Li [1996] studies a similar problem in a general equilibrium setting. She shows that some people will become middlemen through the costly acquisition of skills which allow them to identify product quality. Buyers will pay more for goods bought from the middleman because they believe such goods to be of higher quality.

The other strand of the literature has enriched the basic Rubinstein and Wolinsky [1987] model through the introduction of heterogeneous goods and

idiosyncratic preferences (see Shevchenko [2001], Johri and Leach [2000] and Smith [2002]). These are essentially models of shopping. In the presence search frictions it behooves some portion of the population to hold multiple inventories of heterogeneous goods so that the probability of gains from trade at any match is increased.

The current paper focusses on an additional incentive for people to become intermediaries, that of avoiding production costs. As such, the paper does not provide a model of any particular market. The assertion is rather that all of the roles of middlemen identified in the literature along with that put forward here are simultaneously active in real-world markets. The relative importance of each, however, might vary across sectors.¹

Another literature to which this paper bares comparison is that on *micro* models of money. In a general sense, money can be defined as anything that is worth more in exchange than in consumption. Being intrinsically worthless, fiat money clearly fits this description. An item that has non-negligible value in consumption (for at least some of the population), and is used in exchange is called a commodity money. In the literature on the foundations of monetary economics (*e.g.* Kiyotaki and Wright [1989]), commodity moneys have typically been of this type. That is, goods have different values to different people and they are used as money by those who do not value them in consumption. An exception to this rule has been Burdett *et al* [2001] who show that a general good (from which everyone can derive utility) might be used as money in order to increase the chances of acquiring a more desired specialist good. In this paper, it is shown that goods can be used as money

¹Ollivier [2000] had different groups of people rank occupations and found that real-estate agents were consistently placed low in terms of prestige and perceived usefulness to society relative to other occupations requiring similar levels of education.

even if they are as desirable in consumption to the carrier as the good to be acquired with it. The reason the holder does not eat his inventory is that it improves his bargaining position at subsequent trading opportunities.

2 Model

2.1 Baseline Environment

The economy comprises a continuum of risk neutral individuals who live for ever. The population is normalized to 1. They exist in continuous time and they discount the future at a common rate r .

Following Diamond [1982], to provide a motive for trade, individuals get no utility from the consumption of their own output but get u per unit of anyone else's output consumed.² Trade occurs in a decentralized anonymous market characterized by random matching. Any participant meets another with Poisson arrival rate β .³ Only whole goods can be brought to market and individuals can only carry one good at a time. Goods can, however, be divided for the purpose of consumption but any part of the good not eaten immediately rots (coconuts continue to be a good example).

There is *ex ante* heterogeneity with respect to production costs. A type c individual has production cost $c \sim F$ with support $(0, \bar{c}]$. Production costs are observable. For the baseline environment, we assume that production takes no time and $\bar{c} \leq u$. The model is extended to incorporate time-to-produce in Section 4. To rule out long-term relationships, individuals have

²Discussion of an alternative preference arrangements is provided in Section 3.3.

³In this first environment, either r or β is redundant. Both have been kept in for expositional clarity and to ease comparison with the extended model of Section 4 where they will have distinct roles.

to leave the market to produce; only when they have a good in hand may individuals re-enter by which point their connection with any previous trading partner is lost.⁴ Although there is no obligation to trade, only people with inventory are permitted to remain in the market. Those that leave the market must produce. This last assumption merely simplifies the analysis by preventing people from entering the production location and refusing to produce - effectively taking themselves out of the model. The implications of relaxing this assumption will be explored in Section 3.2.

Apart from the heterogeneity and potential divisibility of the goods, the basic environment is that of Diamond [1982] with immediate production and fixed production costs. The following subsection shows how divisibility of goods generates a much richer set of potential trades than the one-for-one swaps implied by indivisibility. It will be shown later that the heterogeneity is needed in order for that set of trades to be realized in equilibrium.

2.2 Exchange

In any meeting between individuals the outcome is determined according to Nash bargaining. A thorough treatment of Nash Bargaining is found in Osborne and Rubinstein [1990]. Here, it suffices to point out that the implied allocation is assumed to provide utilities (s_1^*, s_2^*) such that

$$(s_1^*, s_2^*) \in \arg \max_{(s_1, s_2) \in S} (s_1 - T_1)^{\frac{1}{2}} (s_2 - T_2)^{\frac{1}{2}} \quad (1)$$

⁴There is a huge literature on matching to form long-term relationships, usually in a labor market context (see Pissarides [2000]). This paper is focused on how trading partners meet in the first place. Restricting attention to one-time exchange of goods allows abstraction from the complications caused by long-term relationships for steady state flows into and out of the market.

where S is the set of utilities (s_1, s_2) associated with every feasible and *ex post* individually rational agreement between individuals 1 and 2.⁵ The T_i 's are the utilities associated with disagreement for each individual i , also called their 'outside-options'. Feasibility requires that $s_1, s_2 \geq 0$ and $s_1 + s_2$ cannot exceed the maximum utility associated with either:

consumption of whatever goods are immediately available to the participants plus their continuation values

or,

the combined value to disagreement, $T_1 + T_2$,

whichever is the larger.

We assume that $(T_1, T_2) \in S$ - they can agree to disagree. Individual rationality requires that for (s_1, s_2) to be in S , $s_i \geq T_i$, $i = 1, 2$.

Market equilibria, characterized below, are patterns of trade from which no one wishes to deviate. To apply the Nash Bargaining Rule, (1), we fix an equilibrium and consider its implications for the behavior of individuals in any possible meeting.

(i) **Producer-producer meetings**

We will call a *producer* anyone who is regularly producing goods in order to trade them and return to production. Whenever two producers meet, the natural outcome is that they simply swap their goods. This is also the Nash solution. To see this, notice that in any equilibrium $S = [V_1, u - c_1 + V_1] \times [V_2, u - c_2 + V_2]$ where the c_i 's are their respective production costs and the V_i 's represent the equilibrium value to being a producer of type c_i . The intuition for this result is that because individuals derive no value from

⁵Osborne and Rubinstein [1990] only impose feasibility on S .

the consumption of their own output, utility is non-transferable. The Nash axioms that underlie (1) include efficiency which precludes wastage.

(ii) **Producer-middleman meetings**

A *middleman* is anyone who, rather than eating it, holds onto a good they did not produce. This makes sense only if the value to holding onto the good exceeds that of consumption, production and returning to the market with ones own good. That is, if

$$\tilde{V}_i \geq u - c_i + V_i. \quad (2)$$

where \tilde{V}_i represents the value to a type c_i person of being a middleman.

Notice that as both parties derive utility from consuming the good brought to the table by the middleman there is something to potentially negotiate over. This can happen if the producer is prepared to hand over her whole good for part of the one carried by the middleman. The analysis proceeds by assuming this is true. We will then verify that this is indeed true as long as condition (2) holds. Utility is now (locally) transferable and the Nash Bargaining Rule (1) simply states that the participants divide the surplus (*i.e.* the gains from trade) equally. Given the stationarity of the environment and condition (2) any one who carries a good they did not produce prior to trade will also prefer to hold on to any intact good they acquired through trade (rather than eat it). The match surplus is therefore:

$$\begin{aligned} & u - c_j + V_j + \tilde{V}_i - V_j - \tilde{V}_i \\ = & u - c_j. \end{aligned}$$

Verification that under condition (2) the middleman will obtain at least the producer's good intact simply requires that $u - c_j > 0$.

The upshot is that as long as condition (2) is true when a middleman trades with a producer:

- The producer consumes $\frac{1}{2}(u - c_j) + c_j$ utils worth of the middleman's good
- The middleman gets to walk off with the producer's good and consume $\frac{1}{2}(u - c_j)$ utils worth of the good he was carrying.

Of course, so far it has still not been established that condition (2) is ever true. This will be addressed below.

(iii) Middleman-middleman meetings

Both people are holding goods they can eat, since trade could make neither better off, the match surplus is zero. There is no trade.

2.3 Market Equilibria

Equilibria are patterns of trading from which, in steady-state, no one wants to deviate. Attention is restricted to steady-state equilibria so that the value to being a producer or a middleman does not change over time. Consequently, anyone choosing to be a middleman will be so for ever. To eliminate the trivial equilibrium (where no one does anything) I assume that people have been initially endowed with a unit of their production good. In a steady-state, of course, any middleman would have long since traded that good for one they can eat so that everyone who intended to be a middleman carries a good produced by someone else. As middlemen do not produce, we will write the value to being a middleman as $V_m = \tilde{V}_i$ for all i . It is also convenient to write the asset value to being a type c producer as $V(c)$. So that V_i , in the above notation would be replaced by $V(c_i)$. The continuous time asset value equation for a producer of type c is then

$$rV(c) = \alpha\beta \max\{u - c, V_m - V(c)\} + \frac{1}{2}(1 - \alpha)\beta(u - c) \quad (3)$$

where α is the proportion of people in the market who are producers.⁶

The value of α is taken as given by the participants but will be endogenously determined below. The first term highlights the fact that each producer has the option, after every meeting with another producer, of choosing to become a middleman. The second term relates to possible meetings with middlemen at which the type c producer gets half of the surplus, $(u - c)$.

Given α , $V_m \geq 0$, equation (3) implies that $V(\cdot)$ is strictly decreasing. So, there exists a $\hat{c} < u$ such that $u - \hat{c} = V_m - V(\hat{c})$ and all individuals with $c < \hat{c}$ are producers.⁷ Given \hat{c} ,

$$rV_m = \frac{1}{2}\alpha\beta\mathbb{E}_{\{c < \hat{c}\}}(u - c) \quad (4)$$

where \mathbb{E} is the expectations operator. That is to say the middlemen get half the match surplus from meetings with producers. They never produce and meetings with other middlemen generate no trade.

Definition 1 *A market equilibrium is a tuple $\{V(\cdot), V_m, \hat{c}, \alpha\}$ that satisfies equations (3) and (4), consistent with:*

- *optimal career choice: $(u - \hat{c}) = V_m - V(\hat{c})$.*

⁶The simplicity of this expression is a consequence of the Poisson process for meetings. It can be derived as follows:

$$V(c) = \left(\frac{1}{1+r\Delta}\right) \left[\beta\Delta \left(\alpha \max\{u - c + V(c), V_m\} + (1 - \alpha)\frac{1}{2}(u - c) + V(c) \right) + (1 - \beta\Delta)V(c) \right] + o(\Delta)$$

where Δ is a short time period and $o(\Delta)$ represents the error in the equation due to the possibility of two meetings within the time period Δ . By the assumptions of the Poisson process, $\lim_{\Delta \rightarrow 0} o(\Delta)/\Delta = 0$. Equation (3) follows from taking the limit as Δ approaches 0.

⁷For large values of V_m , $u - \hat{c} = V_m - V(\hat{c})$ implies $\hat{c} < 0$. In this case as (by assumption) no individual has negative production costs we can impose that $\hat{c} = 0$.

- *rational expectations*: $\alpha = F(\hat{c})$.

In equilibrium, the critical value of the cost parameter above which people optimally choose to be a middleman has to be consistent with the implied probability with which individuals actually encounter with middlemen.

Replacing α with $F(\hat{c})$, from (3), for $c < \hat{c}$, we get

$$rV(c) = \frac{1}{2}(1 + F(\hat{c}))\beta(u - c) \quad (5)$$

and from (4),

$$rV_m = \frac{1}{2}F(\hat{c})\beta \int_0^{\hat{c}} (u - c) \frac{dF(c)}{F(\hat{c})} = \frac{\beta}{2} \int_0^{\hat{c}} (u - c) dF(c).$$

From these equations and the definition of \hat{c} we get:

$$(u - \hat{c}) [2r + \beta(1 + F(\hat{c}))] = \beta \int_0^{\hat{c}} (u - c) dF(c)$$

Integration by parts yields,

$$(2r + \beta)(u - \hat{c}) = \beta \int_0^{\hat{c}} F(c) dc. \quad (6)$$

This is similar to the “reservation price” equations from the search literature (*e.g.* Mortensen [1985]) and is a characterization of equilibrium.

Proposition 2 *There exists a unique market equilibrium, $\hat{c} < u$.*

Proof. A direct implication of equation (6). ■

3 Analysis and welfare

First, it should be clear that while \hat{c} always exists, it need not be the case that equilibrium always implies the existence of middlemen. For that we require

$\hat{c} < \bar{c}$. Indeed, if F is degenerate (i.e. in the limit as $\bar{c} \rightarrow 0$) \hat{c} converges from above to

$$\underline{\hat{c}} \equiv \frac{(2r + \beta)u}{2(r + \beta)} > 0 \quad (7)$$

This means that when everyone is the same there are no middlemen.⁸ Moreover, $\underline{\hat{c}}$ is the lowest value of \hat{c} for any F . This is simply because any other distribution of production costs necessarily first-order-stochastically dominates the degenerate distribution at 0. That is, $\int_0^c G(\nu)d\nu < c$ for any $c > 0$ and any probability distribution function G defined on the strictly positive real line.

Straightforward differentiation reveals that \hat{c} is increasing (decreasing) in r (β). This means that as market frictions decrease, more people move into intermediation. As the expected time until the next meeting falls (or everyone gets more patient) the rate of return on a middleman's inventory increases. That is, for a given value of \hat{c} , $V_m - V(c)$ increases in proportion to β but $u - c$ which is always available to a type c middleman does not change.

Utilitarian welfare, W , is

$$W = \alpha \mathbb{E}_{\{c|c < \hat{c}\}} V(c) + (1 - \alpha)V_m$$

When $\hat{c} \geq \bar{c}$, there are no middlemen in equilibrium so

$$W = \mathbb{E}_c V(c) = \frac{\beta}{r} \int_0^{\bar{c}} (u - c) dF(c) \quad (8)$$

When $\hat{c} < \bar{c}$ from (5) we have

$$W = F(\hat{c}) \frac{\beta}{2r} \int_0^{\hat{c}} (1 + F(\hat{c}))(u - c) \frac{dF(c)}{F(\hat{c})} + (1 - F(\hat{c})) \frac{\beta}{2r} \int_0^{\hat{c}} F(\hat{c})(u - c) \frac{dF(c)}{F(\hat{c})}.$$

⁸Notice that because we can always re-normalize u , setting the infimum of the support of F to 0 is done without loss of generality. So this statement is general.

That is,

$$W = \frac{\beta}{r} \int_0^{\hat{c}} (u - c) dF(c). \quad (9)$$

Welfare is simply the present value of flow output less costs incurred in the economy.

Comparing (8) and (9) reveals that middlemen contribute nothing to society. Theirs is a parasitic existence, they simply clog up the system extracting rents from any producer they might meet. The net loss of welfare from permitting this form of intermediation is

$$\frac{\beta}{r} \int_{\hat{c}}^{\bar{c}} (u - c) dF(c).$$

The source of the inefficiency is that in their decision to become middlemen individuals do not take account of the effect of their choice on their future trading partners.

Of course, to consider the implications of an effective ban on being a middleman we have to consider what happens along the transitional path. Here the path is trivial; middlemen simply eat their inventory produce and re-enter the market. The economy moves instantaneously to the new ‘no-middleman steady-state’ adding another

$$\int_{\hat{c}}^{\bar{c}} (u - c) dF(c).$$

to total welfare.

In the absence of any compensatory transfer schemes, there are winners and losers from this policy. Let $V^n(c)$ represent the steady-state value to being a type c individual when there is an effective ban on middleman activities. Then,

$$rV^n(c) = \beta(u - c).$$

Those who were initially producers now consume a whole unit of output at every meeting rather than having to share with middlemen. They each gain $\beta(1 - F(\hat{c}))(u - c)/2r$. Middlemen, forced to abandon their trade, get to eat their inventory but now have to produce to survive. They get $(u - c) + V^n(c) - V_m$. Since for the marginal middleman $c = \hat{c}$,

$$V^n(\hat{c}) - V(\hat{c}) = \beta(1 - F(\hat{c}))(u - \hat{c})/2r > 0$$

Some ex-middlemen are, therefore, made better off. They choose to be middlemen in the *laissez-faire* economy because their returns from production were reduced by the presence of other middlemen. While $\hat{c} < \bar{c}$, a ban on middlemen would be binding for some individuals with values of c close to \bar{c} , or no one would have chosen to be a middleman in the first place. However, because everyone is made worse off by the existence of other middlemen, it is possible for the ban on intermediation to be Pareto improving. The following example illustrates

3.1 Example

Half of the population is high productivity with $c = 0$ and half is low productivity with $c = \bar{c}$. The object is to consider what happens to the equilibrium allocation and welfare at different values of \bar{c} .

As long as $\bar{c} < \hat{c}$, there are no middlemen and

$$\hat{c} = \frac{2(2r + \beta)u + \beta\bar{c}}{2(r + \beta)}.$$

When $\bar{c} = 0$, $\hat{c} = \underline{\hat{c}}$. As \bar{c} increases so does \hat{c} but to a lesser extent until⁹

$$\hat{c} = \bar{c} = \tilde{c} \equiv \frac{4r + 2\beta}{4r + 3\beta}u.$$

It should be clear from inspection of (6) that in this case, further increases in \bar{c} do not affect \hat{c} . Let $W(\bar{c})$ represent the (*laissez-faire*) equilibrium welfare when the high cost people have production cost \bar{c} and let $W^n(\bar{c})$ represent the steady-state welfare under a ban on middleman activity. Then ,

$$\bar{c} \leq \tilde{c} \Rightarrow W(\bar{c}) = W^n(\bar{c}) = \frac{\beta}{2r}(2u - \bar{c}) \quad (10)$$

$$\tilde{c} \leq \bar{c} < u \Rightarrow W(\bar{c}) = \frac{\beta u}{2r} < W^n(\bar{c}) = \frac{\beta}{2r}(2u - \bar{c}) \quad (11)$$

When $\bar{c} > \tilde{c}$, (so all the high cost individuals are middlemen) the value to being a producer is $3\beta u/4r$. The value to being a middleman is $\beta u/4r$. Under a ban on middlemen, the steady-state value to being a low cost person is $\beta u/r$. The high cost people get $\beta(u - \bar{c})/r$. If $\bar{c} > 3u/4$, the high cost people will prefer the *laissez-faire* allocation. But, for values of \bar{c} in the range $(\tilde{c}, 3u/4)$ a ban on middlemen is a strict Pareto improvement.¹⁰

3.2 Voluntary production

Recall that so far, people who visit the production location must produce. If instead production is voluntary, there is an extra decision to be made (whether to produce or not). For any individual of type c , production is

⁹When $F(\cdot)$ contains mass points, as in this example, purifiable mixed strategy equilibria are possible. Here, if $\bar{c} = \tilde{c}$, there are a continuum of mixed strategy equilibria associated with every proportion in $[0, 1]$ of the high cost individuals are middlemen.

¹⁰If $\bar{c} = \tilde{c}$, there are a continuum of mixed strategy equilibria which are Pareto rankable according to the share of high cost individuals who choose to produce.

worthwhile if $V(c) > c$. To see whether this requirement might preclude the emergence of middlemen, notice that from (5), for all c ,

$$V(c) > \frac{\beta(u - c)}{2r}.$$

So, as long as

$$\bar{c} < \frac{\beta u}{2r + \beta} \tag{12}$$

everyone who enters the production location will choose to produce. That is, under restriction (12) equilibria when production is enforced are also equilibria when production is voluntary. The question now, is whether (12) rules out those parameter configurations that would imply the emergence of intermediaries.

From (6), $\hat{c} < \bar{c}$ if and only if

$$(2r + \beta)(u - \bar{c}) - \beta \int_0^{\bar{c}} F(c) dc < 0 \tag{13}$$

Now, pick any distribution, with $\bar{c} < u$ for which (13) holds. As \bar{c} is a constant, it is always possible to pick a high enough value of β such that (12) holds and since \hat{c} decreases with β , (13) must also hold.

The upshot is that for the earlier analysis to go through, the extent of discounting between meetings, r/β , has to be small enough. In the case of the example of Section 3.1, the existence of middlemen with voluntary production requires

$$\frac{4r + 2\beta}{4r + 3\beta}u < \bar{c} < \frac{\beta}{2r + \beta}u$$

such values of \bar{c} are possible as long as $r/\beta < 0.183$.¹¹

¹¹By comparison, in a labor market context it is reasonable to use an annual discount rate of 4% and a matching rate of one per month (see Blanchard and Diamond [1989]). These figures imply $r/\beta = 0.0033$. More generally we expect market matching rates to be much bigger than discount rates.

While it may, be of interest to investigate what happens in this model if some people refuse to produce, this goes beyond the scope of the current paper.¹²

3.3 Alternative preference arrangement

A special feature of the model analyzed so far is that individuals get utility from consuming everyone's produce but their own. Here we briefly consider an alternative preference arrangement.

There are red and blue people, named after the color of their production good. Red people only eat blue goods and *vice-versa*. There is the same number of each color people and within each color there is the same distribution of productive abilities. In this economy, sufficiently unproductive individuals will become middlemen. For example, a red middleman will trade her initial inventory for a blue good. Rather than eat it, she looks for a red producer, persuades that producer to hand over his whole good for part of the one she is carrying. She then consumes the remainder of the blue good and goes off with the red one. In this way, she avoids ever having to produce.

4 Time to produce

In the baseline model analyzed above, production has no opportunity cost in terms of lost time in the market. This section of the paper extends the model to incorporate time spent in production. The point here is to check the robustness and efficiency of market equilibria with middlemen to this extension. In keeping with Diamond's [1982] model, producers' receive production

¹²A paper that does explore the implications of the choice between market and non-market production in a micro model of money is Camera and Vesely [2004].

opportunities according to a Poisson process; parameter γ . For ease of exposition however, the production cost is incurred on entry to the production location. Again, to abstract from the complications caused by people refusing to participate in the economy, individuals who arrive at the production location must produce.¹³

As production takes time, steady-state finds a positive share of the population at the production location. This raises the issue of how the market matching rate, β , depends on the number of people, N , in the market. It is assumed that $\beta = \beta(N)$ where $\beta(\cdot)$ is positive, continuously differentiable and monotonic. Attention is restricted to cases where

$$\beta'(\cdot) \begin{cases} = 0 & \text{constant returns to scale (CRS)} \\ > 0 & \text{increasing returns to scale (IRS)} \end{cases}$$

A maintained component of the environment is *ex ante* heterogeneity with attention restricted to continuous distributions in production costs.¹⁴ Again, Nash bargaining implies a swap of goods when both parties to a meeting are producers. Middlemen in this environment remain at all times in the market. A prerequisite for being a middleman is that the asset value of that activity, V_m , exceeds that of the alternative which is to consume and move to the production location. When middlemen meet producers, Nash bargaining implies a 50/50 split of the surplus by dividing the good brought by the middleman. The constraint on the allocation implied by the fact that the producer does not want to eat his own good will never bind.¹⁵ As before,

¹³It can be shown that the equilibria explored here are robust to voluntary production over a positive proportion of the parameter space.

¹⁴Continuity of $F(\cdot)$ is imposed here to avoid mixed strategy equilibria and ease comparative static analysis.

¹⁵Nor does it affect the agreed allocation - a consequence of Nash's axiom of independence of irrelevant alternatives.

this is because no one would become a middleman in the first place if she cannot ensure that the agreements involve her walking off with the whole good brought by the producer. Thus,

$$rV_t(c) = \alpha\beta(N) \max\{u - c + V_p(c) - V_t(c), V_m - V_t(c)\} + \frac{1}{2}(1 - \alpha)\beta(N) [u - c + V_p(c) - V_t(c)] \quad (14)$$

$$rV_m = \frac{1}{2}\alpha\beta(N) \mathbf{E}_{\{c|u-c+V_p(c) \geq V_m\}} [u - c + V_p(c) - V_t(c)] \quad (15)$$

$$rV_p(c) = \gamma [V_t(c) - V_p(c)] \quad (16)$$

where $V_t(c)$ is the value to being a producer of type c in the market, $V_p(c)$ is the value to being a type c producer in the production location and α is the proportion of people in the market who are producers. Following the proof of Lemma 1, it is straightforward to show that if $u - c + V_p(c) > V_m$, $V_t(\cdot)$ is continuous and everywhere decreasing. Consequently, from (16), given α, N there exists a unique production cost, \hat{c} such that

$$u - c + V_p(c) \begin{cases} < \\ = \\ > \end{cases} V_m \text{ as } c \begin{cases} > \\ = \\ < \end{cases} \hat{c}.$$

Using this and suppressing arguments in $V_p(\cdot)$ and $V_t(\cdot)$ we have

$$V_t[2r + \beta(N)(1 + \alpha)] = \beta(N)(1 + \alpha)(u - c + V_p)$$

$$rV_m = \frac{\alpha\beta(N)}{2} \int_0^{\hat{c}} (u - c + V_p - V_t) \frac{dF(c)}{F(\hat{c})}$$

These lead to

$$(2r + \beta(N))(u - \hat{c}) = \frac{\alpha\beta(N)}{F(\hat{c})} \int_0^{\hat{c}} F(c) dc$$

In steady-state, the flow of individuals into and out of the production location have to be equal. That is

$$(1 - N)\gamma = \alpha\beta(N)N.$$

Middlemen represent $1 - F(\hat{c})$ of the total population. And, as they never leave the market, they represent $1 - \alpha$ share of the market population. So

$$(1 - \alpha)N = 1 - F(\hat{c}).$$

Thus,

$$\alpha = \frac{\gamma F(\hat{c})}{\beta(N)(1 - F(\hat{c})) + \gamma}.$$

Definition 3 *A Steady-state Market Equilibrium in the extended model, is a pair $\{\hat{c}, \hat{N}\}$, such that*

$$(2r + \beta(\hat{N})) \left[\beta(\hat{N})(1 - F(\hat{c})) + \gamma \right] (u - \hat{c}) = \beta(\hat{N})\gamma \int_0^{\hat{c}} F(c) dc \quad (17)$$

and

$$\beta(N)[\hat{N} - (1 - F(\hat{c}))] = \gamma [1 - \hat{N}] \quad (18)$$

4.1 Constant Returns To Scale: $\beta(N) = \beta$ for all $N > 0$

Proposition 4 *Market Equilibrium exists and is unique.*

Proof. Under CRS, the system (17), (18) is recursive. LHS of (17) is decreasing in \hat{c} on $[0, u]$ achieving 0 at $\hat{c} = u$. RHS of (17) is increasing in \hat{c} on $[0, u]$ achieving 0 at $\hat{c} = 0$. Then

$$\hat{N} = [(1 - F(\hat{c}))\beta + \gamma] / (\beta + \gamma). \quad (19)$$

■

Straightforward comparative static analysis reveals that \hat{c} increases with r . Middlemen are more prominent in economies with more patient populations. Again this reflects the requirement for middlemen to forgo current consumption in order to maintain their inventory. A consequence of this is that a more patient population means that at any point in time, more people are in the market (*i.e.* as r falls, \hat{N} increases).

As the arrival rate of production opportunities, γ , increases, \hat{c} falls. That is, as the expected length of a spell in the production location decreases more middlemen emerge. Shorter time to produce means that value to any meeting in the market increases - there are more rents to be captured. As γ can be viewed as a measure of technological progress, this result suggests that more middlemen will emerge in those economies that are better able to support them. From (19) the population in the market unambiguously increases with γ . As γ approaches infinity, the model reverts back to that with instantaneous production.

Another form of technological progress in this model is reductions in the production costs. As long as the fall in c is the same for everyone this is equivalent to an increase in u which leads to an increase in \hat{c} - a fall in the number of middlemen and an increase in the market population, \hat{N} .¹⁶ This reflects the increase in the opportunity cost of being a middleman. While a middleman's return from any meeting rises with u , she only gets a part of that increase. On the other hand, producers see the whole increase in u whenever they trade with other producers. However, it is simple to show that the change in \hat{c} is less than the change in u .¹⁷ The increased number of

¹⁶Recall that setting the infimum of the distribution of production costs to zero was a normalization.

¹⁷That is to say, if the exercise were to actually to lower everyone's production cost by the same amount, rather than increase u , \hat{c} would fall but by less than the amount that

producers off-sets the relative disadvantage felt by middlemen. The difference between this result and the previous one is that changes in γ affect everyone equally. Increases in u on the other hand, are more strongly felt by those producers with higher values of c .

The introduction of positive spell lengths in the production location, means that changes in β have ambiguous effects on \hat{c} . In particular, the sign of the derivative of \hat{c} with respect to β is equal to the sign of $\{\beta^2(1 - F(\hat{c})) - 2r\gamma\}$. That is, \hat{c} falls with β for low values of β (or high values of r, γ) and increases with β when meeting is already very frequent. As β approaches both 0 and infinity, \hat{c} converges to u , from below. Moreover, as

$$\frac{d\hat{c}}{d\beta} = 0 \Rightarrow \frac{d^2\hat{c}}{d\beta^2} > 0$$

\hat{c} is in fact quasi-convex in β . That is, there is a unique value, $\hat{\beta}$ of the market matching rate that minimizes \hat{c} . So $\hat{\beta}^2(1 - F(\hat{c})) = 2r\gamma$. As $1 - F(\hat{c})$ is always finite, in the limit as γ approaches infinity, and production time goes to zero, $\hat{\beta}$ approaches infinity. This result is consistent with the monotone relationship found between β and \hat{c} in the model with instantaneous production.

When β is relatively low, trading partners are hard to come by and few people are prepared to make the investment required to become a middleman; increasing β means more middlemen because the cost of investment becomes less severe. When the market meeting rate is already large, however, the gains from trade for any finite γ become relatively small and any further increase in β further diminishes the return to being a middleman.

The present value of steady-state equilibrium welfare is

$$W = \frac{\alpha\beta\hat{N}}{r} \int_0^{\hat{c}} (u - c) \frac{dF(c)}{F(\hat{c})} = \frac{\beta\gamma}{r(\beta + \gamma)} \int_0^{\hat{c}} (u - c) dF(c).$$

production costs had been reduced by.

Steady-state welfare under a ban on middlemen is¹⁸

$$W^n = \frac{\beta\gamma}{r(\beta + \gamma)} \int_0^{\hat{c}} (u - c) dF(c)$$

When production takes time, the transition from the steady-state with middlemen to the steady-state without middlemen is no longer trivial. As before, a ban on intermediation means that all middlemen consume their inventory and incur their production cost, but now the number of people in the market takes time to reach its steady-state value. During transition there is an extra inflow to the market from the block of individuals who entered the production location together and leave it gradually, as they find production opportunities. The population in the market therefore converges on $\gamma/(\beta + \gamma)$ from below.

Let $w(t)$ represent instantaneous flow welfare at date t during transition. If the ban occurs at time 0, it will be welfare improving if $w(0) \geq rW$ and $w'(\cdot)$ is positive. When the ban is implemented, all the individuals with $c \geq \hat{c}$ (the ex-middlemen) leave the market. Those who remain in the market are all producers with $c < \hat{c}$. So

$$w(0) = \beta N(0) \int_0^{\hat{c}} (u - c) \frac{dF(c)}{F(\hat{c})}$$

where $N(0)$ is the market population immediately after the ban. Clearly, $N(0) = \alpha \hat{N}$ so $w(0) = rW$. As time passes, the ex-middlemen begin to re-enter the market as producers. As this process leaves those who were already producers unaffected, their contribution to flow welfare remains constant. The ex-middlemen contribute an increasing amount to welfare as their

¹⁸Although an explicit notion of equilibrium under a ban on middlemen has not been defined, as no decisions are being made, it should be clear that behavior simply involves individuals producing searching and swapping goods.

number in the market increases overtime. So, at every point in the transition, flow welfare exceeds that of the *laissez-faire* equilibrium.

When production takes time, middlemen remaining in the market means they, increase the total number of market interactions that take place. They do not, however, improve welfare because the matching rate for productive people is unaffected by the number of middlemen.

4.2 Increasing returns to scale, $\beta'(N) > 0$

Here we investigate the possibility that if matching becomes more efficient when there are more people in the market, the existence of middlemen can be welfare enhancing.

Proposition 5 *There exists a market equilibrium under increasing returns to scale in the market meeting technology.*

Proof. Equations (17), (18) are a non-linear system in $\{\hat{c}, N\}$ on $[0, u] \times [0, 1]$. The graph of equation (18) is monotonically decreasing from $(1, \gamma/(\beta + \gamma))$ to $(0, 1)$. As $\beta(\cdot)$ is continuous, a direct application of the implicit function theorem is that the value of \hat{c} as implied by (17) is continuous in N . As N only enters (17) through its effect on β , we know from the previous discussion on comparative statics under CRS that the implied domain of \hat{c} is $[0, 1]$. The result then follows from the intermediate-value theorem for values of N between $\gamma/(\beta + \gamma)$ and 1. ■

This result says nothing about uniqueness. Although multiple steady-state equilibria have not been ruled out, none have been found.

In any steady-state equilibrium, utilitarian welfare is

$$W = \frac{\beta(\hat{N})\gamma}{r(\beta(\hat{N}) + \gamma)} \int_0^{\hat{c}} (u - c) dF(c).$$

Steady state welfare under a ban on middlemen is

$$W^n = \frac{\beta(N^n)\gamma}{r(\beta(N^n) + \gamma)} \int_0^{\bar{c}} (u - c) dF(c).$$

Here N^n is the number of people in the market when there are no middlemen.

That is, N^n solves

$$N^n = \frac{\gamma}{(\beta(N^n) + \gamma)}$$

In any equilibrium where $\hat{c} < \bar{c}$, $N > N^n$.

To evaluate the possible benefits of the *laissez-faire* economy over a ban on intermediation, it is necessary to consider the transition. During the transition to the no-middlemen steady-state, after the initial consumption of inventories, present value of welfare will track from W to W^n .

We know from the comparative statics in β in the CRS case that there exists some minimum value, \hat{c}' , of \hat{c} that does not depend on N nor the functional form of $\beta(\cdot)$. Now,

$$W > \frac{\beta(N)\gamma}{r(\beta(N) + \gamma)} \int_0^{\hat{c}'} (u - c) dF(c)$$

and we can always pick a functional form for β so that

$$\frac{\beta(N)\gamma}{r(\beta(N) + \gamma)} \int_0^{\hat{c}'} (u - c) dF(c) > W^n$$

In this case, welfare immediately after the banned middlemen consume their inventories must be strictly less than W . The difference will be increasing in the difference between $\beta(N)$ and $\beta(N^n)$. The welfare benefit of inventory consumption is given by

$$(1 - F(\hat{c})) \int_{\hat{c}}^{\bar{c}} (u - c) \frac{dF(c)}{(1 - F(\hat{c}))} = \int_{\hat{c}}^{\bar{c}} (u - c) dF(c).$$

So long as the functional form for β makes $\beta(N) - \beta(N^n)$ large enough, a ban on intermediation will not be welfare enhancing.

In this context, then, middleman activities can be beneficial to society. Individuals would, of course, prefer to avoid trading with them if possible. This provides a new, market based justification for the existence of middlemen. They have no special skills and their matching opportunities are the same as for anyone else in the economy but by remaining in the market, they increase the efficiency of the meeting technology.

5 Conclusion

This paper has analyzed a simple variant of the Diamond [1982] model. By adding divisibility of goods and *ex ante* heterogeneity in productive abilities, it has shown that middlemen can spontaneously emerge to take advantage of market frictions. They trade on the basis of specialization and yet, in the baseline economy, their activities are welfare reducing. As such the model provides a simple example of a large economy in which some individuals should be prohibited from acting in their own (private) best interests.

As market frictions diminish, the number of middlemen increases. This runs contrary to the popular belief that the internet will ring the death knell for market intermediaries. The reason it happens is that as trading opportunities get more frequent, middlemen see a faster return on their inventory investment. The emergence of intermediaries therefore increases with the economy's ability to support them.

If we allow for time taken in production and increasing returns to scale in the market meeting technology, middlemen have been shown to be potentially beneficial to society. Moreover, the way in which they contribute, by increasing the effectiveness of the meeting technology, is novel to this paper.

The paper also provides an example of commodity money in which the

carrier could obtain as much utility from its consumption as he obtains from the good he intends to buy with it. This happens because the possession of an intact good that he can consume strengthens the middleman's bargaining power in subsequent meetings.

In principle, it is possible to write down a model that incorporates all the forms of intermediary that have been documented in the literature and calibrate it to a particular industry. While this might resolve the empirical issue as to which role is most important, such a model would provide little by way of insight as to the wider implications of each role. The point of the paper has been to highlight the simplicity of the change to the familiar Diamond environment that creates the incentive for individuals to avoid producing and assess the implications of such activities for social welfare.

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