Marriage, Commitment and Divorce in a Matching Model with Differential Aging

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Abstract

This paper analyses a matching model of the marriage market with directed, on-the-job search. Everyone is born attractive but "matures" according to a Poisson process into unattractiveness. Marriages between equally attractive people are stable but mixed marriages are not. The implied patterns of marriage and divorce are consistent with those identified in the empirical literature. When the utility from being single is low, in order to commit to their relationship, attractive people in mixed marriages may voluntarily divest of their attractiveness.

Key Words: Marriage, Matching, directed search

JEL: J12, C78

1 Introduction

As people's extra marital activities are unobserved by their spouses, neither partner can credibly promise not to re-enter the marriage market. This paper looks at how this moral hazard problem interacts with the aging process in determining patterns of marriage, divorce and life-style choice.

A frictional matching model of the marriage market with non-transferable utility and "on-the-job search" is analyzed. In the model, new entrants are single and attractive. While everyone faces the same aging process, realizations of the process differ across individuals so that maturity (i.e. becoming unattractive) will occur sooner for some people than others. This means marriages initially between two attractive people will at some point comprise one attractive and one unattractive partner. Being married to an attractive person provides a higher level of utility than being married to an unattractive person. Mixed marriages are therefore potentially unstable as the attractive spouse might do better by looking for another attractive partner. Moreover, aware of the possibility of being left alone, the less attractive spouse may also re-enter the market in the hope of finding another unattractive person with whom marriage would be more stable.

While it can be mutually beneficial for the partners to agree not to re-enter the marriage market, the attractive partner in a mixed marriage will inevitably renege on the deal. Knowing this, the unattractive partner will renege too. The moral hazard problem therefore serves to further destabilize marriages made vulnerable by differential aging. In this situation, the couple might be willing to incur some cost in order to restore stability to the marriage. To further explore this possibility, I assume that attractive individuals can make an irreversible choice to become unattractive. The results show that when the prospect of being made single is sufficiently unpleasant, the attractive partner will, indeed, choose to divest of his or her appearance in order to stabilize the relationship.

Willis and Weiss [1997], document the observed patters of divorce in the USA. Two key features emerge. First, the hazard rate of divorce is initially increasing, it peaks at some duration and then falls again. The second feature is that marriages between older couples are less likely to end in divorce. While they suggest theories that are consistent with each of these facts individually, the principal reason for looking at the role of stochastic aging is that the implied patterns of divorce are consistent with both facts simultaneously. This is because weddings typically occur between similar individuals. In the model, unattractive couples have stable marriages and attractive couples' marriages remain stable until one or other partner matures. While the marriages remain mixed they are subject to divorce. Eventually, contingent on not having separated, both partners will be unattractive and stability returns to the marriage. Simulations show that, aggregated across society as a whole, this generates a smooth hump-shaped hazard of divorce as function of marital duration (See Figure 3). Also in the model, older people are more likely to be unattractive so that a larger percentage of their marriages are never subject to divorce. This leads to lower divorce rates for marriages among older couples as found in the data. The extent to which attractive partners in mixed marriages divest of their attractiveness reduces the overall divorce rate but the qualitative patterns are unaffected. Still, as long as the moral hazard problem is sufficiently severe, the model predicts that divesting should occur. Some discussion of how people might deliberately mar their own appearance is provided in the text.

Matching environments often exhibit a large number of steady-state equilibria. To help overcome this problem I use a single gender model (as in Shimer and Smith [2000]). Clearly, this means the paper cannot address the significant differences in observed marriage and divorce patterns across genders. My assumption, however does help keep the paper focused on the potential role of aging in precipitating divorce and identifying general circumstances under which individuals might "let themselves go" in order to stabilize their relationship.

Another important modelling choice is that of directed search as in Moen [1997]. In the current context it implies the existence of three potential markets. There are two specialized markets, attractive-only and unattractive-only, and a mixed market. At any point in time a particular market may be active or inactive. Complete market segmentation is said to occur when the mixed market is inactive. Purely directed search would allow any individual to enter whichever market promised the highest lifetime expected utility. To simplify the analysis, however, I impose that individuals may not enter the other type's specialized market. This restriction is not without precedent. Moen [1997], for instance, excludes employed workers from the labor market. In any case, it will be shown below that exclusion is not essential for the main results. It provides a class of market equilibria (those with complete market segmentation) that exhaust the entire parameter space. Under a simple parameter restriction, exclusion is unnecessary.

The specialized markets here would more commonly be called 'social clubs'. Directed search in a marriage market context, therefore, leads to endogenous club formation. Some clubs (e.g. chess, bingo) are open to everyone. Some (e.g. country clubs, night clubs) are exclusive. Exclusion as used in the paper, amounts to having some kind of entry requirement.

There are 3 types of pure strategy equilibria with complete market segmentation. In all of these, symmetric marriages are stable so that each equilibrium is characterized by what happens in mixed marriages. For any given parameter configuration, at most one of these equilibria exists. In a "doting partner" equilibrium, the attractive partner re-enters the marriage market while the unattractive partner does not. This happens if the value to being married to an attractive person is sufficiently high or whenever the prospect of becoming single again is not too bad. As the prospect of singleness becomes more unappealing, the unattractive partner will also re-enter the marriage market (this is called the "sham marriage" equilibrium). If singleness is even more unpleasant, rather than face this prospect, the attractive partner will divest of his attractiveness to restore stability in the marriage (the "Roseanne" equilibrium). There is also a region of the parameter space in which none of these pure strategy equilibria exist. Instead, there is a mixed strategy equilibrium in which some people have sham marriages and some attractive partners divest of their appearance.

The applicability of frictional matching models to the marriage market was recognized by Mortensen [1982,1988], and has been used subsequently to shed light on observed patterns of marriage (see Burdett and Coles [1997]). The idea that couples might benefit from access to a costly commitment device to keep their marriage together was suggested by Burdett *et al* [2004]. They provide a model of partnership formation with on-the-job search. They show that equilibria with too much search, along the lines of my Sham Marriage Equilibrium are possible. In that case some kind of commitment device would be beneficial to the couple and to society. Their model, however, does not incorporate any life-cycle component and therefore has no consistent way that a commitment device can arise. Cornelius [2003] provides a model of good and bad marriage partners who search for each other and can continue to search after marriage. She imposes that marriages between good partners are stable and looks at possible matching patterns. The emphasis of her paper is on how equilibria with and without divorce can coexist. In her model people's types are fixed so that the life-cycle issues raised here are ignored.

Chiappori *et al* [2005] provide a model of the marriage market with frictionless matching and transferable utility (*a la* Becker [1991]). *Ex post* realizations of match quality lead to separation (divorce) as in Jovanovic [1979]. Forward looking agents take divorce policy into account in the determination of intra-household allocations. Steady-state allocations are therefore divorce policy neutral. Changes in divorce policy can destabilize current marriages in which case intra-household transfers can restore stability. Allocations remain efficient. This serves to highlight the difference in approaches. Here, non-transferable utility means that restoration of marital stability requires disposal of a valuable asset - attractiveness.

One paper with directed, on-the-job search but with transferable utility (in a labor market context) is Delacroix and Shi [2006]. Essentially, they look at what happens in the Moen [1997] environment when employed workers are no longer excluded from the job market. Clearly the unique wage equilibrium cannot prevail. As long as some match rents go to firms, others will create vacancies at higher wages to entice workers away from their current employers. As in the current paper, Delacroix and Shi [2006] show that the market endogenously segments its-self (by wage level). Individuals climb a wage ladder one rung at a time. The result highlights the role of permitting participants to direct their search in shaping the aggregate structure of markets. (In the same context, Burdett and Mortensen [1998] show that random search generates a continuous distribution of wages.)

While the option to divest of one's appearance can be viewed as an extension of the basic model of marriage with aging, in the interest of space, the exposition incorporates this choice from the beginning. How the modeled society would look in the absence of this choice will be obvious. The next section describes the modeling environment. Section 3 discusses various types of equilibria and obtains parameter configurations under which each arises. Section 4 provides some discussion of the implications that arise from the model. Section 5 concludes.

2 Model Environment

The infinite horizon, continuous time environment comprises a continuum of *ex ante* homogeneous, risk-neutral individuals. The population is constant with mass normalized to unity. Individual longevity is an exponentially distributed random variable with expected value $(1/\delta)$. People who die are replaced by newborns so that the parameter δ represents both the birth and death rate. Everyone discounts the future at a rate r.

There is *ex post* heterogeneity of two types. People can be single or married and they can be attractive or unattractive. Everyone is born single and attractive. Individuals can remain attractive at a flow cost c, until such time as they "mature". Maturity comes to everyone according to a Poisson process with parameter μ. Mature people and those unwilling to incur the cost of attractiveness are unattractive. Once given up, attractiveness cannot be restored.

Being single provides flow utility s. Marriage provides a flow utility of $m \geq s$ to both partners. Being married to an attractive person provides an additional flow benefit a. As with other work on matching models of the marriage market, (*e.g.* Burdett and Coles [1997] and Burdett *et al* [2004], Coles and Francesconi [2006]) utility is non-transferable. This is an extreme assumption based on the view that within marriage the component of match value that is negotiable is relatively small and that any utility transfer is imperfect.[1] By contrast, matching models of the labor market (e.g. Pissarides

[1]There is a large literature (e.g. Lundberg and Pollak [2001]) that looks at the role of bargaining within marriage. Although a potentially fertile area of research, the matching

[2000], Mortensen [2003], Masters [1999]) assume that utility is transferable through the wage.

In a departure from earlier models of the marriage market, search is directed according to observable characteristics - attractiveness. This approach implies the existence of 3 potential markets: an attractive-only, an unattractive-only, and a mixed market. Within any one market matching is random; people meet potential marriage partners at the Poisson arrival rate β. In such an environment, market formation is endogenous (see Moen [1997], Delacroix and Shi [2006]). For instance, when attractive people wish to direct their search only toward other attractive people and the unattractive do not direct their search toward attractive people then the "attractive-only" market is active.

An additional restriction used here is exclusion: attractive people are excluded from the unattractive peoples' market and *vice-versa.* By comparison, a common exclusion restriction used in labor market models keeps employed workers from re-entering the labor market.[2] Exclusion is only required in one of the 3 pure strategy equilibria discussed below. The implications of dropping exclusion are discussed in section 4. For now it is worth pointing out that we do not necessarily require exclusion for specialized markets (i.e. attractive-only or unattractive-only) to be active. Even in the absence of exclusion, we will see that whenever neither type prefer a mixed marriage to a same-type marriage, only specialized markets will operate.[3]

Being married does not affect the rate at which people who are in any market meet potential partners. Market entry is free but individuals indif-

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literature has thus far abstracted from these issues in the context of marriage markets.

[2]See Delacriox and Shi [2006] for a discusson of the role of this restiction.

[3]Exclusion is not without empirical justification. Many social venues have explicit restrictions on entry (*e.g.*. night clubs, country clubs, the Oscars), many more have implicit ones (*e.g.* raves, pubs)

ferent between market entry and staying at home are assumed to choose the latter. We will call those individuals who are in the market "available". Only monogamous relationships are permitted. If a married person finds a willing new partner with whom marriage is strictly preferred to staying in their current relationship, they will divorce their current spouse and marry the new suitor.

Individuals can potentially be in one of six states. Each state will be summarized by a pair of letters. Thus, an individual in state ij is of attractiveness, $i = a, u$ (attractive, unattractive) with marital status $j = s, a, u$ (single, attractive spouse, unattractive spouse). Marriages are in one of 3 possible states: aa, au, uu.[4]

3 Equilibrium

One way forward here is to write down a system of equations (incorporating all possible contingencies) in the asset values to being in each of the 6 states. We could then solve the system at particular parameter values under the restriction that no individual could make himself better off by a deviation from the implied behavior. Each solution would be an equilibrium.

For simplicity and clarity of exposition, we will, instead, postulate a particular marriage pattern and determine the value to being in each of the 6 states when everyone is bound to that pattern. This marriage pattern is a an *equilibrium* if no individual would prefer to deviate from the behavior

[4]Individual strategies in these kinds of models are always complicated things. As the environment is stationary and we seek stationary equilibria, the state of the world is the propensity for individuals to be in any of the 6 marital states and their propensities to be in each or any of the marriage markets. Individual strategies for attractive people map their current state and that of the world into a market entry and an attractiveness choice. Unattractive individuals do not have an attractiveness choice.

specified by the pattern.[5] Initially, only symmetric pure strategy steady-state equilibria are considered. The restriction that individuals would prefer not to deviate will imply a system of inequalities. These inequalities will in turn specify regions (possibly empty) of the parameter space for which the proposed marriage pattern is an equilibrium.

It is important to address what it might mean for someone to deviate to entering an inactive market. In such a scenario, because their strategies call for it, other people will enter in response to the deviant's behavior. The initial deviation is worthwhile if the implied matching opportunities generate a higher present value of expected lifetime utility than remaining in the active market. We will say that there is *complete market segmentation* if the mixed market is inactive.

Marriage patterns will be identified by capital letters with N representing a generic pattern. We will use V_{ij}^{N} to represent the value to being an individual in state ij when the prevailing marriage pattern is N. The notation n_{ij}^{N} will be used for the steady-state proportion of the population who are in state ij under marriage pattern N. Once the regions of the parameter space for which each of these marriage patterns is an equilibrium has been determined, we will combine the results to provide an overall picture of the model's predictions.

Some general points can be made. Because the matching rate for married people is the same as for unmarried people, in any equilibrium, single people prefer to be married and married people will re-enter the market if there is any possibility of finding a better relationship. (Otherwise they stay at

[5] This method is common in search/matching models and allows us to focus on those patterns of behavior that are known to be consistent with equilibrium. The downside is that we cannot rule out other equilibria. Other possible equilibria are discussed in the next section.

home.) Under complete market segmentation, all meetings lead to marriage. This means that the proportions of individuals in each state (the n_{ij}^N's) do not enter the calculations of the value functions (the V_{ij}^N's). Complete market segmentation also implies that all marriages between similar individuals are stable.[6] This is because when only specialized markets are active, individuals in such marriages cannot do better than they are currently doing.[7]

We focus, for now, on three particular pure strategy marriage patterns which differ in the stability of mixed marriages. In the

Doting Partner (type D) Pattern, the attractive partner is available while the unattractive partner stays at home.

Sham Marriage (type S) Pattern, both partners make themselves available.

Roseanne (type R) Pattern, when either partner matures naturally, the other chooses unattractiveness to commit to the relationship.

As the focus is not on changes in c, the flow cost of remaining attractive, it is set to zero.

3.1 The Doting Partner Marriage Pattern (D)

In marriage pattern D,

(Da) no one voluntarily gives up attractiveness

(Db) unattractive people with attractive partners stay at home

[6] A marriage is termed *stable* if neither partner re-enters the marriage market.

[7] A deviant could consider entering the inactive mixed market but, while same-type marriages dominate mixed marriages for at least one type, entering the mixed market cannot be better than entering the appropriate specialized market.

(Dc) there is complete market segmentation

In the implied steady-state there will be a strictly positive share of the population in each state, $n_{ij}^D > 0$ for $i = a, u$ and $j = s, a, u$.

A single attractive person finds an attractive partner at rate β, matures at rate μ, or dies at rate δ. The implied asset value equation associated with state as is therefore:

$$(r + \delta)V_{as}^D = s + \beta(V_{aa}^D - V_{as}^D) + \mu(V_{us}^D - V_{as}^D). \tag{1}$$

An unattractive single person can find an unattractive partner or die;

$$(r + \delta)V_{us}^D = s + \beta(V_{uu}^D - V_{us}^D). \tag{2}$$

An attractive person with an attractive spouse gets the associated high value of the flow benefit but she or her partner can either mature or pass-on. Since neither partner can do any better, the relationship is stable. So,

$$(r + \delta)V_{aa}^D = a + m + \mu(V_{ua}^D - V_{aa}^D) + \mu(V_{au}^D - V_{aa}^D) + \delta(V_{as}^D - V_{aa}^D). \tag{3}$$

In this marriage pattern, attractive people can find themselves with an unattractive spouse only if their previously attractive partner matures. Such an individual re-enters the market in the hope of finding a new attractive partner. Meanwhile she remains subject to the possibility of maturity, death and the death of her current (loyal) partner.

$$(r + \delta)V_{au}^D = m + \beta(V_{aa}^D - V_{au}^D) + \mu(V_{uu}^D - V_{au}^D) + \delta(V_{as}^D - V_{au}^D) \tag{4}$$

Individuals who mature before their partners do, continue to receive the benefit of marriage to an attractive spouse but also face the possibility of being jilted. Of course, before that happens, their partner could mature or die or they too could die.

$$(r + \delta)V_{ua}^D = a + m + \beta(V_{us}^D - V_{ua}^D) + \mu(V_{uu}^D - V_{ua}^D) + \delta(V_{us}^D - V_{ua}^D) \tag{5}$$

Even though $V^D_{ua} > V^D_{uu}$ unattractive individuals with unattractive spouses do not look for alternative partners. This is because they are excluded from the attractive peoples' market and anyone they might meet in the market to which they are welcome is no better than their current spouse. All that can happen to such an individual is her own death or that of her spouse.

$$(r + \delta)V^D_{uu} = m + \delta(V^D_{us} - V^D_{uu}) \tag{6}$$

Equations (1) to (6) comprise a linear and independent system in the 6 unknowns, V^D_{ij}, $i = a, u$, $j = s, a, u$. Obtaining the unique solution is therefore straightforward but the resulting expressions for the value functions are in general, messy and uninformative. The exceptions are

$$V^D_{us} = \frac{(r + 2\delta)s + \beta m}{(r + \delta)(r + \beta + 2\delta)}, \quad V^D_{uu} = \frac{\delta s + (r + \beta + \delta)m}{(r + \delta)(r + \beta + 2\delta)}. \tag{7}$$

These values clearly reflect the irreversibility of unattractiveness - once unattractive, individuals simply cycle though marriage to other unattractive people and widowhood until they die. As $m > s$, it should be clear that $V^D_{uu} > V^D_{us}$.

For this marriage pattern to be an equilibrium, no one should want to deviate from the specified behavior as long as everyone else conforms. Each of the statements, (Da), (Db) and (Dc) that characterize marriage pattern D behavior imply inequalities between the value functions derived above. These inequalities will be used to obtain conditions on the parameters for which all 3 statements are true. These conditions will specify a region (possibly empty) of the parameter space for which marriage pattern D is an equilibrium.

That no one voluntarily gives up attractiveness (Da) requires $V^D_{as} \geq V^D_{us}$, $V^D_{aa} \geq V^D_{ua}$ and $V^D_{au} \geq V^D_{uu}$. That unattractive people with attractive partners stay at home (Db) requires $V^D_{ua} \geq V^D_{uu}$. Complete market segmentation (Dc), requires that $V_{aa}^D \geq V_{au}^D$.[8] Straightforward algebra reveals that

$$V_{aa}^D - V_{au}^D = \frac{(r+\beta+2\delta)(r+\beta+2\mu+2\delta)a - \beta\mu(m-s)}{(r+\beta+2\delta)(r+\beta+2\mu+2\delta)(r+\beta+\mu+2\delta)}$$

So $V_{aa}^D \geq V_{au}^D$ requires that

$$a \geq \frac{\beta\mu(m-s)}{(r+\beta+2\delta)(r+\beta+2\mu+2\delta)} \tag{8}$$

Now $V_{au}^D \geq V_{uu}^D$ also implies the condition (8) on parameters. To see why this is true, notice that from (4) and (6),

$$V_{au}^D - V_{uu}^D = \frac{\beta(V_{aa}^D - V_{au}^D)}{(r+\mu+2\delta)}$$

This simply reflects the fact that, in this marriage pattern, the premium to attractiveness is solely a consequence of the implied continued ability to match with other attractive people.

Similar analysis finds

$$V_{ua}^D \geq V_{uu}^D \Leftrightarrow a \geq \frac{\beta\,(m-s)}{(r+\beta+2\delta)} \tag{9}$$

This means that for unattractive individuals with an attractive spouse, staying at home is incentive compatible as long as a is large relative to $m-s$. In such a case, the prospect of being left is not so awful and the unattractive partner prefers the less stable relationship with the attractive spouse over the possibility of a stable match with an unattractive souse. Moreover, inspection confirms that whenever (9) is satisfied, so is (8).

[8] To see why this is true, suppose that an attractive person enters the otherwise inactive mixed market, other people will enter as well to take advantage of the new matching opportunities now created. By hypothesis we have $V_{ua}^D \geq V_{uu}^D$. This means that there is a positive probability that any entrant will be unattractive making the initial attractive deviant worse off.

Inequalities $V_{as}^D \geq V_{us}^D$, and $V_{aa}^D \geq V_{ua}^D$ are implied by the other restrictions. To see why, subtract (2) from (1) and rearrange to get

$$V_{as}^D - V_{us}^D = \frac{\beta(V_{aa}^D - V_{uu}^D)}{(r + \beta + \mu + \delta)}.$$

and subtract (5) from (3) to get

$$V_{aa}^D - V_{ua}^D = \frac{\beta(V_{ua}^D - V_{us}^D) + \mu(V_{au}^D - V_{uu}^D) + \delta(V_{as}^D - V_{us}^D)}{(r + 2\mu + 2\delta)}.$$

It is worth pointing out that for marriage pattern D to be an equilibrium requires exclusion. In the absence of exclusion, because $V_{ua}^D \geq V_{uu}^D$, unattractive people would enter the attractive people's market.

3.2 The Sham Marriage Pattern (S)

Marriage Pattern S is similar to Marriage Pattern D except that for unattractive individuals, marriage to an unattractive but loyal spouse is preferred to marriage to an attractive cheater. Specifically:

(Sa) no one voluntarily gives up attractiveness

(Sb) unattractive people with attractive partners re-enter the market

(Sc) there is complete market segmentation

Again the steady state is characterized by a positive proportion of the population in each state.

Using similar logic to the previous marriage pattern, the implied asset value equations are:

$$(r + \delta)V_{as}^S = s + \beta(V_{aa}^S - V_{as}^S) + \mu(V_{us}^S - V_{as}^S) \tag{10}$$

$$(r + \delta)V_{us}^S = s + \beta(V_{uu}^S - V_{us}^S) \tag{11}$$

$$(r+\delta)V_{aa}^S = a + m + \mu(V_{ua}^S - V_{aa}^S) + \mu(V_{au}^S - V_{aa}^S) + \delta(V_{as}^S - V_{aa}^S) \quad (12)$$

$$(r+\delta)V_{au}^S = m + \beta(V_{aa}^S - V_{au}^S) + \mu(V_{uu}^S - V_{au}^S) + (\delta+\beta)(V_{as}^S - V_{au}^S) \quad (13)$$

$$(r+\delta)V_{ua}^S = a+m+\beta[(V_{us}^S - V_{ua}^S) + (V_{uu}^S - V_{ua}^S)] + \mu(V_{uu}^S - V_{ua}^S) + \delta(V_{us}^S - V_{ua}^S) \quad (14)$$

$$(r+\delta)V_{uu}^S = m + \delta(V_{us}^S - V_{uu}^S) \quad (15)$$

Equations (10) to (15) can be solved for the V_{ij}^S's. As life for unattractive singles and unattractive people with unattractive spouses is the same under pattern S as under pattern D, $V_{uu}^S = V_{uu}^D$ and $V_{us}^S = V_{us}^D$ as derived in equation (7).

For Pattern S to be an equilibrium, all participants must voluntarily conform. That no one gives up attractiveness (Sa) requires $V_{aa}^S \geq V_{ua}^S$, $V_{au}^S \geq V_{uu}^S$ and $V_{as}^S \geq V_{us}^S$. That the unattractive partner in mixed marriages is disloyal (Sb) requires $V_{uu}^S \geq V_{ua}^S$. Complete market segmentation (Sc) requires either the attractive or the unattractive people prefer their specialized market to the mixed market (i.e. $V_{aa}^S \geq V_{au}^S$ or $V_{uu}^S \geq V_{ua}^S$).

In fact both requirements for market segmentation are always satisfied in this matching pattern. The inequality $V_{uu}^S \geq V_{ua}^S$ is implied by condition (Sb). And,

$$V_{aa}^S \geq V_{au}^S \Leftrightarrow \beta\,(m-s) + (r+2\beta+2\mu+2\delta)a \geq 0 \quad (16)$$

Now,

$$V_{au}^S \geq V_{uu}^S \Leftrightarrow a \geq \frac{\Omega(m-s)}{(r+\beta+2\delta)(r+2\beta+2\mu+2\delta)} \quad (17)$$

where

$$\Omega = (r+\beta+2\mu+2\delta)(r+\mu+\delta) + 2\beta\mu$$

That $V^S_{aa} \geq V^S_{ua}$ follows from $V^S_{aa} \geq V^S_{au} \geq V^S_{uu} \geq V^S_{ua}$

$$V^S_{uu} \geq V^S_{ua} \Leftrightarrow a \leq \frac{\beta\,(m-s)}{(r+\beta+2\delta)} \tag{18}$$

That $V^S_{aa} \geq V^S_{ua}$ follows from $V^S_{aa} \geq V^S_{au} \geq V^S_{uu} \geq V^S_{ua}$ and $V^S_{as} \geq V^S_{us}$ follows directly from (10) and (11).

Notice that in the parameter range implied by (17) and (18) supporting marriage pattern S as an equilibrium does not require exclusion. As both $V^S_{aa} \geq V^S_{au}$ and $V^S_{uu} \geq V^S_{ua}$ hold, neither type would enter each other's specialized market even if it were permitted.

3.3 The Roseanne Marriage Pattern (R)

Under marriage pattern R,

(Ra) attractive individuals maintain attractiveness while single and while married to another attractive person

(Rb) attractive individuals who find themselves with an unattractive partner divest of their attractiveness

(Rc) there is complete market segmentation

Here no one is ever in a mixed marriage, $n^R_{au} = n^R_{ua} = 0$. Other than this, in steady-state, a positive share of the population is in each state. Since the states ua and au are never visited, the system simplifies to,

$$(r+\delta)V^R_{as} = s + \beta(V^R_{aa} - V^R_{as}) + \mu(V^R_{us} - V^R_{as}) \tag{19}$$

$$(r+\delta)V^R_{us} = s + \beta(V^R_{uu} - V^R_{us}) \tag{20}$$

$$(r+\delta)V^R_{aa} = a + m + 2\mu(V^R_{uu} - V^R_{aa}) + \delta(V^R_{as} - V^R_{aa}) \tag{21}$$

$$(r+\delta)V_{uu}^R = m + \delta(V_{us}^R - V_{uu}^R) \tag{22}$$

Again for Marriage Pattern R to be an equilibrium requires specific relationships between the value functions. That attractive singles and attractive people with attractive spouses retain their looks (Ra) requires $V_{as}^R \geq V_{us}^R$ and $V_{aa}^R \geq V_{uu}^R$. These both follow whenever $a > 0$. To ensure that any attractive individual who finds himself with an unattractive spouse divests of his attractiveness (Rb), requires that $V_{uu}^R \geq V_{au}^R$. But so far V_{au}^R is unknown. To obtain this value, consider what happens to an individual who deviates from the specified behavior and retains her appearance when her partner matures. Then,

$$(r+\delta)V_{au}^R = m + \beta(V_{aa}^R - V_{au}^R) + \mu(V_{uu}^R - V_{au}^R) + (\delta+\beta)(V_{as}^R - V_{au}^R) \tag{23}$$

Equation (23) encompasses every eventuality that can happen to the deviant. Consistent with the assumed pattern of behavior, the deviant, while retaining her looks, can get jilted by her current (now unattractive) partner. This possibility is represented by the second β. If this were missing then $V_{uu}^R \geq V_{au}^R$ could not be true. Retaining her looks affords the option value of re-entering the attractive people's marriage market. To ensure that equation (23) correctly contains the second β, requires that the deviant's partner would enter the unattractive marriage market, i.e. $V_{uu}^R \geq V_{ua}^R$.[9] Here

$$(r+\delta)V_{ua}^R = a + m + \beta[(V_{us}^R - V_{ua}^R) + (V_{uu}^R - V_{ua}^R)] + \mu(V_{uu}^R - V_{ua}^R) + \delta(V_{us}^R - V_{ua}^R) \tag{24}$$

Everyone else is assumed to behave consistently with the specified marriage pattern.

Now,

$$V_{uu}^R \geq V_{ua}^R \Leftrightarrow a \leq \frac{\beta\,(m-s)}{(r+\beta+2\delta)} \tag{25}$$

[9]This amounts to checking that the equilibrium is subgame perfect.

and

$$V_{uu}^R \geq V_{au}^R \Leftrightarrow a \leq \frac{\Gamma(m-s)}{(r+\beta+2\delta)(r+2\beta+\mu+2\delta)} \tag{26}$$

where

$$\Gamma \equiv (r+2\mu+2\delta)(r+\beta+\mu+\delta)-\delta\beta > 0$$

Complete market segmentation (Rc) requires that either $V_{aa}^S \geq V_{au}^S$ or $V_{uu}^S \geq V_{ua}^S$. Under the parameter restrictions implied by (25) and (26) both inequalities always hold. (Inequality $V_{aa}^S \geq V_{au}^S$ holds because $V_{aa}^S \geq V_{uu}^R \geq V_{au}^R$.) This means that exclusion is not required to support matching pattern R as an equilibrium - no one wants to enter a market other than the appropriate specialized one.

3.4 Symmetric pure strategy equilibria with complete market segmentation

Each of the preceding subsections identify conditions on parameters that specify the region of the parameter space for which that marriage pattern is an equilibrium. (In the remainder of the paper these will be referred to as Type D, S and R equilibria respectively.) The object here is to find out how these regions are located with respect to each other and how they change with the parameters. These equilibria are symmetric in the sense that everyone behaves the same way. Asymmetric equilibria are considered in the next subsection.

In (s, a) space, each of the equilibrium conditions is represented by a straight line crossing the s axis at the point $s = m$. Consequently each condition can be characterized by an intercept value on the a axis. This is demonstrated in Figure 1. The line joining a_D to m represents conditions (9), (18) and (25). On the diagram, the line joining a_S to m represents condition (17). The line joining a_R to m represents condition (26).

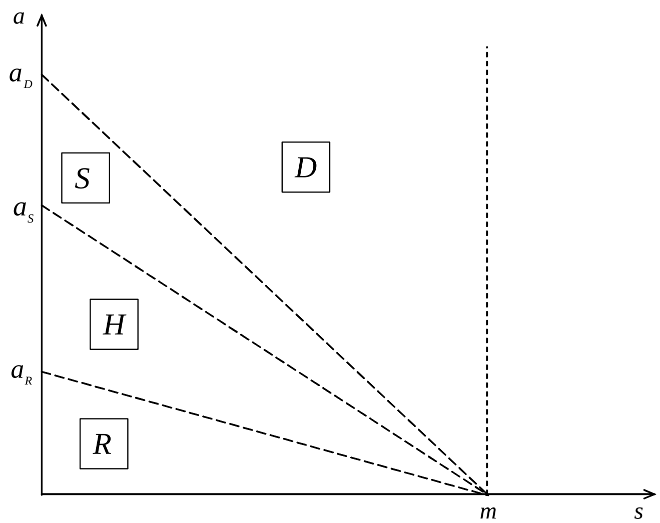

Figure 1: Regions of exisitence of equilibria

Setting $s = 0$ in these conditions yields

$$a_D = \frac{\beta m}{(r + \beta + 2\delta)} \tag{27}$$

$$a_S = \frac{\Omega m}{(r + \beta + 2\delta)(r + 2\beta + 2\mu + 2\delta)} \tag{28}$$

$$a_R = \frac{\Gamma m}{(r + \beta + 2\delta)(r + 2\delta + \mu + 2\beta)} \tag{29}$$

The regions of existence of each type of equilibrium are indicated on Figure 1 by the corresponding letter. The preceding analysis implies that Type D equilibrium exists everywhere above the line joining a_D to m to the left of the $s = m$ line. Type S equilibrium exists between the line joining a_D to m and the line joining a_S to m. Equilibrium type R exists everywhere below both the line joining a_R to m and the line joining a_S to m. As drawn $a_D > a_S > a_R$ but their relative magnitudes actually depend on the other parameters. When $a_S > a_D$ type S equilibrium does not exist for any values of a and s. (The region marked H will be explained below.)

To understand the working of the model, it is helpful to consider why conditions (25), (18) and (9) coincide. Notice, that in all of the equilibria, an unattractive person is subject to being left by an attractive partner. Furthermore, the value to being unattractive and single, V_{us}^N, and unattractive and married to an unattractive person, V_{uu}^N, are the same across all equilibria. This means that if the parameters of the environment are such that in any marriage pattern, unattractive people with attractive spouses are indifferent between looking for another partner and staying at home (*i.e.* $V_{ua}^N = V_{uu}^N$) they will also be indifferent in the other 2 marriage patterns.

As $a_D < m$, whenever $a > m$ loyalty is the optimal strategy for the unattractive partner in mixed marriages. This is because remaining loyal leaves the unattractive partner subject to being made single at rate β and then having to look for an unattractive spouse at rate β. Being disloyal basically halves the wait time to meeting another partner. As long as staying with the attractive person provides more than twice the flow utility of marriage to an unattractive partner, being disloyal is not worthwhile.

Another important result that follows from straightforward but messy algebra is that parameters are such that $a_D > a_S$ (as in Figure 1) if and only if $a_S > a_R$. To see why this is true, fix parameters such that Type S equilibrium exists and condition (17) just binds. Attractive people with unattractive spouses are indifferent between letting themselves go and retaining their looks. We need to understand why the type R marriage pattern cannot be an equilibrium at such parameter values. The reason is that by retaining their looks, consistent with the type S equilibrium, people expose themselves to the possibility of being in state ua at some point in the future and $V_{ua}^S < V_{uu}^S$. If everyone else was following the type R marriage pattern

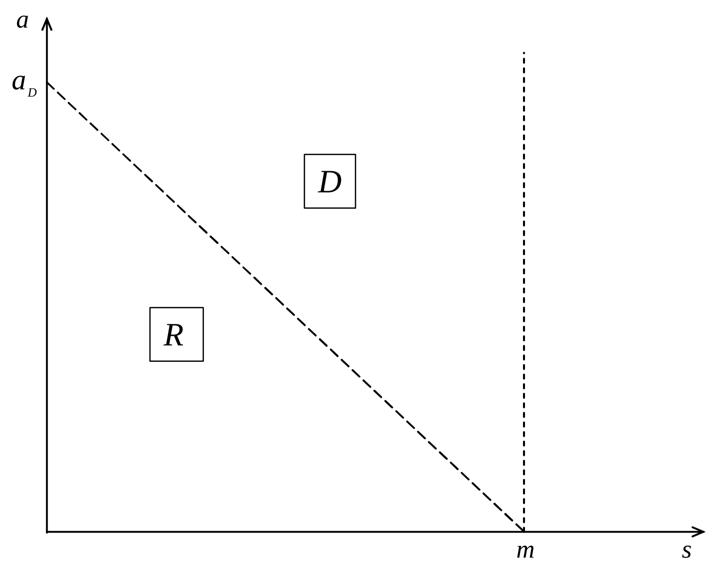

Figure 2: Regions of existence of equilibria for low matching rates.

there would be no chance of entering state ua as any partner they had would divest of their attractiveness. Thus at these parameters $V_{au}^{R} > V_{uu}^{R}$ and a rational individual would deviate from the type R marriage pattern. The locus of points in the parameter space for which $V_{au}^{R} = V_{uu}^{R}$ therefore coincides with that for which $V_{au}^{S} = V_{uu}^{S}$ only when the type S equilibrium ceases to exist. That is when $a_S = a_D$. Figure 2 shows the existence map for these equilibria when parameters are such that $a_R > a_S > a_D$.

3.5 Asymmetric Equilibria

When, as depicted in Figure 1, the parameter values imply $a_R < a_S$ none of the symmetric equilibria discussed above exist in the region marked H. Instead, associated with every point in that region, there is an asymmetric equilibrium in which a proportion $\phi \in (0, 1)$ of those attractive and married people whose partner matures, commit to their current relationship by divesting of their own attractiveness.[10] The remaining $1-\phi$ of such individuals remain attractive even though they are aware that this will drive their current partner to look around for someone else more loyal.

To obtain the value of ϕ associated with a particular set of parameters, we write down the system of steady-state Bellman equations as we have done for the symmetric marriage patterns, and impose additionally that $V_{au}^H = V_{uu}^H$. The system for the general Type H marriage pattern is the same as that for the Type S marriage pattern except that equation (12) becomes:[11]

$$\begin{aligned}(r+\delta)V_{aa}^H &= a+m+\mu[(1-\phi)(V_{ua}^H - V_{aa}^H) + \phi(V_{uu}^H - V_{aa}^H)] \\ &\quad +\mu[(1-\phi)(V_{au}^H - V_{aa}^H) + \phi(V_{uu}^H - V_{aa}^H)] + \delta(V_{as}^H - V_{aa}^H)\end{aligned} \tag{30}$$

It can be shown that ϕ is (generically) a continuous function of the other parameters. For example, if we gradually reduce a, the option value of marriage to another attractive person falls while the value to remaining with the loyal but unattractive spouse remains constant. This tends to increase ϕ

[10] As described, the type H equilibrium is an asymmetric equilibrium in pure strategies. We could also recast it as a symmetric equilibrium in mixed strategies. In that case ϕ would have the interpretation as the probability with which an individual chooses to divest of his or her appearance. Indeed, there is no reason to suppose that individuals will divest immediately their spouse matures. Individuals in state au, here, are always indifferent between committing to their current relationship and retaining their looks in order to re-enter the marriage market. What matters is that at any point in time, a proportion ϕ of those who could have been in state au have chosen to be in state uu.

[11] To be consistent with the interpretation of this equilibrium as asymmetric but pure strategy, equation (30) implicitly assumes that individuals do not know whether they or their current spouse is someone who will commit to the relationship. When viewed as a mixed strategy equilibrium no such assumption is needed.

which partially restores the value to remaining attractive because the possibility of finding oneself in state ua in the future falls.

3.6 Other symmetric pure strategy equilibria

Under the maintained parameter restriction $s < m$, two symmetric equilibria with a single marriage market are known to be possible. There is a "trivial" equilibrium where, everyone gives up attractiveness at birth. This is caused by a pure coordination failure and is common to many search models (see *e.g.* Diamond [1982]). While this equilibrium may be relevant to some societies, we are interested in the interaction between the propensity to remain attractive and divorce and so focus on equilibria where attractiveness is retained beyond birth.

The other possibility falls outside of the primary focus of the paper but a brief description can help illustrate the way the model works. I call this marriage pattern A.

(Aa) both partners in mixed marriages stay home

(Ab) no one divests of their attractiveness

(Ac) all available people are in the mixed market

For this marriage pattern to be an equilibrium requires, from (Aa), that $V^A_{au} \geq V^A_{aa}$ and $V^A_{ua} \geq V^A_{uu}$. Condition (Ab) requires that $V^A_{as} \geq V^A_{us}$, $V^A_{aa} \geq V^A_{ua}$ and $V^A_{au} \geq V^A_{uu}$. Condition (Ac) is a direct consequence of (Aa) and (Ab). Combining these inequalities means that $V^A_{au} \geq V^A_{aa} \geq V^A_{ua} \geq V^A_{uu}$. All singles, attractive and unattractive, are in the market as are people in symmetric marriages. Even though $V^A_{au} \geq V^A_{aa}$, and $V^A_{ua} \geq V^A_{uu}$, symmetric marriages will form between singles. This is because $m > s$ and market (re)entry is costless.

Analysis of this marriage pattern exhibits the complexities that arise when markets are not completely segmented. We now have 12 non-linear equations in the V_{ij}^A's and the n_{ij}^A's as apposed to the 6 linear equations solved for the marriage patterns analyzed above.[12] This precludes determination of the exact restrictions on parameters for which pattern A is an equilibrium. That there are parameter configurations for which it is an equilibrium, however, should be clear. If a is very small, the value to marital stability can easily out weigh the value to marriage to an attractive person. Of course, the mixed marriages are only stable because everyone believes them to be so. This equilibrium is very reminiscent of those found by Burdett at al [2004]. They found multiple equilibria to exist based on beliefs as to the stability of marriages. In the current framework, the counterpart equilibrium, in which mixed marriages are unstable does not exist except in the form of the equilibria discussed above. That is, there can be no equilibrium with an active mixed market in which mixed marriages are unstable. For instability of such marriages, we need either $V_{aa} \geq V_{au}$ or $V_{uu} \geq V_{ua}$. If either of these inequalities holds, at least one group will confine themselves to their specialized market.

[12] For example someone in state aa moves to state as if her current partner dies or her current partner leaves her (for an unattractive person); she moves to state au if she meets an unattractive person or if her current partner matures; she moves to state ua if she matures. The asset equation for state aa is, therfore,

$$(r+\delta)V_{aa}^A = a + m + (\beta\psi_{aa}^A + \delta)(V_{as}^A - V_{aa}^A) + (\mu + \beta\psi_{aa}^A)(V_{au}^A - V_{aa}^A) + \mu(V_{ua}^A - V_{aa}^A)$$

where ψ_{aa}^A is the probability that a meeting including a person in state aa leads to match formation. Thus

$$\psi_{aa}^A = (n_{us}^A + n_{uu}^A)/(n_{aa}^A + n_{as}^A + n_{us}^A + n_{uu}^A)$$

4 Discussion

4.1 Exclusion

With the development of directed (or competitive) search, the idea that specialized markets can form should not be controversial. The notion of exclusion, however, is arbitrary and is made here as a deliberate means of simplifying the analysis.

Under exclusion, the only restriction on parameters required for the equilibria with complete market segmentation to exhaust the parameter space is $m \geq s$. The analysis of equilibrium types S, R an H, however, showed that existence of these equilibria does not require exclusion. This means that as long as a, the flow utility from marriage to an attractive person, is small enough relative to $m - s$, as indicated by condition (25), we could do away with exclusion all together. This would eliminate equilibrium type D but all the remaining analysis would stand.[13]

4.2 Implications of the model

The model has testable implications for the pattern of divorce. The only kind of weddings that can lead to divorce are those between attractive people. While both partners remain attractive they can do no better in the market and they stay at home - the divorce hazard is zero. When one partner matures, divorce becomes a possibility. If the attractive partner does not divest of her attractiveness, it means that she is looking for another attractive person and her partner may also re-enter the (unattractive) marriage market. The divorce hazard for such couples is either β or 2β. Once the second partner

[13] In the absence of exclusion equilibria that look a lot like marriage pattern D will be possible but everyone will search in the mixed market.

r	β	μ	δ	m	a	s
0.05	0.33	0.1	0.025	1	0.25	0.4

Table 1: Parameter Values for the Leading Example

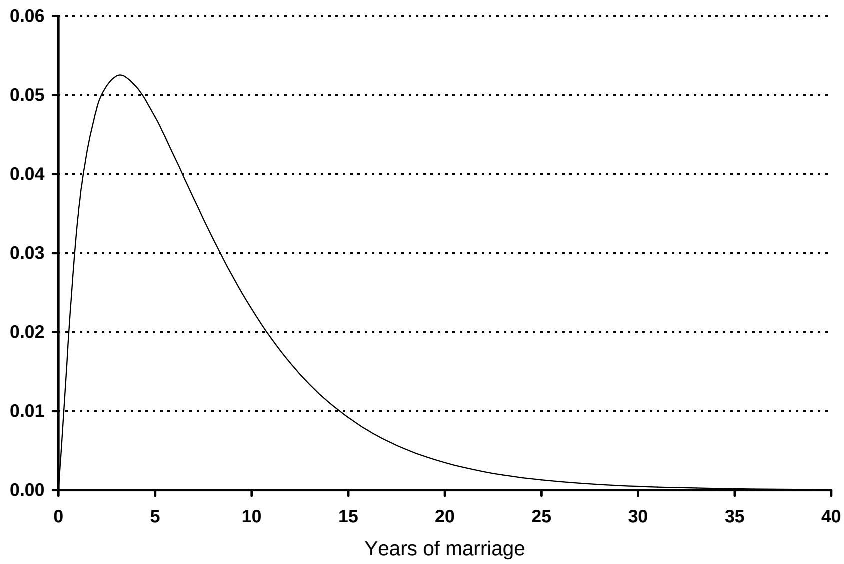

Figure 3: Anualized Hazard Rate of Divorce for Leading Example

matures naturally the divorce hazard drops back to 0. While the implied hazard rate for an individual marriage is discontinuous, the realized pattern of aging differs across people and the implied aggregate divorce hazards are smooth. Figure 3 plots the annualized hazard rate of divorce for the leading example. Table 1 contains the parameters.

In the model only those marriages that start out as *aa* marriages can end in divorce. As older people are more likely to be unattractive, a further implication of the model is that the probability that any marriage ends in divorce falls with the *chronological* age of the individuals getting married.

In the leading example, assuming that individuals initially enter the market at age 20, the divorce rate declines from 62.5% for those marrying at 20 to 15.5% for people who get married at 40.[14]

These phenomena have been documented in the empirical divorce literature. In particular Weiss and Willis [1997] look at divorce hazards in the NLSY data set. They attribute the hump shaped divorce hazard to learning about match quality. They offer no real explanation for why marriages between older people are more stable than those between young people. Differential aging as put forward here is able to explain both facts simultaneously.

In any equilibrium where people use appearance as a commitment device, married people will be less attractive than their unmarried counterparts. How people might divest of their attractiveness is an open question. A *credible* commitment device, however, should be something that is not readily reversed. A clear possibility here is weight-gain. If so, an implication of the model would be that married people are heavier than their single counterparts.

5 Conclusion

The point of this paper has been to demonstrate that in a matching model with differential aging, allowing for surreptitious market re-entry implies patterns of divorce that fit well with the stylized facts. Married individuals are likely to face periods of instability caused by the inability to credibly promise fidelity.

The paper has explored one costly means of addressing this problem: de-

[14] As new marriages are either aa or uu, this 15.5% is the probability that a new marriage involving a 40 year old ends in divorce. Of course, this probability would change if the age of the spouse is known as this would change the likelihood of the marriage being type aa.

liberate aging which restores stability by eliminating the incentive for both partners to seek a new relationship. Essentially, attractiveness which is observable is used as a proxy for market entry which is unobservable. The paper shows that individuals will use such a proxy even though it is costly to both partners. Other observable characteristics of individuals or the match might be used in the same way. For instance, acquiring common property increases the cost of separation; having children can increase the value to remaining in a current marriage; moving to the suburbs might reduce matching rates for either partner (see Gautier, Svarer and Teulings [in progress]).

An important avenue for future research that emerges from this work is to gauge the quantitative importance of aging (or health) shocks in predicting divorce. With the right data one could test this theory of divorce against Jovanovic [1979] type learning about the match. Matching models could also be used to address differences across genders in terms of their observed patterns of marriage and divorce. Of course, this would require constructing a two gender model. If the option to divest of attractiveness were to be removed, such a model which might remain sufficiently tractable for purposes of calibration.

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