Money in a model of prior production and imperfectly directed search

Adrian Masters

September 2010

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
    - Goods are typically made before the seller meets the eventual buyer
    - Goods are dated (e.g. by deterioration, fashion, technology)

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer
  - Goods are dated (e.g. by deterioration, fashion, technology)
  - How much a buyer likes a good is his private information.

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
    - Goods are typically made before the seller meets the eventual buyer
    - Goods are dated (e.g. by deterioration, fashion, technology)
    - How much a buyer likes a good is his private information.
- Inflation tax

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer
  - Goods are dated (e.g. by deterioration, fashion, technology)
  - How much a buyer likes a good is his private information.
- Inflation tax
  - The difference between what money costs the buyer and what a sellers get for it

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer
  - Goods are dated (e.g. by deterioration, fashion, technology)
  - How much a buyer likes a good is his private information.
- Inflation tax
  - The difference between what money costs the buyer and what a sellers get for it
  - Present regardless of trading probability

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer
  - Goods are dated (e.g. by deterioration, fashion, technology)
  - How much a buyer likes a good is his private information.
- Inflation tax
  - The difference between what money costs the buyer and what a sellers get for it
  - Present regardless of trading probability
- Hot potato effect

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer
  - Goods are dated (e.g. by deterioration, fashion, technology)
  - How much a buyer likes a good is his private information.
- Inflation tax
  - The difference between what money costs the buyer and what a sellers get for it
  - Present regardless of trading probability
- Hot potato effect
  - The propensity to off-load cash more quickly if inflation increases

MOTIVATION

- The aim is to assess the welfare effects of inflation in retail markets
- In retail markets:
  - Goods are typically made before the seller meets the eventual buyer
  - Goods are dated (e.g. by deterioration, fashion, technology)
  - How much a buyer likes a good is his private information.
- Inflation tax
  - The difference between what money costs the buyer and what a sellers get for it
  - Present regardless of trading probability
- Hot potato effect
  - The propensity to off-load cash more quickly if inflation increases
  - Reduces wastage of goods

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market
  - goods, once made, are indivisible and perishable

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market
  - goods, once made, are indivisible and perishable
  - buyers have general and idiosyncratic preferences over search goods

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market
  - goods, once made, are indivisible and perishable
  - buyers have general and idiosyncratic preferences over search goods
- It introduces two new decision margins:

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market
  - goods, once made, are indivisible and perishable
  - buyers have general and idiosyncratic preferences over search goods
- It introduces two new decision margins:
  - Sellers decide on the general quality of the good

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market
  - goods, once made, are indivisible and perishable
  - buyers have general and idiosyncratic preferences over search goods
- It introduces two new decision margins:
  - Sellers decide on the general quality of the good
  - Buyers set a reservation idiosyncratic preference shock

MOTIVATION (cont.)

- Model is a variant of Rocheteau and Wright [2005] in which
  - sellers produce search market good at end of centralized market
  - goods, once made, are indivisible and perishable
  - buyers have general and idiosyncratic preferences over search goods
- It introduces two new decision margins:
  - Sellers decide on the general quality of the good
  - Buyers set a reservation idiosyncratic preference shock
- Entry cost for sellers endogenous

LITERATURE

- **Competitive search**

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]
- **Hot potato effect**

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]
- **Hot potato effect**
  - Liu, Wang and Wright [2009]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]
- **Hot potato effect**
  - Liu, Wang and Wright [2009]
  - Nosal [2009]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]
- **Hot potato effect**
  - Liu, Wang and Wright [2009]
  - Nosal [2009]
- **Private information**

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]
- **Hot potato effect**
  - Liu, Wang and Wright [2009]
  - Nosal [2009]
- **Private information**
  - Ennis [2008]

LITERATURE

- **Competitive search**
  - Rocheteau and Wright [2005]
  - Faig and Huangfu [2007]
- **Prior production:**
  - Dutu and Julien [2008]
  - Jafarey and Masters [2003]
- **Hot potato effect**
  - Liu, Wang and Wright [2009]
  - Nosal [2009]
- **Private information**
  - Ennis [2008]
  - Faig and Jerez [2006]

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.
- There are two types of infinite lived individual in the model

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.
- There are two types of infinite lived individual in the model
  - **Buyers:** (Mass 1) Consume the good produced in the night market

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.
- There are two types of infinite lived individual in the model
  - **Buyers:** (Mass 1) Consume the good produced in the night market
  - **Sellers:** (Mass $\bar{n}$) Produce the night market good.

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.
- There are two types of infinite lived individual in the model
  - **Buyers:** (Mass 1) Consume the good produced in the night market
  - **Sellers:** (Mass $\bar{n}$) Produce the night market good.
- Both buyers and sellers produce and consume the day market good

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.
- There are two types of infinite lived individual in the model
  - **Buyers:** (Mass 1) Consume the good produced in the night market
  - **Sellers:** (Mass $\bar{n}$) Produce the night market good.
- Both buyers and sellers produce and consume the day market good
- All buyers enter the night market but for sellers it's a choice.

ENVIRONMENT: Time and Demography

- Time is discrete and continues forever.
- Every period of time is divided into 2 subperiods: day and night.
  - **Day:** Centralized frictionless market for a homogeneous and perfectly divisible good.
  - **Night:** Trade occurs in a decentralized market characterized by search frictions.
- There are two types of infinite lived individual in the model
  - **Buyers:** (Mass 1) Consume the good produced in the night market
  - **Sellers:** (Mass $\bar{n}$) Produce the night market good.
- Both buyers and sellers produce and consume the day market good
- All buyers enter the night market but for sellers it's a choice.
- Both day and night goods are perishable

ENVIRONMENT: Preferences

Date t Instantaneous utility

$$\begin{aligned} \text{buyers:} \qquad & U_t^b = v(x_t) - y_t + \beta_d \varepsilon u(q_t) \\ \text{sellers:} \qquad & U_t^s = v(x_t) - y_t - c(q_t) \end{aligned}$$

x_t is the quantity of the day good consumed
y_t is the quantity of day good produced
q_t is quality of the night good consumed/produced
$v(.)$: increasing, strictly concave,
$\beta_d \leq 1$ is a common discount factor between day and night.
$\varepsilon \sim G(.)$ is match-specific taste shock (support $(0, \bar{\varepsilon}]$)
$u(.)$: increasing, strictly concave, $u(0) = 0$, $u'(0) = \infty$
$c(.)$: increasing, strictly convex, $c(0) = c'(0) = 0$

ENVIRONMENT: Preferences (cont.)

- Special values, x^*, $\tilde{\varepsilon}$ and $\bar{q}$:

  $v'(x^*) = 1$, normalization $v(x^*) = x^*$
  $\tilde{\varepsilon} = \mathbb{E}(\varepsilon)$
  $\beta_d \bar{\varepsilon} u(\bar{q}) = c(\bar{q})$

ENVIRONMENT: Preferences (cont.)

- Special values, x^*, $\tilde{\varepsilon}$ and $\bar{q}$:

  $v'(x^*) = 1$, normalization $v(x^*) = x^*$
  $\tilde{\varepsilon} = \mathbb{E}(\varepsilon)$
  $\beta_d \bar{\varepsilon} u(\bar{q}) = c(\bar{q})$

- Lifetime utility of an individual type $i = b, s$ is

$$\sum_{t=0}^{\infty} \beta^t U_t^i.$$

where,

$$\beta = \beta_n \beta_d$$

$\beta_n \leq 1$ is a common discount factor between night and day.

ENVIRONMENT: Matching (night market)

- In a market where the ratio of sellers to buyers is n,

ENVIRONMENT: Matching (night market)

- In a market where the ratio of sellers to buyers is n,
  - The probability with which a buyer meets a seller is $\alpha(n)$

ENVIRONMENT: Matching (night market)

- In a market where the ratio of sellers to buyers is n,
  - The probability with which a buyer meets a seller is $\alpha(n)$
  - The probability with which a seller meets a buyer is $\alpha(n)/n$

ENVIRONMENT: Matching (night market)

- In a market where the ratio of sellers to buyers is n,
  - The probability with which a buyer meets a seller is $\alpha(n)$
  - The probability with which a seller meets a buyer is $\alpha(n)/n$
- $\alpha(0) = 0$, $\alpha'(n) > 0$, $\alpha''(n) < 0$ and $\lim_{n\to\infty} \alpha(n) = 1$.

ENVIRONMENT: Matching (night market)

- In a market where the ratio of sellers to buyers is n,
  - The probability with which a buyer meets a seller is $\alpha(n)$
  - The probability with which a seller meets a buyer is $\alpha(n)/n$
- $\alpha(0) = 0$, $\alpha'(n) > 0$, $\alpha''(n) < 0$ and $\lim_{n\to\infty} \alpha(n) = 1$.
- For $\alpha(n)$, $\alpha(n)/n$ to be probabilities, $\alpha(n) \le \min\{1, n\}$

ENVIRONMENT: Matching (night market)

- In a market where the ratio of sellers to buyers is n,
  - The probability with which a buyer meets a seller is $\alpha(n)$
  - The probability with which a seller meets a buyer is $\alpha(n)/n$
- $\alpha(0) = 0$, $\alpha'(n) > 0$, $\alpha''(n) < 0$ and $\lim_{n\to\infty} \alpha(n) = 1$.
- For $\alpha(n)$, $\alpha(n)/n$ to be probabilities, $\alpha(n) \leq \min\{1, n\}$
- CRS of underlying technology requires $\alpha(n)/n$ decreasing, guaranteed if $\alpha'(0) = 1$.

EFFICIENCY

- Planner's objective function,

$$W(x, n, q; \bar{n}) \equiv [v(x) - x](1 + \bar{n}) + \beta_d \alpha(n)\tilde{\varepsilon}u(q) - nc(q)$$

EFFICIENCY

- Planner's objective function,

$$W(x, n, q; \bar{n}) \equiv [v(x) - x](1 + \bar{n}) + \beta_d \alpha(n) \tilde{\varepsilon} u(q) - nc(q)$$

- Planner's problem:

$$\max_{x,n,q} W(x, n, q; \bar{n}) \qquad \text{subject to } n \leq \bar{n}.$$

EFFICIENCY (cont.)

- F.O.C's:

$$\begin{aligned} x &: \quad v'(x_p) = 1 \\ n &: \quad \beta_d \alpha'(n_p)\tilde{\varepsilon}u(q_p) - c(q_p) = 0 \\ q &: \quad \beta_d \alpha(n_p)\tilde{\varepsilon}u'(q_p) - n_p c'(q_p) = 0 \end{aligned}$$

EFFICIENCY (cont.)

- F.O.C's:

$$\begin{aligned} x &: \quad v'(x_p) = 1 \\ n &: \quad \beta_d \alpha'(n_p)\tilde{\varepsilon} u(q_p) - c(q_p) = 0 \\ q &: \quad \beta_d \alpha(n_p)\tilde{\varepsilon} u'(q_p) - n_p c'(q_p) = 0 \end{aligned}$$

- $x_p = x^*$, a solution, (n_p, q_p) exists in $(0, \bar{n}] \times (0, \bar{q})$

EFFICIENCY (cont.)

- F.O.C's:

$$\begin{aligned} x &: \quad v'(x_p) = 1 \\ n &: \quad \beta_d \alpha'(n_p)\tilde{\varepsilon}u(q_p) - c(q_p) = 0 \\ q &: \quad \beta_d \alpha(n_p)\tilde{\varepsilon}u'(q_p) - n_p c'(q_p) = 0 \end{aligned}$$

- $x_p = x^*$, a solution, (n_p, q_p) exists in $(0, \bar{n}] \times (0, \bar{q})$
- Dividing the n equation by the q equation and multiply through by q:

$$e_u(q_p) = \eta(n_p) e_c(q_p)$$

where $e_u(.)$ is the elasticity of $u(.)$, $e_c(.)$ is the elasticity of $c(.)$ and $\eta(.)$ is the elasticity of $\alpha(.)$.

Market Economy: Money

- Lack of a coincidence of wants makes money essential in the search market

Market Economy: Money

- Lack of a coincidence of wants makes money essential in the search market
- Money is perfectly divisible and agents can hold any non-negative amount.

Market Economy: Money

- Lack of a coincidence of wants makes money essential in the search market
- Money is perfectly divisible and agents can hold any non-negative amount.
- Aggregate nominal money supply: M_t

Market Economy: Money

- Lack of a coincidence of wants makes money essential in the search market
- Money is perfectly divisible and agents can hold any non-negative amount.
- Aggregate nominal money supply: M_t
- Grows at constant gross rate $\gamma \geq \beta$ so that $M_{t+1} = \gamma M_t$.

Market Economy: Money

- Lack of a coincidence of wants makes money essential in the search market
- Money is perfectly divisible and agents can hold any non-negative amount.
- Aggregate nominal money supply: M_t
- Grows at constant gross rate $\gamma \geq \beta$ so that $M_{t+1} = \gamma M_t$.
- New money is injected or withdrawn by lump-sum transfers

Market Economy: Day market prices

- The day market price of goods normalized to 1

Market Economy: Day market prices

- The day market price of goods normalized to 1
- The relative price of money is denoted ϕ_t.

Market Economy: Day market prices

- The day market price of goods normalized to 1
- The relative price of money is denoted ϕ_t.
- $z_t = \phi_t m_t$ used as choice variable

Market Economy: Day market prices

- The day market price of goods normalized to 1
- The relative price of money is denoted ϕ_t.
- $z_t = \phi_t m_t$ used as choice variable
- In steady-state (aggregate real variables are constant over time) $\phi_{t+1} = \phi_t / \gamma$.

Market Economy: Day market prices

- The day market price of goods normalized to 1
- The relative price of money is denoted ϕ_t.
- $z_t = \phi_t m_t$ used as choice variable
- In steady-state (aggregate real variables are constant over time) $\phi_{t+1} = \phi_t / \gamma$.
- So real value in $t + 1$ of m_t is $\phi_t m_t / \gamma = z_t / \gamma$.

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)
  - q is the quality of the good they will bring to the night market

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)
  - q is the quality of the good they will bring to the night market
  - d is the real price at which they will part with it $(\phi_t p_t)$

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)
  - q is the quality of the good they will bring to the night market
  - d is the real price at which they will part with it $(\phi_t p_t)$
- Buyers use the announced values of (q, d) to direct their search.

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)
  - q is the quality of the good they will bring to the night market
  - d is the real price at which they will part with it $(\phi_t p_t)$
- Buyers use the announced values of (q, d) to direct their search.
- Buyers choose cash holding $z = d$

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)
  - q is the quality of the good they will bring to the night market
  - d is the real price at which they will part with it $(\phi_t p_t)$
- Buyers use the announced values of (q, d) to direct their search.
- Buyers choose cash holding $z = d$
- Submarkets characterized by $\omega = (z, q, n)$ (n is sellers per buyer)

Market Economy: submarkets and directed search

- In the morning, sellers announce their intention to enter and (q, d)
  - q is the quality of the good they will bring to the night market
  - d is the real price at which they will part with it $(\phi_t p_t)$
- Buyers use the announced values of (q, d) to direct their search.
- Buyers choose cash holding $z = d$
- Submarkets characterized by $\omega = (z, q, n)$ (n is sellers per buyer)
- In submarket ω, meeting probabilities are $\alpha(n)$ and $\alpha(n)/n$

Market Economy: value functions

- $W^i(z)$, is value to entering the day market with (current period) real money holding z, $i = s, b$

Market Economy: value functions

- $W^i(z)$, is value to entering the day market with (current period) real money holding z, $i = s, b$
- $V^i(\omega)$ is (present discounted expected) value to entering (night) submarket ω for $i = s, b$

Market Economy: day market

- **Buyers**: choose production, consumption and which night market to enter

$$\begin{aligned} W^b(z) &= \max_{x,y,\hat{\omega}} \left\{ v(x) - y + \beta_d V^b(\hat{\omega}) \right\} \\ \text{subject to } \hat{z} + x &= z + T + y, \qquad y \geq 0 \end{aligned}$$

Market Economy: day market

- **Buyers**: choose production, consumption and which night market to enter

$$\begin{aligned} W^b(z) &= \max_{x,y,\hat{\omega}} \left\{ v(x) - y + \beta_d V^b(\hat{\omega}) \right\} \\ \text{subject to } \hat{z} + x &= z + T + y, \qquad y \geq 0 \end{aligned}$$

- **Sellers**: choose production, consumption and which night market to enter

$$W^s(z) = \max_{x,y,\hat{\omega}} \{ v(x) - y + \max [\beta_d V^s(\hat{\omega}) - c(\hat{q}), \beta W^s(0)] \}$$

$$\text{subject to } \hat{z} + x = z + y, \qquad y \geq 0$$

Day market outcomes

- Ignore non-negativity of y ($v(.)$ can be chosen so it does not bind)

Day market outcomes

- Ignore non-negativity of y ($v(.)$ can be chosen so it does not bind)
- $x = x^*$ for both buyers and sellers

Day market outcomes

- Ignore non-negativity of y ($v(.)$ can be chosen so it does not bind)
- $x = x^*$ for both buyers and sellers
- For sellers, $\hat{z} = 0$

Day market outcomes

- Ignore non-negativity of y ($v(.)$ can be chosen so it does not bind)
- $x = x^*$ for both buyers and sellers
- For sellers, $\hat{z} = 0$
- For buyers, $\hat{z}$ does not depend on z.

Day market outcomes

- Ignore non-negativity of y ($v(.)$ can be chosen so it does not bind)
- $x = x^*$ for both buyers and sellers
- For sellers, $\hat{z} = 0$
- For buyers, $\hat{z}$ does not depend on z.
- $W^i(z) = z + W^i(0)$

Market Economy: night market,

- **Buyers**:

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon \left[\max \left\{ \varepsilon u(q) + \beta_n W^b(0), \beta_n W^b\left(\frac{z}{\gamma}\right) \right\} \right] \\ + (1-\alpha(n))\beta_n W^b\left(\frac{z}{\gamma}\right)$$

Market Economy: night market,

- **Buyers**:

$$V^b(\omega) = \alpha(n)\mathbb{E}_\varepsilon\left[\max\left\{\varepsilon u(q) + \beta_n W^b(0), \beta_n W^b\left(\frac{z}{\gamma}\right)\right\}\right] + (1-\alpha(n))\beta_n W^b\left(\frac{z}{\gamma}\right)$$

- If $\varepsilon_R = \beta_n z/\gamma u(q)$, reservation value of ε.

$$V^b(\omega) = \alpha(n)u(q)S_G(\varepsilon_R) + \beta_n W^b\left(\frac{z}{\gamma}\right)$$

where

$$S_G(\varepsilon_R) = \int_{\varepsilon_R}^{\bar{\varepsilon}} [\varepsilon - \varepsilon_R]\, dG(\varepsilon)$$

Market Economy: night market (cont.)

- **Sellers**:

$$V^s(\omega) = \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\beta_n W^s\left(\frac{z}{\gamma}\right) + \left(1 - \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\right)\beta_n W^s(0)$$

Market Economy: night market (cont.)

- **Sellers**:

$$V^s(\omega) = \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\beta_n W^s\left(\frac{z}{\gamma}\right) + \left(1 - \frac{\alpha(n)}{n}\left[1 - G(\varepsilon_R)\right]\right)\beta_n W^s(0)$$

- or

$$V^s(\omega) = \frac{\alpha(n)}{n\gamma}\left[1 - G(\varepsilon_R)\right]\beta_n z + \beta_n W^s(0)$$

Market Economy: equilibrium

Definition

A symmetric, competitive search equilibrium is a submarket, $\tilde{\omega} = (\tilde{z}, \tilde{q}, \tilde{n})$ such that given all other buyers and sellers enter $\tilde{\omega}$, then $\tilde{\omega}$ solves both the individual buyer's (morning) problem, and the individual seller's (morning) problem subject to

$$\beta_d V^s(\hat{\omega}) - c(\hat{q}) = \beta_d V^s(\tilde{\omega}) - c(\tilde{q}) \begin{cases} = \beta W^s(0) \text{ for } \tilde{n} \leq \bar{n} \\ \geq \beta W^s(0) \text{ for } \tilde{n} = \bar{n} \end{cases}$$

Market Economy: characterization

Degenerate G (a spike at $\bar{\varepsilon}$):

- Under **free entry** (duality implies),

$$\tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\}$$
$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$

Market Economy: characterization

Degenerate G (a spike at $\bar{\varepsilon}$):

- Under **free entry** (duality implies),

$$\tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\}$$

$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$

- Substituting for value functions and eliminating z, $(\tilde{n}, \tilde{q})$ solves

$$\max_{(n,q)\in[0,\bar{n}]\times[0,\infty)} \left\{ \beta_d \alpha(n)\bar{\varepsilon}u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta}\right)[\gamma - \beta] + \beta W^b(0) \right\}$$

Market Economy: characterization

Degenerate G (a spike at $\bar{\varepsilon}$):

- Under **free entry** (duality implies),

$$\tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\}$$
$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$

- Substituting for value functions and eliminating z, $(\tilde{n}, \tilde{q})$ solves

$$\max_{(n,q)\in[0,\bar{n}]\times[0,\infty)} \left\{ \beta_d \alpha(n)\bar{\varepsilon}u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta} \right) [\gamma - \beta] + \beta W^b(0) \right\}$$

- At Friedman rule, $\gamma = \beta$, same as Planner's problem

Market Economy: characterization

Degenerate G (a spike at $\bar{\varepsilon}$):

- Under **free entry** (duality implies),

$$\tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\}$$
$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$

- Substituting for value functions and eliminating z, $(\tilde{n}, \tilde{q})$ solves

$$\max_{(n,q)\in[0,\bar{n}]\times[0,\infty)} \left\{ \beta_d \alpha(n)\bar{\varepsilon}u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta}\right)[\gamma - \beta] + \beta W^b(0) \right\}$$

- At Friedman rule, $\gamma = \beta$, same as Planner's problem
- In general, first-order conditions imply

$$e_u(q) = \left(\frac{\eta(n)\,[\gamma - \beta + \alpha(n)\beta]}{(1 - \eta(n))(\gamma - \beta) + \alpha(n)\beta} \right) e_c(q).$$

Market Economy: characterization

Degenerate G (a spike at $\bar{\varepsilon}$):

- Under **free entry** (duality implies),

$$\tilde{\omega} \in \arg\max_{\omega} \left\{ \beta_d V^b(\omega) - z \right\}$$
$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$

- Substituting for value functions and eliminating z, $(\tilde{n}, \tilde{q})$ solves

$$\max_{(n,q)\in[0,\bar{n}]\times[0,\infty)} \left\{ \beta_d \alpha(n) \bar{\varepsilon} u(q) - nc(q) - \left(\frac{nc(q)}{\alpha(n)\beta} \right) [\gamma - \beta] + \beta W^b(0) \right\}$$

- At Friedman rule, $\gamma = \beta$, same as Planner's problem
- In general, first-order conditions imply

$$e_u(q) = \left(\frac{\eta(n)\left[\gamma - \beta + \alpha(n)\beta\right]}{(1 - \eta(n))(\gamma - \beta) + \alpha(n)\beta} \right) e_c(q).$$

- Equilibrium unique if $u(.)$, $c(.)$ isoelastic and $\eta(n)$ monotone

Market Economy: characterization

General G:

- Under **free entry**,

$$(\tilde{\omega}, \tilde{\varepsilon}_R) \in \arg\max_{\omega, \varepsilon_R} \left\{ \beta_d V^b(\omega) - z \right\}$$

$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$

$$\text{and } \gamma u(q) \varepsilon_R = \beta_n z$$

Market Economy: characterization

General G:

- Under **free entry**,

$$(\tilde{\omega}, \tilde{\varepsilon}_R) \in \arg\max_{\omega, \varepsilon_R} \left\{ \beta_d V^b(\omega) - z \right\}$$
$$\text{subject to } \beta_d V^s(\omega) - c(q) = \beta W^s(0)$$
$$\text{and } \gamma u(q) \varepsilon_R = \beta_n z$$

- Substituting value functions and eliminating z means $(\tilde{n}, \tilde{q}, \tilde{\varepsilon}_R)$ solves

$$\max_{(n,q,\varepsilon_R) \in [0,\bar{n}] \times [0,\infty) \times [0,\bar{\varepsilon}]} \left\{ \left[\alpha(n) \beta S_G(\varepsilon_R) - (\gamma - \beta) \varepsilon_R \right] u(q) / \beta_n \right\}$$
$$\text{subject to: } \quad \frac{\alpha(n)}{n} \beta_d [1 - G(\varepsilon_R)] \varepsilon_R u(q) - c(q) = 0$$

- **Welfare** (due to night market activity)

$$\tilde{W}_m(\gamma) \equiv \alpha(\tilde{n})\beta_d[1 - G(\tilde{\varepsilon}_R)]\mathbb{E}_{\{\varepsilon|\varepsilon \geq \tilde{\varepsilon}_R\}}\varepsilon u(\tilde{q}) - \tilde{n}c(\tilde{q})$$

Market Economy: Policy

- **Welfare** (due to night market activity)

$$\tilde{W}_m(\gamma) \equiv \alpha(\tilde{n})\beta_d[1 - G(\tilde{\varepsilon}_R)]\mathbb{E}_{\{\varepsilon|\varepsilon \geq \tilde{\varepsilon}_R\}}\varepsilon u(\tilde{q}) - \tilde{n}c(\tilde{q})$$

- Planner would set $\tilde{\varepsilon}_R = 0$ – market not generally efficient

Market Economy: Policy

- **Welfare** (due to night market activity)

$$\tilde{W}_m(\gamma) \equiv \alpha(\tilde{n})\beta_d[1 - G(\tilde{\varepsilon}_R)]\mathbb{E}_{\{\varepsilon|\varepsilon \geq \tilde{\varepsilon}_R\}}\varepsilon u(\tilde{q}) - \tilde{n}c(\tilde{q})$$

- Planner would set $\tilde{\varepsilon}_R = 0$ – market not generally efficient
- Using the free-entry constraint,

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d S_G(\tilde{\varepsilon}_R)u(\tilde{q})$$

Market Economy: Policy

- **Welfare** (due to night market activity)

$$\tilde{W}_m(\gamma) \equiv \alpha(\tilde{n})\beta_d[1 - G(\tilde{\varepsilon}_R)]\mathbb{E}_{\{\varepsilon|\varepsilon \geq \tilde{\varepsilon}_R\}}\varepsilon u(\tilde{q}) - \tilde{n}c(\tilde{q})$$

- Planner would set $\tilde{\varepsilon}_R = 0$ – market not generally efficient
- Using the free-entry constraint,

$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d S_G(\tilde{\varepsilon}_R)u(\tilde{q})$$

- At Friedman rule $\dfrac{d\tilde{W}_m(\gamma)}{d\gamma} = 0$ by envelope theorem.

Market Economy: Policy under free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **All margins active**

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **All margins active**
- $$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} > 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$

Market Economy: Policy under free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **All margins active**
- $$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} > 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$
- $$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = 0.$$

Market Economy: Policy under free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **All margins active**
- $$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} > 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$
- $$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = 0.$$
- **Exogenous** q,
$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d S_G(\tilde{\varepsilon}_R)u(q)$$

Market Economy: Policy under free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **All margins active**
- $$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} > 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$
- $$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} = 0.$$
- **Exogenous** q,
$$\tilde{W}_m(\gamma) = \alpha(\tilde{n})\beta_d S_G(\tilde{\varepsilon}_R)u(q)$$
- $$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{n}}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{W}_m}{d\gamma}\right|_{\gamma=\beta} = 0$$

Market Economy: Policy without free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry** $(n = \bar{n})$

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$

Market Economy: Policy without free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry** $(n = \bar{n})$

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$

- $$\tilde{W}_m(\gamma) = \frac{1}{2}\alpha(\bar{n})\beta_d \left[1 - \tilde{\varepsilon}_R^2\right] u(\tilde{q}) - \bar{n}c(\tilde{q}).$$

Market Economy: Policy without free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry** $(n = \bar{n})$

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$

- $$\tilde{W}_m(\gamma) = \frac{1}{2}\alpha(\bar{n})\beta_d \left[1 - \tilde{\varepsilon}_R^2\right] u(\tilde{q}) - \bar{n}c(\tilde{q}).$$

- At the Friedman rule, after substituting for $\frac{d\tilde{\varepsilon}_R}{d\gamma}$ and $\frac{d\tilde{q}}{d\gamma}$,

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} \gtrless 0?$$

Market Economy: Policy without free-entry

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry** $(n = \bar{n})$

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0 \qquad \left.\frac{d\tilde{q}}{d\gamma}\right|_{\gamma=\beta} < 0$$

-
$$\tilde{W}_m(\gamma) = \frac{1}{2}\alpha(\bar{n})\beta_d\left[1 - \tilde{\varepsilon}_R^2\right]u(\tilde{q}) - \bar{n}c(\tilde{q}).$$

- At the Friedman rule, after substituting for $\frac{d\tilde{\varepsilon}_R}{d\gamma}$ and $\frac{d\tilde{q}}{d\gamma}$,

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} \gtrless 0?$$

- positive if $\bar{n}$ small enough.

Market Economy: Policy (cont)

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry** $(n = \bar{n})$ **and quality choice**, $q = \bar{q}$.

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0$$

Market Economy: Policy (cont)

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry ($n = \bar{n}$) and quality choice**, $q = \bar{q}$.
$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0$$
-
$$\tilde{W}_m(\gamma) = \frac{1}{2}\alpha(\bar{n})\beta_d\left[1 - \tilde{\varepsilon}_R^2\right]u(\bar{q}) - \bar{n}c(\bar{q}).$$

Market Economy: Policy (cont)

Under, $u(.)$, $c(.)$ isoelastic, $\eta(n)$ monotone decreasing, G uniform on $(0, \bar{\varepsilon}]$

- **Shutting down seller free-entry** ($n = \bar{n}$) **and quality choice**, $q = \bar{q}$.

$$\left.\frac{d\tilde{\varepsilon}_R}{d\gamma}\right|_{\gamma=\beta} < 0$$

- $$\tilde{W}_m(\gamma) = \frac{1}{2}\alpha(\bar{n})\beta_d \left[1 - \tilde{\varepsilon}_R^2\right] u(\bar{q}) - \bar{n}c(\bar{q}).$$

- At the Friedman rule, after substituting for $\frac{d\tilde{\varepsilon}_R}{d\gamma}$,

$$\left.\frac{d\tilde{W}_m(\gamma)}{d\gamma}\right|_{\gamma=\beta} \geq 0$$

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers
  - Sellers take the money and hand over goods free of charge.

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers
  - Sellers take the money and hand over goods free of charge.
  - Efficiency is restored at the Friedman rule

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers
  - Sellers take the money and hand over goods free of charge.
  - Efficiency is restored at the Friedman rule
  - Model eliminates holding of idle cash balances

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers
  - Sellers take the money and hand over goods free of charge.
  - Efficiency is restored at the Friedman rule
  - Model eliminates holding of idle cash balances
  - Does better than lucky bags away from the Friedman rule

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
    - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
    - Empowered to charge entrance fees to buyers and sellers
    - Free-entry of market makers drives their profits to zero
    - They distribute buyers' entrance fees to sellers
    - Sellers take the money and hand over goods free of charge.
    - Efficiency is restored at the Friedman rule
    - Model eliminates holding of idle cash balances
    - Does better than lucky bags away from the Friedman rule
- **Lotteries**

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers
  - Sellers take the money and hand over goods free of charge.
  - Efficiency is restored at the Friedman rule
  - Model eliminates holding of idle cash balances
  - Does better than lucky bags away from the Friedman rule
- **Lotteries**
  - Don't work - same as price-only model.

Alternative Market Structures

- **Lucky bags:** buyers pay for good before they see it.
  - Model same as if there were no match specific preferences
- **Market Makers** (Faig and Huangfu [2007])**:**
  - Empowered to charge entrance fees to buyers and sellers
  - Free-entry of market makers drives their profits to zero
  - They distribute buyers' entrance fees to sellers
  - Sellers take the money and hand over goods free of charge.
  - Efficiency is restored at the Friedman rule
  - Model eliminates holding of idle cash balances
  - Does better than lucky bags away from the Friedman rule
- **Lotteries**
  - Don't work - same as price-only model.
  - free-entry equilibrium constrained efficient

Conclusions

- This project introduces:

Conclusions

- This project introduces:
    - Production prior to retail

Conclusions

- This project introduces:
    - Production prior to retail
    - Private information over match specific preferences

Conclusions

- This project introduces:
  - Production prior to retail
  - Private information over match specific preferences
- Under seller free-entry the Friedman rule is optimal policy but not necessarily efficient

Conclusions

- This project introduces:
  - Production prior to retail
  - Private information over match specific preferences
- Under seller free-entry the Friedman rule is optimal policy but not necessarily efficient
- When free-entry is shut down, and n is small enough low levels of inflation can improve welfare.