Inflation and welfare in retail markets: prior production and imperfectly directed search

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Abstract

This paper considers the effect of monetary policy and inflation on retail markets: goods are dated and produced prior to being retailed; buyers direct their search on price and general quality; buyers' match specific tastes are private information. Sellers set the same price for all buyers some of whom do not value the good highly enough to buy it. The market economy is typically inefficient as a social planner would have the good consumed. Under free entry of sellers, the Friedman rule is optimal policy. When the upper bound on the number of participating sellers binds, moderate levels of inflation can be welfare improving.

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1 Introduction

In retail markets (i.e. for food, shoes, computers etc.) prices are anonymous in that they are posted and non-negotiable. A second pervasive feature of such markets is that they do not clear at the posted prices - excess goods get marked down, sold wholesale or simply scrapped. This paper provides an analysis of how inflation affects the choices made by participants in retail markets. To capture the salient features of these markets in the model, goods are produced prior to being retailed at posted prices and are dated (e.g. by deterioration, fashion or technology). Furthermore, potential buyers do not know how much they like the good until they see it and their realized preference remains their private information.

In the model, inflation can affect welfare through two channels. The 'inflation tax' effect introduces a wedge between what a buyer gives up to acquire money and what the eventual holder gets for it. This effect is common to all of the existing literature on micro-founded models of money. It implies that the Friedman rule should be optimal policy. The second channel, which is novel to this paper, occurs because inflation ameliorates the private information problem. Having already produced their good, the sellers are keen to off-load it but they are committed to their posted price. By lowering the value of money, inflation makes buyers less picky thereby increasing the probability of a sale. With free-entry, sellers' welfare is driven down to the value of autarky. In this case then, all of the welfare effects due to the choice of the quality threshold at which they buy accrue to the buyers and inflation cannot improve welfare. But, when the number of sellers is fixed, the buyers' choice of quality threshold affects sellers' welfare too. Inflation, by increasing the probability of a sale, generates aggregate welfare gains that can dominate the losses due to the inflation tax.

The model has implications for the effectiveness of monetary policy. In the short run producers may not be able to respond to changes in demand through an increase in their number or perhaps in the quality of the goods they produce. In that case the version of the model with a fixed number of sellers may be most relevant and monetary policy that generates nominal interest rates above zero can be temporarily beneficial. The version of the model with free-entry of producers may, however, be more relevant in the long-run when producers can adjust their production lines and new producers can enter. The model also has implications for the relationship between inflation and the velocity of money. The information channel that affects the propensity for trade to occur means that velocity of money increases with inflation. This relationship has been documented in the data by (Lui et al 2010).

The framework is based on (Rocheteau and Wright 2005, henceforth RW). It incorporates two markets that individuals cycle through each period. One market is characterized by centralized trade and the other is a search market in which the opportunity to trade is probabilistic. Following RW there is ex ante heterogeneity in that some people, called sellers, can produce the good traded in the search market but have no interest in consuming it. Buyers, on the other hand, cannot produce the search market good but they do get utility from its consumption. This absence of a double coincidence of wants in the search market makes money essential. Buyers acquire money in the centralized market in which everyone can produce and consume. All individuals (buyers and sellers) have the same quasi-linear preferences for centralized market goods. This means that all buyers leave the market with the same cash holdings and all sellers leave with none. Individuals cycle through these markets forever.

The point of departure from the existing literature is that sellers are required to produce goods prior to market entry. Goods cannot be altered once made. Of course this requirement would have little relevance if suppliers could store goods indefinitely. Either because they rot (e.g. groceries), they are subject to fashion (e.g. clothes) or they are subject to technological obsolescence (e.g. computers), many of the goods we actually buy are dated. To capture this notion in its simplest form I assume that unsold goods perish at the end of the search market.

Two main variants of the model are considered. In the baseline model preferences of the buyers for the search good are fixed and identical. In the extended model, buyers' preferences are match specific. Both the baseline and the extended model incorporate what RW calls competitive search.[1] Specifically, buyers are able to direct their search according to the quality of the good for sale and its price. In the baseline model, therefore, there are 4 decision margins: buyers decide how much cash to bring, sellers decide on whether or not to participate, the quality of the good to produce, and its price. The extended model introduces a fifth decision margin: buyers set a reservation preference level required to purchase the good.[2]

In the baseline model, the Friedman rule, that equates the gross rate of money growth to the common discount factor, is shown to bring about efficiency on all 4 decision margins. Of course, goods do go to waste but

[1] See (Masters 2010) for a discussion of how the alternative market structures considered in RW affects outcomes in this environment.

[2] This paper uses the term "imperfectly directed" search to capture the notion that buyers may not know everything about a good before they go shopping for it. By contrast, (Menzio 2007) has a notion of "partially directed" search in which (equivalently) sellers cannot commit to prices but use them to signal the quality of their goods.

no social planner who is subject to the same search frictions as the market economy could do any better than monetary policy that follows the Friedman rule.

In the extended model, a buyer's true preference towards the good carried by any seller he meets is his private information. Sellers cannot, therefore, make the price contingent on the buyer's type; they post a single anonymous price. Of course, they can only recoup their cost of production when that price is strictly positive. As long as the distribution of buyers' preference shocks attaches positive probability to every neighborhood of zero (i.e. there is a chance that the buyer hardly likes the good) then buyers will sometimes reject goods in favor of holding on to their cash. Still, since the good will go to waste if not consumed, a social planner would have the seller hand the good over to the buyer no matter how little the latter likes it. Monetary policy cannot, therefore, be fully efficient. It will be shown, however, that as long as there is free-entry of sellers, the Friedman rule represents optimal monetary policy.

To check the robustness of this result, the analysis considers what happens if each of the decision margins are shut down. Making the quality of the good exogenous does not affect the optimality of the Friedman rule. Increasing inflation away from the Friedman rule in this case, still has buyers being less picky but now sellers reduce participation which reduces welfare more than the improved transaction rate increases it. When the free-entry margin is shut down, however, inflation causes buyers to become less picky and sellers produce lower quality goods. The net effect of changes in inflation at the Friedman rule is ambiguous. If the upper bound on market entry for sellers binds sufficiently, increases in inflation away from the Friedman rule can improve welfare.

To understand what is going on in the model, recall that there are two channels by which inflation can affect welfare. The inflation tax effect is always present. If the rate of trading increases the inflation tax is more heavily born by sellers but the overall welfare impact is unaffected. The information channel is always operative too but only impacts welfare when the set of sellers is fixed. This is because under free-entry, sellers are driven down to the value of autarky so that inflation cannot affect their welfare. Overall welfare is effectively that of the buyers. The buyers have no control over the impact of the inflation tax but the information channel relies on their choice of the utility threshold at which they buy goods. Because aggregate welfare and buyers' welfare coincide, their privately optimal choices are also socially optimal under the information constraints faced by the economy. When monetary policy moves away from the Friedman rule only the inflation tax effect has any bearing on welfare.

When the number of sellers is fixed, they share in the expected gains from trade. Now when buyers set the utility threshold at which they buy goods, they do not take account of the effect of their choice on sellers. As inflation reduces the value of money, the information problem becomes less severe and trade volume increases which benefits sellers. In this case, both the inflation tax and information channels have welfare implications. The overall impact of inflation depends on which effect dominates.

Other papers that have incorporated a requirement on sellers to produce prior to market entry are (Jafarey and Masters 2003) and (Dutu and Julien 2008). Both of these papers consider indivisible money. (Jafarey and Masters 2003) uses a random search environment with match specific preference shocks to address the relationship between prices and output under various sources of economic growth. (Dutu and Julien 2008) analyzes a directed search framework and is concerned with the existence of a monetary equilibrium. The baseline model of the current paper is essentially (Dutu and Julien 2008) with divisible money. The extended model of the current paper is essentially (Jafarey and Masters 2003) with divisible money and competitive search.

In the current paper it is the reduction in the value of money associated with inflation that ameliorates the private information problem. This reduction in value is often referred to as the 'hot potato' effect. In concurrent research two papers have attempted to capture the hot potato effect. (Lui et al 2010) point out that models of endogenous search intensity cannot generate the effect because the reduction of gains from trade that come from the inflation tax, make buyers less eager to trade at all.[3] They show, however, that if buyers face an extensive margin decision (i.e. whether to enter the market or not), inflation causes less buyers to acquire cash but, due to an increase in their matching probability, those that do enter trade faster. (Nosal 2010) constructs a RW style model with random matching and point-of-sale production in the decentralized market. Successful buyers in one period have to join a queue for entry in to the subsequent decentralized market. This generates an opportunity cost of trading. With match-specific preferences buyers put off purchasing goods they do not like enough because there are no bilateral gains from trade. Buyers do not take account of the impact of their refusal to trade on the other buyers. Inflation lessens the externality by making buyers less picky.

(Faig and Jerez 2005) is specifically focused on retail markets. Their

[3] (Li 1994,1995) is able to generate a hot-potato effect by endogenizing search intensity but, as (Lui et al 2010) point out, it emerges a consequence of the indivisibility of money in his environment.

model does not incorporate fiat money. Rather, they use book money but because preference shocks are match specific, sellers offer anonymous price/quantity schedules. Point of sale production means that there are no unsold goods. It also means that realized prices do depend on the realized preferences of buyers. (Faig and Jerez 2006) essentially incorporates fiat money into (Faig and Jerez 2005) again prices are ex ante anonymous but buyers types are revealed ex post by the price and quantity at which they choose to trade. In contrast, having goods produced before they are retailed means that some goods remain unsold. Moreover, in the current paper, everyone who buys the good pays the same price for it. This result is shown below to be robust to sellers offering lotteries (i.e. posting general mechanisms). Offering different probabilities of getting the good for different prices could target buyers with different preferences for the good but the ex post indivisibility of the good means that viable combinations do not affect the expected gains from trade. The optimal mechanism pools buyers into two groups - those who acquire the good with probability 1 and those who acquire the good with probability 0.

The remainder of the paper is organized as follows. Section 2 considers the baseline environment. Section 3 then incorporates match specific preferences. Section 4 looks at the role of inflation in the model in more detail by analyzing a version with functional form restrictions under which equilibrium is unique. Section 5 considers some alternative market structures including the possibility that sellers post general mechanisms. Section 6 concludes.

2 Baseline Model

2.1 Environment

Time is discrete and continues forever. Every period of time is divided into 2 subperiods: day and night. During the day there is a centralized frictionless market for a homogeneous and perfectly divisible good. In the night, trade occurs in a decentralized market characterized by search frictions. There are two types of individual in the model: those who consume the good produced in the night market and those who can produce the night market good. Following RW they will be referred to as buyers and sellers respectively. The measure of buyers is normalized to 1 while the measure of sellers is $\bar{n}$. All the buyers enter each night market but only a subset of the sellers do. In contrast to RW, for sellers to indicate a willingness to trade at night they must produce a good. In any period t, the measure of sellers, n_t, who enter the night market will be controlled by a free-entry condition. The day good is non-storable. The night good can be held by the seller for that night but cannot be stored through the ensuing day.

The net instantaneous utility of a buyer at date t is

$$U_t^b = v(x_t) - y_t + u(q_t),$$

where x_t is the quantity of the day good consumed, y_t is the quantity of day good produced and q_t is quality of the night good consumed. Both instantaneous utility functions u and v are strictly increasing and strictly concave. I also require that $u(0) = 0$, $u'(0) = \infty$ and that there exists an x^* such that $v'(x^*) = 1$. The function $v(.)$ is normalized so that $v(x^*) = x^*$.

The instantaneous utility of a seller at date t is

$$U_t^s = v(x_t) - y_t - c(q_t),$$

where q_t is now the quality of the good produced and $c(.)$ is the associated cost function. Sellers have to produce before they enter the night market. They cannot augment their production after the beginning of the night market which means that they are fully committed to their choice of q_t. I assume the cost function is strictly increasing, strictly convex and $c(0) = c'(0) = 0$. I also assume $\lim_{q\to\infty} c'(q) = \infty$. This implies a unique $\bar{q} > 0$ such that $u(\bar{q}) = c(\bar{q})$ and a unique q^* in $[0, \bar{q}]$ such that $u'(q^*) = c'(q^*)$. There is a common discount factor, $\beta < 1$ between periods so that the lifetime utility of an individual type $i = b, s$ is $\sum_{t=0}^{\infty} \beta^t U_t^i$.

The night market is characterized by trading frictions. Specifically, the measures of buyers and sellers who get a trading opportunity are both equal to $\alpha(n)$ where n is the measure of sellers who enter the market. I assume that $\alpha(n) \leq \min\{1, n\}$, $\alpha(0) = 0$, $\alpha'(n) > 0$, $\alpha''(n) < 0$ and $\lim_{n\to\infty} \alpha(n) = 1$. These largely reflect the requirement that the matching probability of the buyers, $\alpha(n)$, does not exceed 1. The matching probability of the sellers (with goods in hand) is $\alpha(n)/n$ which will also be less than 1 under these restrictions. Constant returns to scale in matching is a further desirable (and commonly assumed) feature of the underlying matching technology. Here, this amounts to $\alpha(n)/n$ decreasing in n or $\alpha(n) \geq \alpha'(n)n$. Given the previous assumptions, this is guaranteed if $\alpha'(0) = 1$ which will be imposed in the sequel. The matching function $\alpha(n) = \min\{1, n\}$ has a discontinuous derivative at $n = 1$ and is not, therefore, covered by the following analysis. Subsection 4.5 considers this case.

2.2 Efficiency in the baseline model

The Central Planner weights all individuals equally in the welfare function. (Money provides no utility so it does not feature in the Planner's problem.)

As recognized in RW, these models do not feature any transitional dynamics so the Planner can and will always choose a stationary path for consumption. Equal treatment implies that contingent on type (buyer or seller) everyone produces and consumes the same amount. The Planner is subject to the same trading frictions as the market but does not require *quid pro quo* for exchange to occur. As utility functions are strictly increasing and goods are perishable, all output brought to any trading opportunity will be consumed. Unlike in RW, however, the Planner cannot avoid output going to waste. The Planner maximizes

$$\begin{aligned} W(x,n,q;\bar{n}) &\equiv (v(x)-x)(1+\bar{n})+\alpha(n)u(q)-nc(q) \qquad (1)\\ \text{subject to } n &\leq \bar{n}. \end{aligned}$$

The first order conditions for an internal solution are

$$\begin{aligned} x &: v'(x_p)=1 \\ q &: \alpha(n_p)u'(q_p)-n_pc'(q_p)=0 \qquad (2)\\ n &: \alpha'(n_p)u(q_p)-c(q_p)=0 \qquad (3) \end{aligned}$$

The choice of $x_p = x^*$ is clearly optimal (by assumption) and independent of q and n. As $W(x^*,.,.;\bar{n})$ is not necessarily concave, the existence of n_p and q_p are more problematic.

Claim 1 *The optimal choices, n_p and q_p respectively of the number of active sellers and output per seller exist in* $(0,\bar{n}]\times(0,q^*)$.

Proof. This is special case of Claim 3 below. ■

The differentiability of $W(x,n,q;\bar{n})$ and the fact that the first order conditions are necessary for an interior solution means that whenever $n_p<\bar{n}$, (n_p,q_p) solves (2) and (3). Otherwise, q_p is the unique solution to

$$\alpha(\bar{n})u'(q_p)-\bar{n}c'(q_p)=0.$$

2.3 Baseline Model Market Economy

In the absence of a Central Planner, the coincidence of wants problem makes some form of money essential. Here, money is perfectly divisible and agents can hold any non-negative amount. The aggregate nominal money supply M_t grows at constant gross rate γ so that $M_{t+1} = \gamma M_t$. New money is injected (or withdrawn if $\gamma < 1$) by lump-sum transfers (taxes) in the day (centralized) market. Following RW I assume these transfers go only to buyers, but this is not essential for the results. What matters is that transfers do not depend on the choices individuals make. Also, attention is restricted to policies where $\gamma \geq \beta$. For $\gamma < \beta$ there is no equilibrium.

In the day market the price of goods is normalized to 1 at each date t, while the relative price of money is denoted ϕ_t. Let $z_t = \phi_t m_t$ be the real value of an amount of money m_t. I will focus throughout on steady-state allocations in which aggregate real variables are constant over time. This means that $\phi_{t+1} = \phi_t/\gamma$. It will be useful to use individual real money balances, z_t, as the individual's choice variable for money holding rather than m_t.

Once a seller chooses the quality, q, of the good she intends to bring to the night market and the *real* price, d, at which she intends to sell it, neither can be changed.[4] These price quality pairs (q, d) are observed by all buyers who use them as a basis for directing their search. Thus, a seller's choice of a pair, (q, d), opens a submarket for goods of quality q at price d to which other sellers and buyers may enter. If n represents the ratio of sellers to buyers in that submarket, then $\alpha(n)$ and $\alpha(n)/n$ will be respectively the matching probability of buyers and sellers in that submarket. Submarkets are therefore

[4] The nominal price of a good with real price d is d/ϕ_t.

indexed by $\omega \in \Omega = \mathbb{R}^3_+$ where $\omega = (d, q, n)$.

In the morning, sellers simultaneously announce (q, d). On the basis of this knowledge, buyers decide how much money they will need for the night market and trade in the day market accordingly. As the terms of trade are predetermined and there is an opportunity cost to holding money (foregone daytime consumption), buyers only bring enough with them to acquire the a good in the submarket they have chosen to enter; $z = d$. So that when sellers pick d they are effectively also choosing the cash holdings of anyone they meet. In the sequel I use $\omega = (z, q, n)$. As sellers have no use for money in the night market they enter with zero cash balances.

Let $V^i(\omega)$ be the value to entering submarket ω for $i = s, b$ (respectively the seller and the buyer). We have

$$\begin{aligned} V^b(\omega) &= \alpha(n)\left[u(q) + \beta W^b(0)\right] + \left[1 - \alpha(n)\right]\beta W^b\left(\frac{z}{\gamma}\right) \qquad (4) \\ V^s(\omega) &= \frac{\alpha(n)}{n}\beta W^s\left(\frac{z}{\gamma}\right) + \left[1 - \frac{\alpha(n)}{n}\right]\beta W^s(0) \end{aligned}$$

where $W^i(z)$, $i = s, b$ are the values to entering the day market with current period real money holding z (i.e. at the price level prevailing in the morning) for sellers and buyers respectively.[5] Thus

$$\begin{aligned} W^b(z) &= \max_{x,y,\hat{\omega}} \left\{v(x) - y + V^b(\hat{\omega})\right\} \qquad (5) \\ \text{subject to } \hat{z} + x &= z + T + y \end{aligned}$$

[5] The term z/γ enters the day market entry value functions. To see why, suppose someone holds m_t units of money at the end of period t. Then, let $z_t = \phi_t m_t$. Carrying this much cash into the next period corresponds to $\phi_{t+1} m_t$ in period $t+1$ real balances which, in steady-state, equals z_t/γ.

where T is the real transfer/tax and $\hat{\omega} \equiv (\hat{z}, \hat{q}, \hat{n})$. For sellers,

$$\begin{aligned} W^s(z) &= \max_{x,y,\hat{\omega}} \{v(x) - y + \max[V^s(\hat{\omega}) - c(\hat{q}), \beta W^s(0)]\} \qquad (6) \\ \text{subject to } x &= z + y \end{aligned}$$

where the third term in the maximand refers to the sellers' ability to decide between market participation and autarky.[6]

Definition 2 *A symmetric, competitive search equilibrium is a submarket, $\tilde{\omega} = (\tilde{z}, \tilde{q}, \tilde{n})$ such that given all other buyers and sellers enter $\tilde{\omega}$, then $\tilde{\omega}$ solves both the individual buyer's problem, (5), and the individual seller's problem, (6) subject to*

$$V^s(\hat{\omega}) - c(\hat{q}) = V^s(\tilde{\omega}) - c(\tilde{q}) \begin{cases} = \beta W^s(0) \text{ for } \tilde{n} \leq \bar{n} \\ \geq \beta W^s(0) \text{ for } \tilde{n} = \bar{n} \end{cases}$$

The restriction to symmetry here means that there is a unique market to which all buyers and sellers go. This choice is motivated by RW who provide the analysis for the more general case but then restrict attention to equilibria with a unique active market. As non-negativity of y, does not bind (by assumption) it is immediate from (5) and (6), that $W^i(z) = z + W^i(0)$ for $i = b, s$ and that the amount of money brought into the night market is independent of the amount brought into the day market that morning.

It is now well known (see RW, Masters [2009], or Rogerson et al [2005]) that competitive search equilibrium in this environment will be isomorphic to that in the "dual" economy. In the dual here, buyers will commit to and advertise both prices and goods qualities at which they are willing to trade

[6] Notice that I do not make any non-negativity restrictions on x or y. The literature (see RW) has established that only the restriction on y can bind and that appropriate parameter restrictions can always be made to avoid that possibility.

while sellers produce and search accordingly. This latter economy is preferred here for expositional purposes. The corresponding buyer's problem is then:

$$\begin{aligned} &\tilde{\omega} \in \arg\max_{\omega} \left\{ V^b(\omega) - z \right\} \\ &\text{subject to } V^s(\omega) - c(q) = V^s(\tilde{\omega}) - c(\tilde{q}) \\ &\text{where free-entry implies} \begin{cases} V^s(\tilde{\omega}) - c(\tilde{q}) = \beta W^s(0) \text{ for } \tilde{n} < \bar{n} \\ V^s(\tilde{\omega}) - c(\tilde{q}) \geq \beta W^s(0) \text{ for } \tilde{n} = \bar{n} \end{cases} \end{aligned} \tag{7}$$

Substituting in for the value functions and eliminating z using the constraint, reduces problem (7) to

$$(\tilde{n}, \tilde{q}) \in \arg \max_{(n,q)\in[0,\bar{n}]\times[0,\infty)} \left\{ \begin{array}{c} \alpha(n)u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta}\right)[\gamma - \beta] \\ +[\gamma - \beta + \alpha(n)\beta]\left[V^s(\tilde{\omega}) - c(\tilde{q}) - \beta W^s(0)\right] \\ +\beta W^b(0) \end{array} \right\} \tag{8}$$

The reason for recasting the problem should now be clear. Except for constant terms, problem (8) in the market economy and the Planner's problem, (1), coincide when $\gamma = \beta$. That is, the Friedman rule is efficient in this baseline economy. Of course goods carried by sellers who do not match still go to waste but a social planner subject to the same matching frictions could not do any better than the market economy does at the Friedman rule.

In general the objective function in problem (8) is not concave so that a unique solution is not assured.

Claim 3 *A solution, $(\tilde{n}, \tilde{q})$, to problem (8) exists within* $(0, \bar{n}] \times (0, q^*)$

Proof. See Appendix ■

The concavity of the objective function with respect to q means that if $\tilde{n} = \bar{n}$, the solution is unique. In that case $\tilde{q}$ solves,

$$\alpha^2(\bar{n})\beta u'(q) - \bar{n}\left[\gamma - \beta + \alpha(\bar{n})\beta\right] c'(q) = 0.$$

For any interior solution $(\tilde{n}, \tilde{q})$ to problem (8) we have,

$$\alpha^2(n)\beta u'(q) - n\left[\gamma - \beta + \alpha(n)\beta\right] c'(q) = 0 \qquad (9)$$

$$\alpha'(n)\alpha(n)\beta u(q) - \left[(1-\eta(n))(\gamma - \beta) + \alpha(n)\beta\right] c(q) = 0 \qquad (10)$$

where $\eta(n) \equiv \frac{\alpha'(n)n}{\alpha(n)}$ is the elasticity of the matching function with respect to the number of sellers in the submarket.

Equation (9) reflects the extent to which inflation distorts the choice of good quality. If $\gamma = \beta$ it reduces to the Planner's first order condition for q. Equation (10) comes from the free-entry of sellers. The first term represents the benefit of entry with a good of quality q. It reflects the contribution to the matching probability of buyers made by an entering seller which is partially captured by sellers in the price they get for the good. Free entry means that sellers equate the value of entry to the cost. The cost increases with the inflation rate, γ, because of losses due to the inflation tax.

Rearranging equations (9) and (10), dividing (9) by (10), and multiplying through by q yields

$$e_u(q) = \left(\frac{\eta(n)\left[\gamma - \beta + \alpha(n)\beta\right]}{(1-\eta(n))(\gamma - \beta) + \alpha(n)\beta} \right) e_c(q). \qquad (11)$$

Here e_u and e_c are elasticities of the utility and cost function respectively. It is straightforward to show that if $\eta(n)$ decreases monotonically so do the contents the parentheses in (11).[7] Consequently, a sufficient condition for the uniqueness of equilibrium is that the utility and cost functions be isoelastic

[7] The elasticity of the matching function cannot increase monotonically as $\eta(0) = 1$ and $\eta(n) \in [0, 1]$ for all n.

and $\eta'(n) < 0$.[8] Under the Friedman rule, $\gamma = \beta$, equation (11) reduces to

$$e_u(q) = \eta(n) e_c(q). \tag{12}$$

As the contents the parentheses in (11) are increasing in γ, money is not super-neutral. Increasing the rate of money growth increases the wedge between the elasticities of cost and utility. If they are isoelastic and $\eta'(n) < 0$, participation by sellers, n, will fall as the rate of money growth increases. It then follows from the second order conditions that if (q, n) is an optimum, q also decreases with γ.

3 Match-specific heterogeneity

This section incorporates the idea that search may be imperfectly directed in that all aspects of a good may not be known prior to seeing it. Thus, a buyer's instantaneous utility from consumption of a good of general quality q becomes $\varepsilon u(q)$ where $\varepsilon \sim G(.)$ is realized after the buyer and seller meet. The distribution $G(.)$ is assumed to be continuous on $(0, \bar{\varepsilon}]$ and differentiable with density, $g(.)$. The realized value of ε is the private information of the buyer. For this extended environment $\bar{q}$ is redefined such that $\bar{\varepsilon} u(\bar{q}) = c(\bar{q})$.

[8] Examples of functions that satisfy the requirements on $\alpha(n)$ are

$$\begin{aligned} \alpha(n) &= 1 - e^{-n} \\ \alpha(n) &= \frac{n}{(1 + n^{\rho})^{1/\rho}} \quad 0 < \rho < \infty \\ \alpha(n) &= \tanh(n) \end{aligned}$$

or any convex combination of any 2 of them. In all such cases $\eta(n)$ declines monotonically from 1 to 0.

3.1 Efficiency

As utility functions are strictly increasing, goods are perishable and $\varepsilon > 0$, whether or not the planner observes the realized value of ε is moot - all output brought to any trading opportunity should be consumed. The Planner's objective function becomes

$$W(x, n, q; \bar{n}) \equiv (v(x) - x)(1 + \bar{n}) + \alpha(n)\hat{\varepsilon}u(q) - nc(q)$$

where $\hat{\varepsilon}$ is the unconditional expected value of ε. The efficient outcome is identical to that of the baseline model with $u(q)$ replaced by $\hat{\varepsilon}u(q)$.

3.2 Market Economy

Borrowing from the preceding analysis, we know that as long as sellers set the same real price, z, buyers will bring that amount of money into the night market (or nothing). Let $\omega \equiv (z, q, n)$. The value to being a buyer in the night market is then

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon\left[\max\left\{\varepsilon u(q) + \beta W^b(0), \beta W^b\left(\frac{z}{\gamma}\right)\right\}\right] + (1-\alpha(n))\beta W^b\left(\frac{z}{\gamma}\right)$$

where $W^b(.)$ is the value function for buyers in the day market and is identical to that derived for the baseline model. This is because they choose whether to give up their cash for the good only after they meet the seller and realize the extent to which they want it. From earlier analysis we know that $W^b(z) = z + W^b(0)$ so

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon\left[\max\left\{\varepsilon u(q) - \beta\frac{z}{\gamma}, 0\right\}\right] + \beta W^b\left(\frac{z}{\gamma}\right)$$

Given, ω, let $\varepsilon_R = \beta z/\gamma u(q)$ which represents the reservation match specific preference shock for the buyer above which she will purchase the good and below which she will not. It follows that

$$V^b(\omega) = \alpha(n)u(q)S_G(\varepsilon_R) + \beta W^b\left(\frac{z}{\gamma}\right)$$

where $S_G(.)$ is the "surplus function" of distribution G.[9]

The value to being a seller in the night market is then,

$$V^s(\omega) = \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\beta W^s\left(\frac{z}{\gamma}\right) + \left(1 - \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\right)\beta W^s(0)$$

This is because the probability that a seller gets to trade in the night market is equal to the probability, $\frac{\alpha(n)}{n}$, he meets a buyer, times the probability, $1 - G(\varepsilon_R)$, with which the buyer is wiling to give up her cash for the good he carries. As $W^s(z) = z + W^s(0)$ we obtain

$$V^s(\omega) = \frac{\alpha(n)}{n\gamma}\left[1 - G(\varepsilon_R)\right]\beta z + \beta W^s(0)$$

Equilibrium is still described by Definition 2. Substituting for the value functions and eliminating z yields

$$(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R) \in \arg\max_{(n,q,\varepsilon_R)\in[0,\infty]\times[0,\infty)\times[0,\bar{\varepsilon}]} \left\{\left[\alpha(n)\beta S_G(\varepsilon_R) - (\gamma - \beta)\varepsilon_R\right]u(q)/\beta\right\} \tag{13}$$

$$\text{subject to:} \quad \frac{\alpha(n)}{n}[1 - G(\varepsilon_R)]\varepsilon_R u(q) - c(q) = X$$

where $X \equiv \frac{\alpha(\tilde{n})}{\tilde{n}}[1 - G(\tilde{\varepsilon}_R)]\tilde{\varepsilon}_R u(\tilde{q}) - c(\tilde{q})$ which is 0 under free-entry and positive when $\tilde{n} = \bar{n}$. Simple inspection of the constraint in problem (13) and the fact that $X \geq 0$, indicate that for any solution $\tilde{q} \leq \bar{q}$. The choice set

[9] That is,

$$S_G(\varepsilon_R) \equiv \int_{\varepsilon_R}^{\bar{\varepsilon}} \left[\varepsilon - \varepsilon_R\right] dG(\varepsilon) = \int_{\varepsilon_R}^{\bar{\varepsilon}} \left[1 - G(\varepsilon)\right] d\varepsilon$$

where the final equality comes from integration by parts.

is therefore compact and continuity of the objective function implies that a solution exists.[10] The first-order conditions of problem (13) imply

$$\frac{\varepsilon_R[1-G(\varepsilon_R)](1-\eta(n))}{S_G(\varepsilon_R)\eta(n)} = -\left\{\frac{\frac{\alpha(n)}{n}[1-G(\varepsilon_R)]\varepsilon_R u'(q) - c'(q)}{\left[\alpha(n)S_G(\varepsilon_R) - (\frac{\gamma-\beta}{\beta})\varepsilon_R\right] u'(q)}\right\} \tag{14}$$

and

$$\frac{\varepsilon_R[1-G(\varepsilon_R)](1-\eta(n))}{S_G(\varepsilon_R)\eta(n)} = \frac{\frac{\alpha(n)}{n}\left[1-G(\varepsilon_R) - g(\varepsilon_R)\varepsilon_R\right]}{\left[\alpha(n)(1-G(\varepsilon_R)) + \frac{\gamma-\beta}{\beta}\right]} \tag{15}$$

The LHS of equations (14) and (15) is the shadow price on the constraint. It is the ratio of the sellers' loss associated with one additional seller in a market to the buyers' gain due to the entry of that additional seller. The curly brackets in equation (14) contain the ratio of the sellers' to the buyers marginal benefit from an increase in q. The RHS of (15) is the ratio of the sellers' to the buyers marginal loss from an increase in ε_R. It is immediate from equation (14) that in any non-trivial equilibrium, $\varepsilon_R > 0$. A Social Planner, however, could choose $n = \tilde{n}$ and $q = \tilde{q}$ and achieve a higher degree of welfare by setting $\varepsilon_R = 0$ – the market equilibrium cannot be efficient.

3.3 Monetary Policy

The objective of monetary policy is to bring about the best social outcome achievable within the market economy through the manipulation of γ, the gross growth rate of the money supply. Thus we view $(\tilde{\omega}, \tilde{\varepsilon}_R)$ as a function

[10] In this model, depending on the form of $G(.)$, the possibility that $\tilde{n}$ and $\tilde{q}$, and hence $\tilde{z}$, are all zero cannot be ruled out.

of γ. The the contribution to welfare from night market activity is[11]

$$\tilde{W}_m(\gamma) \equiv \alpha(\tilde{n})[1 - G(\tilde{\varepsilon}_R)]\mathbb{E}_{\{\varepsilon|\varepsilon \geq \tilde{\varepsilon}_R\}}\varepsilon u(\tilde{q}) - \tilde{n}c(\tilde{q})$$

This is because the number of exchanges is equal to the measure of buyers, 1, times the probability they meet a buyer, $\alpha(n)$, times the probability they will find the good attractive enough to give up their cash for it. And, $\mathbb{E}_{\{\varepsilon|\varepsilon \geq \varepsilon_R\}}\varepsilon u(q)$ represents the expected utility of the buyer when trade occurs. Thus

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\left[S_G(\tilde{\varepsilon}_R) + (1 - G(\tilde{\varepsilon}_R))\tilde{\varepsilon}_R\right]u(\tilde{q}) - \tilde{n}c(\tilde{q}). \tag{16}$$

Using the constraint in problem (13) we obtain

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})S_G(\tilde{\varepsilon}_R)u(\tilde{q}) + \tilde{n}X \tag{17}$$

The optimal policy problem is to maximize $\tilde{W}_m(\gamma)$ over γ given $\tilde{n}$, $\tilde{q}$ and $\tilde{\varepsilon}_R$ solve problem (13). Under free-entry, $X = 0$ so when $\gamma = \beta$, the optimal policy problem is identical to maximizing the solution to problem (13) with respect to γ. By the envelope theorem, then

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = 0.$$

While the second order condition has not been verified, in all of the examples that follow, under free-entry moving away from the Friedman rule lowers $\tilde{W}_m(\gamma)$. The Friedman rule therefore implements optimal monetary policy under free-entry and inflation has only second-order welfare effects.

This result reflects the universality of the Friedman rule. In this model, buyers get to choose how much money to bring with them and make purchasing decisions based on private information which is not fully revealed in

[11] Since $x = x^*$ at every period, the day market contribution to welfare is not affected by monetary policy.

equilibrium. Sellers get to choose whether to enter the market or not and if they choose to enter, the quality of the good to bring with them. Still, optimal monetary policy follows a simple rule that sets the gross rate of money growth equal to the discount factor of the population. New to this extension of the baseline line environment however is the buyers' purchasing decision. We know that a social planner would prefer that goods change hands in every meeting but, because prices cannot be made contingent on a buyer's realized preference for the good, some trades do not occur.

Even though the information channel is active, it has no impact on welfare because with free-entry, sellers' welfare is unaffected by monetary policy. The social and private benefits to buyers of setting a reservation utility level are equal. Minimizing the inflation tax effect is the sole prerogative of policy.

When the population of participating sellers is fixed or the upper bound on the number of sellers binds (so that $n = \bar{n}$), equation (17) still applies. But, X is not fixed with respect to γ. In such cases, the Friedman rule may fail to implement optimal monetary policy. Further discussion of this possibility is provided in the next section.

4 Discussion

4.1 Interior solution (free-entry)

As long as the upper bound on n does not bind, there is an interior solution to problem (13) and equilibrium is characterized by equations (14), (15) and the free-entry condition,

$$\alpha(n)[1 - G(\varepsilon_R)]\varepsilon_R u(q) - nc(q) = 0. \tag{18}$$

Manipulating equation (14) and dividing it by (18) yields

$$\alpha(n)\beta S_G(\varepsilon_R)\left[e_u(q) - \eta(n)e_c(q)\right] = \varepsilon_R(\gamma - \beta)(1 - \eta(n))e_u(q) \tag{19}$$

Which means that at the Friedman rule, the elasticities equation (12) still holds. In addition at the Friedman rule, equation (15) implies

$$\frac{\eta(n)}{1-\eta(n)} = \frac{\varepsilon_R[1 - G(\varepsilon_R)]^2}{S_G(\varepsilon_R)\left[1 - G(\varepsilon_R) - g(\varepsilon_R)\varepsilon_R\right]}$$

Beyond this, equations (14), (15) and (18) have not been amenable to general analysis. Equation (19), however, points to an obvious simplification.

Claim 4 *If $u(.)$ and $c(.)$ are isoelastic, $G(.)$ is uniform on $[0,1]$ and $\eta'(n) < 0$, $\tilde{\varepsilon}_R$ is decreasing and $\tilde{n}$ is increasing in γ at the Friedman Rule.*

Proof. See Appendix ■

Increased inflation lowers the reservation utility at which buyers will part with their money. Essentially, because holding on to money is costly due to inflation, buyers are less picky about what to purchase. Both the effect on $\tilde{\varepsilon}_R$ and $\tilde{n}$ taken alone should therefore increase overall welfare. However, it is straightforward to verify that the effect of increased inflation on $\tilde{q}$ is sufficiently negative that it exactly offsets the welfare gains through $\tilde{\varepsilon}_R$ and $\tilde{n}$.

4.2 Exogenous q

Given the foregoing, an obvious question is: if we shut down the quality margin will increased inflation close to the Friedman rule improve welfare? If $q = \hat{q}$ is exogenous, sellers simply enter with a good of quality $\hat{q}$ or stay home. Of course the price, z, is still endogenous along with n and ε_R.

Claim 5 *With q exogenous and uniform $G(.)$, both $\tilde{\varepsilon}_R$ and $\tilde{n}$ decrease with γ at the Friedman Rule. The Friedman Rule continues to be optimal policy.*

Proof. See Appendix ■

With q held fixed, inflation has the opposite effect on sellers' propensity to enter the market. When the quality of the good is a choice variable, sellers reduce the quality of their goods due to the lower value of money but increase participation because the chance of trading increases. Here, although buyers get less picky, sellers reduce participation because money is less valued and they cannot adjust the quality of the good to be sold.

4.3 Exogenous n

When there is no free-entry, competition for buyers still prevails across potential markets. However, as we seek a symmetric (single market) equilibrium, $\tilde{n} = \bar{n}$. The goods quality, q, the reservation utility, ε_R, and the price, z, remain endogenous. The equilibrium conditions are therefore (14) and (15) with $n = \bar{n}$.

Claim 6 *When the number of sellers is fixed and $G(.)$ is uniform, both $\tilde{\varepsilon}_R$ and $\tilde{q}$ are decreasing with γ at the Friedman Rule. The Friedman rule may not be optimal policy.*

Proof. See Appendix ■

Simulations indicate that when $\bar{n}$ is sufficiently fewer than the number of sellers that would participate under free-entry, welfare increases with inflation.[12] When the upper bound on n binds, sellers obtain some of the gains

[12] For example if $\alpha(n) = 1 - e^{-n}$, $u(q) = 4q^{\frac{1}{2}}$, $c(q) = q^2$, $\beta_d = 0.95$, and $\beta_n = 0.96$ then at the Friedman rule, $\gamma = \beta$, with free entry, $\tilde{n} = 2.337$, $\tilde{q} = 0.296$, $\tilde{z} = 0.258$, $\tilde{\varepsilon}_R = 0.125$.

from trade but also feel the impact of the private information problem. Inflation ameliorates the private information problem by making buyers less picky. The overall impact of inflation on welfare depends on which effect dominates.

4.4 Exogenous n and q

If $n = \bar{n}$ and $q = \hat{q}$, equilibrium is characterized by the first-order conditions of problem (13) with respect to n and ε_R. An immediate implication is that $\tilde{\varepsilon}_R$ solves equation (15) with $n = \bar{n}$. If G is uniform on $(0, 1]$,

$$\frac{d\tilde{\varepsilon}_R}{d\gamma} = \frac{-2\tilde{\varepsilon}_R(1-\eta(\bar{n}))}{\alpha(\bar{n})\beta\left[\eta(\bar{n}) + 2(1-2\tilde{\varepsilon}_R)\right] + 2(1-\eta(\bar{n}))(\gamma-\beta)} < 0. \qquad (20)$$

When the population of sellers and the quality of goods are fixed, inflation makes buyers less picky. Which means, from equation (25), that welfare increases with inflation. The effect of inflation on the real price of night market goods, z, can be positive or negative depending on the magnitudes of $\alpha(\bar{n})$ and $\eta(\bar{n})$. What equation (20) tells us is that z can never fall sufficiently that buyers get pickier as inflation increases.

4.5 Absence of matching frictions

The foregoing analysis only considers environments in which even those on the short side of the market meet a potential trading partner with probability less than one. In the absence of matching frictions, $\alpha(n)$ becomes $\min\{n, 1\}$.[13]

If we now set $\bar{n} = 0.2337$, we have $\tilde{n} = 0.2337$ (the upper bound on n binds), $\tilde{q} = 0.381$, $\tilde{z} = 1.042$, $\tilde{\varepsilon}_R = 0.444$. And, the elasticity of welfare with respect to γ (at the Friedman rule), is 0.119.

[13] This corresponds to the matching function $\alpha(n) = n/(1 + n^\rho)^{1/\rho}$ as ρ approaches ∞. This matching function was first used by (den Haan et al 2000). (Stevens 2007) provides

This case was excluded earlier because of its non-differentiability at $n = 1$. This is however, an important case to consider because many people may not believe matching frictions to be relevant in retail markets.

First, consider what happens if the total number of potential sellers, $\bar{n} > 1$. From problem (13) any candidate equilibrium $(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R)$ in which $\bar{n} \geq \tilde{n} > 1$, must, solve

$$
\begin{aligned}
(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R) \in \arg \max_{(n,q,\varepsilon_R) \in [0,\bar{n}] \times [0,\infty) \times [0,\bar{\varepsilon}]} \{ [\beta S_G(\varepsilon_R) - (\gamma - \beta)\varepsilon_R] \, u(q)/\beta \} \\
\text{subject to:} \quad \frac{1}{n}[1 - G(\varepsilon_R)]\varepsilon_R u(q) - c(q) = 0
\end{aligned}
$$

Lower values of n make sellers better off while leaving buyers unaffected. This means that markets in which the expected number of sellers per buyer exceeds 1 cannot be active; $\tilde{n} = 1$. Thus, for $\bar{n} \geq 1$ the free-entry condition for sellers applies and the earlier efficiency results go through - monetary policy cannot be fully efficient but the Friedman Rule is optimal policy.

When $\bar{n} < 1$ we have,

$$
\begin{aligned}
(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R) \in \arg \max_{(n,q,\varepsilon_R) \in [0,\bar{n}] \times [0,\infty) \times [0,\bar{\varepsilon}]} \{ [n\beta S_G(\varepsilon_R) - (\gamma - \beta)\varepsilon_R] \, u(q)/\beta \} \\
\text{subject to:} \quad [1 - G(\varepsilon_R)]\varepsilon_R u(q) - c(q) = [1 - G(\tilde{\varepsilon}_R)]\tilde{\varepsilon}_R u(\tilde{q}) - c(\tilde{q})
\end{aligned}
$$

In any candidate symmetric equilibrium for which $\tilde{n} < 1$, higher values of n make buyers better off while leaving sellers unaffected. Buyers will continually look for markets in which $n = 1$ and sellers will freely enter those markets. Integrating across all active markets, however, has to imply that on average $\tilde{n} = \bar{n}$. The upshot is that no symmetric equilibrium exists. Instead there are two active markets, one in which buyers bring no money and sellers refuse to enter and one with $n = 1$ where trade takes place. Buyers are driven

microfoundations.

down to their value of autarky. This means that in the market with trade $[\beta S_G(\varepsilon_R) - (\gamma - \beta)\varepsilon_R]\, u(q) = 0$. At the Friedman Rule trade shuts down; $\varepsilon_R = 1$ and $q = 0$. Away from the Friedman rule, sellers can induce buyers to enter the trading market by offering some return on money which in equilibrium equals that of holding onto it for the next day market. When $\gamma = \beta$, however, competition among the sellers prevents them from offering a return that exceeds the buyer's discount rate. Moving away from the Friedman rule will improve welfare as trade begins to takes place.

5 Alternative market structures

5.1 Lucky bags

The most straightforward institutional arrangement that would return the environment to essentially that of the baseline model is to make buyers pay for the good before observing it. In this arrangement, the general quality of the good is still universally known but sellers require payment before they allow the buyer to realize his match-specific taste shock. Hotwire.com currently uses a similar approach to sell hotel rooms.

5.2 Market makers

A common device used to motivate competitive search (see Moen 1997) are "market makers" who announce the set of submarkets that will be open that night. (Faig and Huangfu 2007) point out that in such an environment there is nothing to prevent market makers from charging entry fees. Moreover, because entry fees are paid for sure, they reduce the need for buyers to carry idle cash balances. Their paper addresses this issue in the RW model. It incorporates competitive entry of market makers, so that their profits are driven to zero and entry fees are simply transfers from buyers to sellers. In the competitive search equilibrium with market makers, they are shown to completely eliminate the transaction price. Furthermore, the market maker equilibrium is always at least as efficient as the competitive search equilibrium without market makers. In the current paper, because they reduce transactions prices, entry fees can also help to address the inefficiency generated by private information.

To reduce the extent of exposition, attention is restricted to configurations in which the free-entry margin is active (i.e. $\tilde{n} < \bar{n}$). We know that the Friedman rule represents optimal policy in this case so any further welfare gains will be attributable to the entrance fees. Let Z_i, $i = b, s$ be the real entrance fees for buyers and sellers respectively. A typical element of Ω, the set of all potential submarkets, is now $\omega = (Z_b, Z_s, z, q, n)$. Here, z represents the total real cash balances carried by buyers into the night market. So the transaction price is $z - Z_{b.}$ For a buyer who chooses to enter submarket ω,

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon \left[\max\left\{\varepsilon u(q) + \beta W^b(0), \beta W^b\left(\frac{z - Z_b}{\gamma}\right)\right\}\right] + (1 - \alpha(n))\beta W^b\left(\frac{z - Z_b}{\gamma}\right)$$

This implies

$$V^b(\omega) = \left[\alpha(n)S_G(\varepsilon_R) - \varepsilon_R\right]u(q) + \beta W^b\left(0\right)$$

where now $\varepsilon_R = \beta(z - Z_b)/\gamma u(q)$.

For a seller in submarket ω, after recognizing that competition between market makers drives their profit to zero (so $Z_s = -Z_b/n$) we have,

$$V^s(\omega) = \frac{\beta}{n\gamma}\left[\alpha(n)(1 - G(\varepsilon_R))\left(z - Z_b\right) + Z_b\right] + \beta W^s(0)$$

Equilibrium is characterized by the solution to

$$\begin{gathered}
\tilde{\omega} \in \arg\max_{\omega} \left\{ V^b(\omega) - z \right\} \\
\text{subject to: } V^s(\omega) - c(q) = \beta W^s(0) \qquad (21) \\
\gamma u(q)\varepsilon_R = \beta(z - Z_b) \\
\varepsilon_R \geq 0
\end{gathered}$$

Claim 7 *Market makers set entrance fees such that transaction price is zero. The Friedman Rule is Efficient.*

Proof. See Appendix ■

Thus the result of (Faig and Huangfu 2007) extends to the environment where preferences are match specific. By forcing buyers to pay on entry to the market, market makers solve the private information problem. Moreover, away from the Friedman rule welfare in the market maker model is higher than under lucky bags. Having to pay Z_b for sure rather than z only when a seller is met eliminates the carrying of idle cash balances.

5.3 Lotteries

The model of Sections 2 and 3 restrict attention to non-probabilistic terms of trade. It should be clear that when every meeting leads to a transaction as in the model of Section 2, there can be no advantage in offering lotteries. However, when some buyers refuse to transact at the posted price because they do not value the good sufficiently, it is conceivable that such an individual will buy a chance to consume at a sufficiently low price. This turns out not to be true.

Claim 8 *If sellers can post lotteries they offer a two-point price distribution. The first offers the good with zero probability and zero transfer of money. The second offers the good with probability one associated with a transfer, z, that coincides with the equilibrium price from Section 3*

Proof. See Appendix ■

So, even if sellers are permitted to advertise a menu of lotteries the realized terms of trade are unaffected from those identified in Section 3. This result should be seen in contrast to (Faig and Jerez 2006) where, despite ex ante anonymous pricing, a buyer's realized preference for the good is revealed ex post. This is because in their model, production occurs at the point of sale and the price schedule offered means that buyers with a higher preference for the good will purchase more of it. Here though, the quality (or size) of the good is fixed ex ante, there is no way that a seller can set a schedule that offers a different price for each type of buyer.

6 Conclusion

This paper provides insight into the effect of monetary policy and inflation on participants in retail markets. It provides an analysis of a model that has three features that add realism to those considered in the literature. These are that goods are produced prior to being retailed, that goods are dated, and that buyers cannot base their choice over where to shop on every aspect of the good they wish to purchase. As the realized preference for a good is a buyer's private information, a market economy is necessarily inefficient. Sellers need to set prices that allow them to recoup their costs but those prices can turn out to be too high for those buyers who do not value the good highly enough. In this context, the Friedman rule represents optimal policy as long as there is free-entry of sellers. If there is an upper bound on the number of sellers which binds sufficiently, moderate levels of inflation can be welfare improving.

7 Appendix

7.1 Proof of Claim 3

Define $\Phi(n,q)$ as

$$\Phi(n,q) = \alpha(n)u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta}\right)[\gamma - \beta].$$

Continuity of $\Phi(n,q)$ immediately implies that it achieves a maximum on $[0,\bar{n}] \times [0,q^*]$. So we have to show that (i) this maximum must also solve problem (8) (ii) it cannot exist on any boundary except $n = \bar{n}$.

Assertion (i) follows from the strict concavity of $\Phi(n,q)$ with respect to q and the fact that $\alpha(n) < n$ for any $n > 0$ means that $\left.\frac{\partial \Phi}{\partial q}\right|_{q=q^*} < 0$. So for any $n \in [0,\bar{n}]$, and $q > q^*$, $\Phi(n,q)$ can always be increased by reducing q to be less than q^*.

Assertion (ii) follows because

$$\begin{aligned}
\lim_{q\to 0} \Phi(n,q) &= 0 \text{ for all } n \geq 0 \\
\lim_{q\to 0} \frac{\partial \Phi(n,q)}{\partial q} &= \lim_{q\to 0}\left\{\alpha(n)u'(q) - nc'(q) - \left(\frac{nc'(q)}{\alpha(n)\beta}\right)[\gamma - \beta]\right\} > 0 \text{ for all } n > 0
\end{aligned}$$

This means that $\Phi(n,q)$ achieves some strictly positive value for some $q > 0$ ruling out the portion of the q axis in the interval $(0,\bar{n}]$. Now, for all $n > 0$ we know that $\alpha(n) < n$ so $\left.\frac{\partial \Phi(n,q)}{\partial q}\right|_{q=q^*} < 0$ and $\Phi(n,q)$ is strictly concave in q. This rules out the portion of $q = q^*$ in the interval $(0,\bar{n}]$.

As $\lim_{n\to 0} n/\alpha(n) = 1/\alpha'(n) = 1$,

$$\lim_{n\to 0}\Phi(n,q) = -c(q)\left(\frac{\gamma-\beta}{\beta}\right)$$

so $\Phi(n,q)\leq 0$ for $n=0$ and $q\in[0,q^*]$. This means that $\Phi(n,q)$ cannot attain a maximum on the n axis.

7.2 Proof of Claim 4

If $u(.)$ and $c(.)$ have elasticities e_u and e_c respectively then (15), (18) and (19) become

$$2\varepsilon_R(1-\eta(n))[\gamma-\beta+\alpha(n)\beta(1-\varepsilon_R)]-\alpha(n)\beta\eta(n)(1-\varepsilon_R)(1-2\varepsilon_R)=0 \quad (22)$$

$$\alpha(n)[1-\varepsilon_R]\varepsilon_R u(q)-nc(q)=0 \quad (23)$$

$$\alpha(n)\beta\eta(n)(1-\varepsilon_R)^2[e_u-\eta(n)e_c]-2\varepsilon_R\left(\gamma-\beta\right)(1-\eta(n))e_u=0 \quad (24)$$

The system is block recursive. Equations (22) and (24) give n and ε_R. Then, q can be obtained from (23). To obtain comparative statics at the Friedman rule we differentiate the system (22) and (24) and then impose $\gamma=\beta$. Under this restriction, (22) and (24) reduce to $\varepsilon_R=\eta(n)/2$ and $e_u=\eta(n)e_c$ respectively which means $(\tilde{n},\tilde{q},\tilde{\varepsilon}_R)$ is unique and

$$\begin{aligned}\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} &= -\frac{\eta(\tilde{n})(2+\eta(\tilde{n}))(1-\eta(\tilde{n}))}{\alpha(\tilde{n})\beta(\eta(\tilde{n})-2)^2}<0\\ \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} &= -\frac{4(1-\eta(\tilde{n}))\eta^2(\tilde{n})}{\alpha(\tilde{n})\beta\eta'(\tilde{n})(\eta(\tilde{n})-2)^2}>0\end{aligned}$$

7.3 Proof of Claim 5

Under uniform $G(.)$, It is straightforward to show that equilibrium conditions are

$$\begin{aligned} 2\varepsilon_R(1-\eta(n))[\gamma-\beta+\alpha(n)\beta(1-\varepsilon_R)]-\alpha(n)\beta\eta(n)(1-\varepsilon_R)(1-2\varepsilon_R) &= 0 \\ \alpha(n)[1-\varepsilon_R]\varepsilon_R u(\bar{q})-nc(\hat{q}) &= 0 \end{aligned}$$

Which imply

$$\begin{aligned} \left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} &= \frac{\eta^2(\tilde{n})(1-\eta(\tilde{n}))}{\alpha(\tilde{n})\beta\left[2\eta'(\tilde{n})\tilde{n}-\eta(\tilde{n})(2-\eta(\tilde{n}))\right]}<0 \\ \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} &= -\frac{4\tilde{n}(1-\eta(\tilde{n}))\eta(\tilde{n})}{\alpha(\tilde{n})\beta(2-\eta(\tilde{n}))\left[2\eta'(\tilde{n})\tilde{n}-\eta(\tilde{n})(2-\eta(\tilde{n}))\right]}<0. \end{aligned}$$

Then,

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta}=\alpha'(\tilde{n})(1-\tilde{\varepsilon}_R)^2\left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta}-2\alpha(\tilde{n})(1-\tilde{\varepsilon}_R)\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta}=0$$

7.4 Proof of Claim 6

Differentiating the system (14) and (15) under uniform $G(.)$ and evaluating at $\gamma=\beta$ yields

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta}=\frac{-\eta(\bar{n})(1-\eta(\bar{n}))}{\alpha(\bar{n})\beta(2-\eta(\bar{n}))}<0$$

$$\left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta}=\frac{-4(1-\eta(\bar{n}))(3-\eta(\bar{n}))u'(\tilde{q})c'(\tilde{q})}{\alpha(\bar{n})\beta(2-\eta(\bar{n}))^2\left[u'(\tilde{q})c''(\tilde{q})-u''(\tilde{q})c'(\tilde{q})\right]}<0.$$

Again, inflation makes buyers less picky even though sellers reduce the quality of their good.

Under uniform G, with $n=\bar{n}$,

$$\tilde{W}_m(\gamma)=\frac{1}{2}\alpha(\bar{n})\left[1-\tilde{\varepsilon}_R^2\right]u(\tilde{q})-\bar{n}c(\tilde{q}). \tag{25}$$

So that at the Friedman rule, after substituting for $\frac{d\tilde{\varepsilon}_R}{d\gamma}$ and $\frac{d\tilde{q}}{d\gamma}$,

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = \frac{\eta(\bar{n})(1-\eta(\bar{n}))}{\beta(2-\eta(\bar{n}))}\left[\eta(\bar{n})u(\tilde{q}) - \frac{(3-\eta(\bar{n}))c'(\tilde{q})\left(u'(\tilde{q})\right)^2}{u'(\tilde{q})c''(\tilde{q})-u''(\tilde{q})c'(\tilde{q})}\right].$$

The sign of which is ambiguous.

7.5 Proof of Claim 7

After substituting for the value functions and eliminating Z_b, the appropriate Lagrangian is

$$\begin{aligned}\mathcal{L} = &\left[\alpha(n)S_G(\varepsilon_R) - \varepsilon_R\right]u(q) - z \\ &+ \mu\left[\beta z - \gamma u(q)\varepsilon_R\left\{1-\alpha(n)(1-G(\varepsilon_R))\right\}\right] - \mu\gamma n c(q) + \lambda\varepsilon_R\end{aligned}$$

where μ and λ are the co-state variables.

The first-order condition for z implies $\mu = \beta^{-1}$. Which means the first-order condition for ε_R reduces to

$$\frac{u(q)}{\beta}\left\{-\gamma\varepsilon_R\alpha(n)g(\varepsilon_R) - \left[1-\alpha(n)(1-G(\varepsilon_R))\right](\gamma-\beta)\right\} + \lambda = 0. \tag{26}$$

Suppose the solution for $\tilde{\varepsilon}_R > 0$. Then, the contents of the curly brackets in equation (26) are strictly negative which implies $\lambda > 0$. Complementary slackness, however, requires that $\lambda\varepsilon_R = 0$ – the only viable solution is $\tilde{\varepsilon}_R = 0$. This means that $Z_b = z$ and the transaction price is zero. The remaining equilibrium conditions are,

$$\begin{aligned}\beta z - \gamma n c(q) &= 0 \\ \alpha'(n)\hat{\varepsilon}u(q) - \frac{\gamma}{\beta}c(q) &= 0 \\ \alpha(n)\hat{\varepsilon}u'(q) - \frac{\gamma}{\beta}nc'(q) &= 0.\end{aligned}$$

These clearly coincide with the Planner's optimality conditions at the Friedman rule.

7.6 Proof of Claim 8

Due to the revelation principle we can restrict attention to sellers advertising incentive compatible and individually rational direct revelation mechanisms. Let $(\pi(\varepsilon_j), \tau(\varepsilon_j))$ represent the terms of trade contract intended for a buyer whose realized preference shock is ε_j. Here $\pi(\varepsilon_j) \in [0,1]$ is the probability with which the buyer gets the good having paid $\tau(\varepsilon_j) \in [0, z]$ in real terms for the chance to do so. The essential problem faced by the buyer is,

$$\varepsilon_i = \arg\max_{\varepsilon_j} \pi(\varepsilon_j)\varepsilon_i u - \beta\tau(\varepsilon_j)/\gamma$$

where u replaces $u(q)$ for some given q. Incentive compatibility will require that

$$\pi'(\varepsilon_i)\varepsilon_i u = \beta\tau'(\varepsilon_i)/\gamma \quad \text{and} \quad \pi'(\varepsilon_j) > 0.$$

Let

$$\xi(\varepsilon_i) = \pi(\varepsilon_i)\varepsilon_i u - \beta\tau(\varepsilon_i)/\gamma.$$

Individual rationality will require that $\xi(\varepsilon_i) \geq 0$. Now,

$$\xi'(\varepsilon_i) = \pi'(\varepsilon_i)\varepsilon_i u + \pi(\varepsilon_i)u - \beta\tau'(\varepsilon_i)/\gamma = \pi(\varepsilon_i)u$$

where the last equality comes from incentive compatibility. Taking z as given for now, the sellers problem is,

$$\max_{(\pi(.),\tau(.))} \int_0^{\bar{\varepsilon}} \tau(\varepsilon)dG(\varepsilon) = \max_{\pi(.)\in[0,1]} \frac{\gamma}{\beta} \int_0^{\bar{\varepsilon}} \pi(\varepsilon)\varepsilon u - \xi(\varepsilon)dG(\varepsilon)$$

subject to $\xi'(\varepsilon) = \pi(\varepsilon)u$ and $\tau(\varepsilon) \leq z$. The linearity of the objective function with respect to π implies a bang-bang solution.

Now, define $\hat{\varepsilon}$ as the least solution to $\varepsilon g(\varepsilon) = 1 - G(\varepsilon)$. It is straightforward to verify that if $\hat{\varepsilon}u\gamma \geq \beta z$, then

$$(\pi(\varepsilon), \tau(\varepsilon)) = \begin{cases} (0,0) & \text{for} \quad \varepsilon \in \left[0, \frac{\beta z}{u\gamma}\right) \\ (1, z) & \text{for} \quad \varepsilon \in \left[\frac{\beta z}{u\gamma}, \bar{\varepsilon}\right] \end{cases}$$

If $\hat{\varepsilon}u\gamma < \beta z$ then

$$(\pi(\varepsilon), \tau(\varepsilon)) = \begin{cases} (0,0) & \text{for} \quad \varepsilon \in [0, \hat{\varepsilon}) \\ (1, \frac{\hat{\varepsilon}u\gamma}{\beta}) & \text{for} \quad \varepsilon \in [\hat{\varepsilon}, \bar{\varepsilon}] \end{cases}$$

In the latter case, however, we know that a buyer would not bring more money that she might need to purchase the good. That is, z would be equal to $\hat{\varepsilon}u\gamma/\beta$. In the general model, z, q, n and $(\pi(\varepsilon), \tau(\varepsilon))$ will be chosen simultaneously but, as they are all advertised, competitive search equilibrium values are taken as given.

8 References

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