Optimal Unemployment Insurance in a Matching Equilibrium

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Abstract

This paper considers the optimal design of unemployment insurance (UI) within an equilibrium matching framework when wages are determined by strategic bargaining. Unlike the Nash bargaining approach, reducing UI payments with duration is welfare increasing. A co-ordinated policy approach, however, one that chooses job creation subsidies and UI optimally, implies a much greater welfare gain than one which considers optimal UI alone. Once job creation subsidies are chosen optimally, the welfare value of making UI payments duration dependent is small.

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1 Introduction

This paper considers optimal unemployment insurance (UI) in an equilibrium matching framework where UI payments distort wages. In contrast to the standard Nash bargaining approach (e.g. Millard and Mortensen (1997), Davidson and Woodbury (1997), Cahuc and Lehmann (2000), Fredriksson and Holmlund (2001)), this paper adopts the strategic wage bargaining approach. This is important as strategic bargaining deals explicitly with the non-stationarity implied by a duration dependent UI system. For reasons analogous to those given in Shavell and Weiss (1979), we show that a duration dependent UI program can increase welfare. This occurs as UI payments at medium durations, particularly those around the one-year mark, raise the option value of remaining unemployed at short unemployment durations. In Shavell and Weiss (1979) this leads to low search effort. Here by raising the reservation wage of unemployed workers, the UI program leads to high equilibrium wages. Decreasing UI payments with duration is welfare increasing as it reduces the distortion of UI payments on wage levels. In contrast, the axiomatic Nash bargaining approach implies UI payments should increase with duration.

This paper also shows that job creation subsidies can generate significant welfare improvements. Increasing the stock of vacancies makes the unemployed better off through a thick market externality, and so job creation subsidies are an effective way of insuring workers against unemployment risk. Furthermore once optimal UI is co-ordinated with optimal job creation subsidies, we show that making UI payments duration dependent cannot yield significant welfare improvements. We shall argue that, in a matching equilibrium, this insight applies equally to the endogenous search effort approach.

Following Shavell and Weiss (1979), there is a large principal-agent literature which considers optimal UI (recent contributions include Hopenhayn and Nicolini (1997), Werning (2001)). As that literature does not consider equilibrium, however, the policy implications are potentially misleading. For example a matching equilibrium implies higher job search effort by one worker reduces the job finding prospects of others. It is not obvious that a micro-policy, whose aim is to increase the job search effort of a laid-off worker and so reduce the cost of layoff insurance, is an appropriate macro-policy. For one thing it ignores the welfare of the unemployed. Of course it is well known there are also thick market externalities, where higher aggregate job search effort increases the return to creating a vacancy. Optimal policy requires taking these two countervailing externalities into account; e.g. Hosios (1990) when all are risk neutral.

There is currently little consensus of results on optimal UI in a matching equilibrium. Millard and Mortensen (1997), Fredriksson and Holmlund (2001) argue UI payments should decrease with duration, Cahuc and Lehmann (2000) argue UI payments might increase with duration while Davidson and Woodbury (1997) argue payments should not vary with duration. This ambiguity arises, at least in part, because that literature adopts a Nash bargaining approach which implies payments might increase with duration. The reason is perhaps clearest in Cahuc and Lehmann (2000) who motivate the Nash bargaining framework by assuming an insider/outsider wage determination process. Currently employed workers (insiders) negotiate wages and the employer is not allowed to wage discriminate between insiders and outsiders. Cahuc/Lehmann argue that early UI payments might be kept low - so that insiders have a low value of being laid-off and so negotiate relatively low wages - while later UI payments are more generous so as to improve the welfare of the longer-term unemployed.

This paper instead assumes strategic bargaining where, in the absence of a union, an unemployed worker negotiates directly with a potential employer. An important feature of strategic bargaining is that the negotiated wage depends on the unemployed worker's option value of remaining unemployed.[1] For plausible parameter values, we show that this option value effect implies that UI payments around the one-year duration mark distort average negotiated wages the most and the distortion falls slowly with duration thereafter. In contrast to the Nash bargaining approach, Shavell-Weiss type arguments suggest that UI payments should decrease with duration in an optimal program.

Another contribution of this paper is that it considers a different but equally plausible market failure. Rather than assume the Planner does not observe job search effort, it is assumed instead that the Planner does not observe job offers (and job search effort is fixed exogenously). The two types of moral hazard are related but distinct. The search effort distortion implies a quantity distortion - insured workers choose too little job search effort. In contrast unobserved job offers implies a price distortion - the option of rejecting a job offer and continuing to receive UI raises a worker's reservation wage (e.g. Mortensen (1977), van den Berg (1990), Albrecht and Vroman (2001)). In a Pissarides (2000) context as considered here, this distortion leads to higher negotiated wages which drives down equilibrium job creation rates. As in the efficiency wage literature, distortions on wages can yield

[1] Millard and Mortensen (1997), Fredriksson and Holmlund (2001) argue that renegotiation constraints bind ex-post; i.e., once the employed worker is entitled to full UI coverage, the worker renegotiates the wage. But that ignores that the worker's renegotiation threat is to quit into unemployment, and workers who quit are typically not entitled to receive UI.

large macroeconomic effects.

Nevertheless the theoretical parallels between the two approaches are close. Lemma 5 in Fredriksson and Holmlund (2001) establishes that regardless of worker bargaining power, the optimal UI program implies incomplete insurance and low aggregate search effort. Here with unobserved job offers, the optimum implies incomplete insurance and high (reservation) wages. Both distortions, however, generate the same underlying policy trade-off - improved insurance leads to marginally lower vacancy creation rates. The search effort approach implies fewer vacancies because of the thick market externality - lower aggregate search effort implies it takes longer to fill a vacancy and so fewer vacancies are created by firms. Here higher wages drive down vacancy creation rates directly.

An important feature of the analysis, however, is that it considers other policy instruments. Given the underlying policy trade-off is between better unemployment insurance and lower equilibrium vacancy creation rates, job creation subsidies play an important role. Such a policy approach has been ignored by the previous optimal UI literature. The Planner's objective here is to maximise a (steady state) Utilitarian welfare function and is allowed three policy variables; (i) a UI scheme which pays a job seeker $b(\tau)$ at duration τ,(ii) an employment tax x, and (iii) a job creation subsidy s, where those instruments must satisfy budget balance.

If there is no disutility to work, the paper shows that the Planner can achieve the First Best allocation using the following mix of policies (i) a constant UI program $b(.) = \bar{b}$, (ii) an employment tax which extracts all match rents (i.e. firms make zero profit ex-post and workers are hired at wage $w = \bar{b}$), (iii) full job creation subsidisation. The insight is that the Planner targets the vacancy creation distortion by subsidising equilibrium vacancy creation rates, which is financed by a tax on employment. In essence the Planner solves the hold-up problem (where firms must invest in a vacancy before hiring a worker) by taxing away all ex-post rents and then using those rents to fully subsidise job creation rates at the socially optimal level. As there is no match surplus ex-post, workers negotiate $w = \overline{b}$ and there is full insurance. The optimal choice of $\overline{b}$ reflects the shadow value of labour.

If there is a strictly positive disutility to work, $c > 0$, then the first best outcome is not incentive compatible. Full insurance requires wage $w = \overline{b}$ but job seekers then prefer to remain unemployed (and so avoid the work cost $c > 0$). Restricting attention to constant UI programs, it is shown that the above tax policy remains optimal - the government extracts all match rents with an employment tax and uses those revenues to fully subsidize job creation investments. But insurance is incomplete as a wage gap $w - b$ has to exist to compensate workers for their disutility of work $c > 0$. However for reasonable parameter values it is argued that the welfare loss relative to the First Best is small and so a duration dependent UI program cannot significantly improve welfare.

Interesting issues arise if we rule out vacancy creation subsidies - say the government does not observe the vacancy creation process. In that case offering vacancy creation subsidies may generate perverse incentives. For example, a firm might claim it has created a vacancy [and so claim the subsidy] even though it has no intention of hiring a worker. If the government insists that the firm must hire a worker to claim the subsidy, an employer might nominally fire an employee, claim the vacancy creation subsidy, and then re-hire that worker.

Assuming job creation subsidies are not properly implementable, a duration dependent UI program becomes optimal. In particular we find that, for plausible parameter values, UI payments around the one-year duration mark distort average negotiated wages the most. Furthermore as the average duration of unemployment is around 13 weeks in the U.S., the insurance value of such payments is relatively small. Given the Planner (at the optimum) trades-off better quality insurance against keeping reservation wages low, the Planner can afford to pay relatively generous UI for short durations and still provide reasonable insurance against unemployment risk (where most workers are re-employed within 6 months) but reduces UI payments at long durations to stop (reservation) wages being driven too high.

Simulations formally establish that with strategic wage bargaining and no job creation subsidies, decreasing UI payments with duration increases total welfare. In contrast, simulations with insider/outsider Nash bargaining find that increasing UI payments with duration increases welfare. To the extent that one believes the strategic bargaining approach is a more appropriate device for determining equilibrium wages, then the simulations of Millard and Mortensen (1997), Cahuc and Lehmann (2000), Fredriksson and Holmlund (2001) understate the optimal rate of decrease of UI payments with duration.

A second finding, however, is that in the absence of the additional job search distortion, the suggested optimal rate of decrease of UI payments is small and the corresponding welfare improvement is also small. The underlying implication seems to be that, in a matching equilibrium, the principal policy aim is to generate efficient job creation rates. If the Planner cannot use vacancy creation subsidies directly, then instead the Planner implements a gross wage (defined as the average worker wage plus any employment tax x) which induces (constrained) efficient vacancy creation rates; i.e. the most efficient way to insure workers against unemployment risk is to have efficient re-employment rates. Cutting UI payments with duration generates some welfare benefit in the third best problem, but the gains appear slight.

2 The Model.

Time is continuous and has an infinite horizon. Throughout only steady states are considered. There is a continuum of identical workers with mass normalized to one, and all workers are infinitely lived. Each worker is either employed or unemployed where U is the measure of unemployed workers. There is also a continuum of vacancies with measure $V > 0$, where V will be determined endogenously via a standard free entry condition.

There are matching frictions where a matching function $M = M(U, V)$ describes the aggregate contact rate between the unemployed job seekers and the firms holding vacancies. M is strictly increasing in both arguments, continuous, concave and homogenous of degree 1 with $M(0, V) = M(U, 0) = 0$ and $M_V(U, 0) = \infty$ for $U > 0$. $\phi = V/U$ denotes labor market tightness. Symmetry implies an unemployed worker receives a job offer according to a Poisson process with parameter α_w where

$$\alpha_w = \frac{1}{U}M(U, V) = M(1, \frac{V}{U}) \equiv m(\phi)$$

and m is a strictly increasing, concave function of ϕ with $m(0) = 0, m'(0) = \infty$. Similarly, a vacancy contacts a worker at rate α_f where

$$\alpha_f = \frac{1}{V}M(U, V) = \frac{U}{V}M(1, \frac{V}{U}) \equiv \frac{m(\phi)}{\phi}.$$

Firms are risk neutral while workers are risk averse. All have the same discount rate r. A worker employed at wage w obtains flow payoff $[u(w) - c]$, where u is a strictly increasing, concave and twice differentiable function. $c \geq 0$ measures the disutility to work. A firm who employes a worker at wage w obtains flow profit $(\pi - w - x)$, where x is the employment tax levied by the government.

There are idiosyncratic job destruction shocks where each job dies according to an independent Poisson process with parameter $\delta > 0$. As all jobs are equally likely to be destroyed, assume that all employed workers are entitled to full UI coverage.

The UI program is a benefit function $b(.)$ which pays a worker who has been unemployed for duration τ a (flow) UI payment $b(\tau)$. Assume $b(.)$ is a positive function and that $\lim_{\tau \to \infty} b(\tau) = \underline{b}$ exists. An important assumption is that the government does not observe job offers and so a worker who rejects a job offer remains entitled to continued UI. Also assume that the UI program only covers workers against job destruction shocks - workers who quit receive no UI payments.

Wages are determined by bargaining. Should a firm and worker reach agreement, assume they sign an enforceable contract which specifies a fixed wage w until the job is exogenously destroyed.[2] When the job is destroyed, assume the firm goes bankrupt [with a zero payoff] while the worker returns to the pool of unemployed workers, but with duration $\tau = 0$; i.e. finding work implies the worker becomes re-entitled to full UI coverage.

In principle, the wage may be subject to renegotiation by mutual agreement. But as there are no productivity shocks (apart from pure job destruction shocks) and given a worker who quits is not entitled to receive UI, the wage is never renegotiated in equilibrium - the worker is better off employed at the negotiated wage (when that wage was negotiated, the worker held the option of continuing UI support) than quitting into unemployment with no UI support.

[2] As workers are risk-averse, a constant wage is the optimal contract.

To ensure a dynamically consistent equilibrium, equilibrium wages are determined using the strategic bargaining approach with period $dt > 0$ (small) between offers. Suppose therefore at date t, a firm is negotiating with a worker who has current unemployment duration τ (which is observed by the firm). At the start of this time period, with probability θ Nature chooses the firm ($i = f$) to make a wage offer, and with probability $1 - \theta$ chooses the worker ($i = w$) to make the offer. Given the wage offer w_i made by agent $i = w, f$, the other agent $-i$ either accepts the offer or rejects it. If the offer is accepted, a contract is signed at the offered wage w_i and the match is consummated. If the offer is rejected, there is a one-period delay during which the firm makes zero profit but, as the government does not observe job offers, the worker receives UI payment $b(\tau)dt$ from the government, and so obtains utility payoff $u(b(\tau))dt$ during that delay. To avoid a spurious re-entitlement effect, assume that jobs are not subject to job destruction shocks during bargaining.[3] Hence bargaining resumes after the one period delay with updated unemployment duration $\tau + dt$.

For tractability, assume workers have no savings - otherwise we need to track the distribution of worker assets over time. No savings also simplifies

[3]Otherwise should the job be destroyed during delay, which occurs with probability δdt,then next period the worker remains unemployed with payoff $V_u(\tau + dt)$. In contrast, if the worker had accepted employment, then the instantaneous re-entitlement assumption implies the worker would instead obtain $\overline{V}$. This creates an additional return of $\delta[\overline{V} - V_u(\tau + dt)]dt$ to reaching immediate agreement. This effect is clearly spurious. UI schemes typically require workers to be employed for a certain length of time to become re-entitled to full UI. The re-entitlement value of being employed for a short period dt should therefore only be $0(dt)$, and so this effect should only have value $0(\delta dt^2)$. As this is negligible in the limiting equilibrium as $dt \to 0$,we can rule it out simply by assuming there is no job destruction while bargaining.

the bargaining game. For example, Coles and Hildreth (2000) describe a strategic bargaining game assuming a firm sells out of its inventory of finished goods during delay. Their results suggest that if a worker's savings are common knowledge, then the more savings a worker has the higher the wage he/she can negotiate [as delay to agreement is less costly]. However if the worker has hidden savings, which is the more reasonable case, we know from the bargaining literature with asymmetric information that a continuum of equilibria may then be possible [e.g. Ausubel and Deneckere (1989)]. Ruling out savings behavior by assumption avoids such complications. Of course this assumption implies the model overstates the value of UI - workers could otherwise self insure against layoff risk using a precautionary savings strategy [see Costain (1996), Werning (2001), Lentz (2002)].

Following Pissarides (2000) a firm pays a flow cost $a > 0$ to keep a vacancy open. With free entry, the number of vacancies adjusts so that the expected discounted value of creating a vacancy is zero.

The next section determines equilibrium wages, denoted $w^*(\tau; .)$, which (among other things) will depend on the worker's unemployment duration τ at the time of agreement. Given those wages, Section 4 describes a Matching Equilibrium and Section 5 then discusses optimal policy.

3 The Wage Bargaining Equilibrium.

If a firm and worker negotiate a contract at agreed wage w, their respective expected discounted payoffs are $\pi_f(w) = (\pi - w - x)/(r + \delta)$ and $\pi_w(w) = [u(w) - c + \delta\overline{V}]/(r + \delta)$ where given a job destruction shock, the worker obtains expected lifetime payoff $\overline{V}$ by re-entering the unemployment pool. Although $\overline{V}$ will be determined endogenously, the firm and worker take its value as given while bargaining. They also take $b(.)$ as given.

If $b(\tau)$ is too large, a worker with unemployment duration τ will prefer remaining unemployed to starting work. As we are only interested in markets where a gain to trade exists between firms and workers, it will never be optimal for the government to pay such large b. The following Claim describes a restriction on b so that at any duration τ, the jointly efficient outcome between an employer and a job seeker is that the job seeker starts work immediately. This condition will be referred to as the Shrinking Pie property and corresponds to a ceiling on benefits paid.

Claim 1: The bargaining game has the Shrinking Pie property if $b(.)$ satisfies

$$u(b(\tau)) \leq \frac{r}{r+\delta}[u(\pi - x) - c + \delta\overline{V}] \text{ for all } \tau. \tag{1}$$

Proof is in the Appendix.

The proof in the Appendix establishes that if $b(\tau)$ does not satisfy (1) at τ, then it is jointly efficient for an unemployed worker with duration τ and a potential employer to defer starting work. Given the Shrinking Pie condition is satisfied, Coles and Muthoo (2003) establish that for any alternating offers bargaining game, a subgame perfect equilibrium exists for any $dt > 0$ (though multiple equilibria are possible). They also establish that if payoffs evolve continuously over time then, as $dt \to 0$, all equilibria converge in outcome to the same limiting equilibrium. That limiting equilibrium implies efficient trade [i.e. agreement is always reached immediately] and is consistent with a Markov perfect equilibrium where agents use history independent bargaining strategies. We refer to an equilibrium of this class as an Immediate Trade Equilibrium (ITE). Coles and Wright (1998) assumes this convergence result and describes the ITE when the value of the "pie" is time varying. Theorem

1 describes the equilibrium wage agreement when instead inside payoffs are time varying.

Theorem 1.

If $b(.)$ satisfies the shrinking pie property (1), then in the limit as $dt \to 0$ an ITE implies w^* is the solution to the differential equation

$$u'(w)\frac{dw}{dt} = r(r+\delta)\left\{\theta\left[\frac{u(w) - c + \delta\overline{V}}{r+\delta} - \frac{u(b(t))}{r}\right] - (1-\theta)u'(w)\left[\frac{\pi - w - x}{r+\delta}\right]\right\}, \tag{2}$$

subject to the boundary condition $w \to \underline{w}$ as $t \to \infty$, where $\underline{w}$ is defined by

$$\theta\left[\frac{u(\underline{w}) - c + \delta\overline{V}}{r+\delta} - \frac{u(\underline{b})}{r}\right] = (1-\theta)u'(\underline{w})\left[\frac{\pi - \underline{w} - x}{r+\delta}\right]. \tag{3}$$

We omit a proof as the argument is a straightforward generalisation of Coles and Wright (1998). Coles and Masters (2003) show that when workers are risk neutral, the above differential equation has a similar structure to the one describing the optimal reservation wage strategy with sequential search (e.g. Mortensen (1977), van den Berg (1990)). In both frameworks, the option of continuing to receive further UI increases the value of remaining unemployed and so raises the worker's (reservation) wage. The difference is that here, if an offer is rejected, the worker prefers to continue bargaining rather than continue search.[4] Nevertheless, the qualitative impact of a duration dependent UI program on (reservation) wages is identical - wages decline (continuously) as the worker's entitlement to further UI expires.

As equilibrium wages in (2) are described by a non-linear differential equation, closed form solutions exist only for two special cases.

[4]For dt small enough, the Shrinking Pie condition implies it is always better to keep bargaining and reach agreement next period than search for an alternative match.

3.1 Illustrative Case I - Risk Neutral Workers.

Suppose workers are risk neutral, and without further loss of generality assume $u(w) = w$. Theorem 1 implies w^* satisfies the linear differential equation

$$\frac{dw^*(\tau)}{dt} - rw^*(\tau) = \left\{\theta\left[r[\delta\overline{V} - c] - (r+\delta)b(\tau)\right] - (1-\theta)r\left[\pi - x\right]\right\},$$

subject to the boundary condition (3). Integration implies

$$w^*(\tau) = \theta(r+\delta)\int_{\tau}^{\infty} e^{-r(t-\tau)}b(t)dt + (1-\theta)[\pi - x] - \theta[\delta\overline{V} - c]. \quad (4)$$

The equilibrium wage $w^*(\tau)$ is composed of three terms. First, the option of receiving further UI directly raises the worker's reservation wage, and so raises the negotiated wage. In essence, the firm is forced to compensate the worker for foregone UI payments. Note, $b(.)$ decreasing implies the negotiated wage w^* falls with unemployment duration. Second if $\theta < 1$ (i.e. the worker has some bargaining power), the worker extracts part of the firm's production rents $[\pi - x]$. Third if $\theta > 0$ (i.e. the firm has some bargaining power), the firm extracts part of the worker's employment rents which depend on $\overline{V}$, the value of becoming re-entitled to full UI coverage. This entitlement effect is well known; e.g. Mortensen (1977) who argues that higher benefits make employment more attractive relative to non-insured unemployment.

Further insight is obtained by solving explicitly for the re-entitlement effect. The value of being unemployed satisfies

$$rV_u(t) - \frac{dV_u(t)}{dt} = b(t) + \alpha_w[\pi_w(w^*(t)) - V_u(t)],$$

where $\pi_w(w)$ is the worker's expected value of employment at wage w, and $w = w^*(t)$ in an ITE. Integrating, putting $t = 0$ and noting that $\overline{V} = V_u(0)$

implies

$$\overline{V} = \int_0^\infty e^{-(r+\alpha_w)t} b(t)dt + \int_0^\infty \alpha_w e^{-(r+\alpha_w)t} \pi_w(w^*(t))dt; \tag{5}$$

i.e. the value of being laid-off is the expected discounted value of total UI receipts when laid-off plus the expected value of re-employment. Also define the fair employment tax

$$x = \delta \int_0^\infty e^{-(r+\alpha_w)t} b(t)dt, \tag{6}$$

where the integral describes the expected discounted cost of UI payments given a worker is laid-off. Substituting out $\overline{V}$ in the wage equation (4) now implies the following.

Claim 2. In an ITE with risk neutral workers and a fair employment tax, an unemployed worker with duration τ negotiates wage

$$\begin{aligned} w^*(\tau) &= c - x + (1-\theta)(\pi - c) + \theta(r+\delta)\int_\tau^\infty e^{-r(t-\tau)} b(t)dt \\ &\quad -\delta\theta \int_0^\infty e^{-(r+\alpha_w)t} \alpha_w \pi_w(w^*(t))dt. \end{aligned} \tag{7}$$

and obtains expected payoff

$$\begin{aligned} \pi_w(w^*(\tau)) &= (1-\theta)\frac{\pi - c}{r+\delta} + \theta \int_\tau^\infty e^{-r(t-\tau)} b(t)dt \\ &\quad + \frac{\delta}{r+\delta}(1-\theta)\int_0^\infty \alpha_w e^{-(r+\alpha_w)t} \pi_w(w^*(t))dt. \end{aligned} \tag{8}$$

Proof follows by using the expressions obtained for w^*, $\overline{V}$ and x.

The worker's equilibrium payoff, $\pi_w(.)$, depends on three terms. The second term depends on the worker's option value of continuing to receive further UI at the point of hire, while the third term describes the (net) re-entitlement effect. Comparing with (5), the last term in (8) depends on the expected value of re-employment once the worker is laid-off at some future date (where the subsequent hiring date t is exponentially distributed with parameter α_w). The crucial insight is that $\pi_w(.)$ does not depend on the tax rate x, nor on expected UI receipts. The wage equation, (7), implies the worker pays for the employment tax through a lower wage, but that term is washed out in $\pi_w(.)$ by the entitlement to receive UI when laid-off (given a fair employment tax). The UI scheme, however, is not neutral. A worker hired at duration τ extracts rents $\theta \int_{\tau}^{\infty} e^{-r(s-\tau)} b(s) ds$ from the new employer. But that worker then becomes re-entitled to full UI coverage. The re-entitlement effect implies that when laid off in the future, the worker becomes re-employed at some (exponentially distributed) duration t and extracts rents $\theta \int_{t}^{\infty} e^{-r(s-t)} b(s) ds$ from that future employer. As the last term in the wage equation (7) shows, the current hiring firm extracts part of those future expected rents through a lower hiring wage. The re-entitlement effect therefore implies a transfer of rents from future hiring firms to current hiring firms. For reasonable parameter values this transfer is not insignificant ; e.g. $r = 5\%$ per annum and $\delta = 25\%$ (expected employment duration of 4 years) imply $\delta/(r+\delta) = 5/6$.[5]

3.2 Illustrative Case II - Constant UI Schemes.

Suppose instead workers are risk averse but $b(\tau) = b$ for all τ. The (unique) solution for w^* defined by Theorem 1 is $w^* = w$ for all τ, where by (3),

$$\theta \left[\frac{u(w) - c + \delta \overline{V}}{r+\delta} - \frac{u(b)}{r} \right] = (1-\theta) u'(w) \left[\frac{\pi - w - x}{r+\delta} \right]. \qquad (9)$$

[5] Coles and Masters (2004) establish that this inter-temporal transfer effect is employment stabilising over the cycle.

The asset pricing equation for V_u implies

$$rV_u = u(b) + \alpha_w[\frac{u(w) - c + \delta\overline{V}}{r+\delta} - V_u]$$

and as $\overline{V} = V_u$, we can solve for $\overline{V}$ as

$$r\overline{V} = \frac{(r+\delta)u(b) + \alpha_w[u(w) - c]}{r + \delta + \alpha_w}. \tag{10}$$

(9) and (10) are two equations which jointly determine the equilibrium wage w and the value of being laid off $\overline{V}$ given a constant UI program. It is now possible to establish the following claim [we omit the proof].

Claim 3. [Constant UI programs]

If $u(b) < u(\pi - x) - c$, then an ITE exists where $(w, \overline{V})$ satisfy (9), (10) and the Shrinking Pie property is satisfied.

A constant UI program implies there are no re-entitlement effects on finding work, and so the Shrinking Pie condition described in Claim 1 reduces to the one given in Claim 3. It requires only that $u(b)$, the flow payoff by being unemployed, is less than the payoff to being employed and extracting all firm rents.

4 A Matching Equilibrium

To assess the value of various UI programs, this section determines α_w endogenously by defining a Matching Equilibrium as described in Pissarides (2000). As the basic structure is well known we quickly sketch the appropriate equilibrium conditions (assuming the Shrinking Pie assumption (1)).

Steady State Unemployment, U, satisfies the flow condition $\delta(1 - U) = \alpha_w U$, and so

$$U = \frac{\delta}{\delta + m(\phi)},$$

where $\alpha_w = m(\phi)$ and ϕ is labour market tightness.

Steady State Vacancies, V, are determined by the standard (zero profit) free entry condition which, with random matching, is given by

$$a - s = \alpha_f \int_0^\infty [\alpha_w e^{-\alpha_w \tau}] \frac{\pi - w^*(\tau) - x}{r + \delta} d\tau, \tag{11}$$

where a describes the flow cost of creating a new vacancy and s is the job creation subsidy offered by the government. (11) equates the expected flow cost to vacancy creation to its expected flow return, where a firm contacts a worker at rate α_f and, conditional on a contact, that worker's duration τ is exponentially distributed with parameter α_w and, conditional on τ, the firm negotiates wage $w^*(\tau)$ which implies expected profit $\pi_f = (\pi - x - w^*(\tau))/(r + \delta) > 0$.

The value of being laid off. $V_u(.)$ satisfies the asset pricing equation

$$rV_u(t) - \frac{dV_u(t)}{dt} = u(b(t)) + \alpha_w [\frac{u(w^*(t)) - c + \delta \overline{V}}{r + \delta} - V_u(t)].$$

Integrating and noting that $\overline{V} \equiv V_u(0)$ implies

$$\overline{V} = \int_0^\infty e^{-(r+\alpha_w)t} \left\{ u(b(t)) + \alpha_w \frac{u(w^*(t)) - c + \delta \overline{V}}{r + \delta} \right\} dt$$

which can be rearranged as

$$r\overline{V} = \frac{(r + \delta)(r + \alpha_w)}{(r + \delta + \alpha_w)} \int_0^\infty e^{-(r+\alpha_w)t} \left\{ u(b(t)) + \frac{\alpha_w}{r + \delta} [u(w^*(t)) - c] \right\} dt. \tag{12}$$

Budget balance. Assuming the good is non-storable, budget balance requires that [real] government transfers satisfy

$$[1 - U]x = sV + U \int_0^\infty \alpha_w e^{-\alpha_w \tau} b(\tau) dt. \tag{13}$$

We can now define a Matching Equilibrium.

Definition: Given policy parameters $(b(.), s)$, a Matching Equilibrium is defined as a vector $\{w^*(.), \overline{V}, U, V, \phi, x\}$ where w^* is given by (2) with boundary condition (3), $\overline{V}$ is given by (12), $U = \delta/(\alpha_w + \delta)$, $V = \phi U$, ϕ satisfies (11) with $\alpha_w = m(\phi), \alpha_f = m(\phi)/\phi$, and the employment tax x satisfies budget balance (13).

Given such a Matching Equilibrium, we discuss the Planner's optimal choice of $(b(.), s)$.

5 Optimal Policy.

We first characterize the First Best in which the Planner forces workers to accept job offers and the Planner also chooses personal consumption and the level of vacancies. The Second Best and Third Best problems then consider policies which maximise a steady state Utilitarian welfare function in a decentralized Matching Equilibrium, where the Planner does not observe job offers.

5.1 The First Best Market Outcome.

Optimal co-insurance with Utilitarian preferences implies that all workers consume the same amount, denoted w^{FB}. The Planner's first best problem is to choose vacancies V to maximise the steady state Utilitarian welfare function

$$W = u(w^{FB}) - c(1 - U)$$

where feasible consumption implies

$$w^{FB} = \pi[1 - U] - aV,$$

and steady state unemployment satisfies

$$\delta(1-U) = M(U, V).$$

As is well known, it is useful to recast the problem in terms of an optimal labour market tightness $\phi = V/U$, noting that $V = \phi U$, steady state $U = \delta/(\delta + m(\phi))$ and feasible consumption becomes $w^{FB} = (m\pi - a\phi\delta)/(m+\delta)$. Straightforward algebra then establishes the following Claim.[6]

Claim 4. If c is not too large, the Planner's first best implies $(w^{FB}, \phi^{FB}) = (w, \phi)$ satisfying

$$w = \frac{m\pi - a\phi\delta}{m+\delta} \qquad \text{(Feasible Consumption)}$$

$$\pi - w = \frac{a\delta}{m'} + \frac{\delta c}{u'(w)}, \qquad \text{(First Best Vacancy Creation)}$$

where $m = m(\phi)$.

The optimal vacancy creation condition compares the benefit of marginally increasing employment, which generates additional production surplus $\pi - w$, against the costs, which include the additional vacancy creation costs to maintaining a marginally higher V/U ratio in steady state, and the (monetised) marginal disutility of work.

Figure 1 graphs these two equations in (ϕ, w) space. As drawn in Figure 1, the Feasible Consumption locus (FC^{FB}) passes through the origin and is single peaked. To plot the vacancy creation curve, suppose for the moment that $c = 0$ and so consider the locus

$$w = \pi - \frac{a\delta}{m'}.$$

[6] Note that the First Best equates marginal utilities but does not equate payoffs. As a result employed workers are worse off than unemployed workers. Without matching frictions, a Planner might use employment lotteries to equate expected payoffs, though note that such lotteries do not affect a Utilitarian welfare function.

This locus passes through $(\phi, w) = (0, \pi)$, is strictly decreasing in ϕ and passes through the peak of the FC locus, denoted $(\overline{\phi}, \overline{w})$. $c > 0$ implies the vacancy creation locus, labelled VC^{FB} in Figure 1, lies strictly below this curve. Note that as $c \rightarrow 0$, the First Best solution converges to $(\overline{\phi}, \overline{w})$ where $\overline{w}$ is the highest possible wage compatible with budget balance and steady state (see Albrecht and Vroman (2001) for further discussion). We will compare this First Best outcome to the Second Best solution below.

5.2 The Second Best Problem.

Suppose now that wages and job creation rates are endogenously determined in a Matching Equilibrium. In the Second Best problem we restrict attention to a constant UI program, one where $b(\tau) = b$ for all τ, and suppose the Planner chooses policy instruments (b, s, x) to maximise the steady state Utilitarian welfare function

$$W = (1 - U)[u(w) - c] + Uu(b),$$

where free entry implies firms make zero profit.

Given a constant UI program, Claim 3 above describes the equilibrium wage, w^*, and $\overline{V}$. Given the definition of a Matching Equilibrium, substitute out U and V using $U = \delta/(\delta + m(\phi))$, $V = \phi U$ and also substitute out $\overline{V}$ using Claim 3. Letting $A = [(1 - \theta)r(r + m + \delta)]/[\theta(r + m)(r + \delta)]$, which is a positive constant, the Second Best problem reduces to the following.

Definition. The Second Best Problem implies the programming problem

$$\max_{b,s,x,\phi,w} W = \frac{m}{\delta + m}[u(w) - c] + \frac{\delta}{\delta + m}u(b),$$

where w, ϕ, x satisfy

$$u(w) - c - u(b) = Au'(w)[\pi - w - x], \tag{14}$$

$$a - s = \frac{m}{\phi(r+\delta)}[\pi - w - x], \tag{15}$$

$$mx = \delta b + \delta\phi s, \tag{16}$$

with $m = m(\phi)$, and the shrinking pie condition [Claim 3] requires

$$u(b) \leq u(\pi - x) - c \qquad \text{(Shrinking Pie)}$$

Note, the Planner has two degrees of freedom; he can choose policy instruments (b, s) freely where a Matching Equilibrium requires x satisfies budget balance (16), wage bargaining implies w satisfies (14) and ϕ is determined by the free entry condition (15). Claim 3 describes the Shrinking Pie condition which has to be satisfied in equilibrium. It requires that a gain to trade exists between an unemployed worker and a firm holding a vacancy [otherwise the worker will remain unemployed].

We first establish that the optimal vacancy creation subsidy implies full subsidisation.

Lemma 1. The Second Best implies $s = a$, $x = \pi - w$ and $u(w) - c = u(b)$.

Proof is in Appendix A.

The proof shows that for any policy (b, s, x) with $s < a$, a welfare improving policy exists where the Planner increases both b and s. The increase in b improves the insurance properties of the Market Equilibrium, and the increase in subsidy s ensures that vacancy creation rates are kept high. As $s > a$ is not feasible in a Matching Equilibrium (the free entry condition implies firms must make a loss by hiring a worker and, formally, the Shrinking Pie constraint fails), optimality in a Matching Equilibrium implies $s = a$ (full job creation subsidisation). Hence employment tax $x = \pi - w$ (by (15); zero ex-post profit for firms) and b satisfies $u(b) = u(w) - c$ (by (14); workers obtain zero surplus through becoming employed).

Theorem 2. The Second Best implies $s = a$, $x = \pi - w^{SB}$, $u(b) = u(w^{SB}) - c$, where $(\phi^{SB}, w^{SB}) = (\phi, w)$ satisfying:

$$w = \frac{m\pi - a\phi\delta + \delta(w - b)}{m + \delta} \qquad \text{(Budget Balance)}$$

$$w = \pi - \frac{a\delta}{m'}. \qquad \text{(Vacancy Creation)}$$

Proof: Using Lemma 1 to substitute out $s = a$ and $x = \pi - w$, the Second Best problem reduces to

$$\max_{b,w,\phi} W = u(w) - c$$

subject to

$$u(w) - c = u(b) \qquad \text{(No Match Surplus)}$$

$$m[\pi - w] = \delta b + \delta a\phi. \qquad \text{(budget balance)}$$

where the (No Match Surplus) condition with $x = \pi - w$ implies the Shrinking Pie condition is satisfied with equality. This optimization problem has a standard structure and the usual Lagrangian approach yields the Vacancy Creation decision stated.

We now compare the equilibrium wage in the Second Best outcome against the First Best outcome.

Lemma 2. Optimality implies $w^{SB} > w^{FB} > b$.

Proof. Figure 2 plots, for the Second Best case, the budget balance locus, labelled BB^{SB}, and the vacancy creation locus, labelled VC^{SB}, as described in Theorem 2. Comparing with the First Best case, the balanced budget curve corresponds to an upward shift in the Feasible Consumption locus. This occurs as for given labour market tightness ϕ, unemployed workers are allocated $b < w$ (where $b = \widehat{b}(w) = u^{-1}(u(w) - c)$)) and the reduced UI payments imply employed workers can consume more. The vacancy creation locus, VC^{SB}, corresponds to the unlabeled locus in Figure 1, and implies a shift to the right. As unemployed workers have incomplete insurance, the Utilitarian Planner compensates by increasing the vacancy rate to reduce the number unemployed. These two shifts both imply an increase in w, hence $w^{SB} > w^{FB}$. Given that, optimality of the first best program then implies $w^{FB} > b$. This completes the proof of Lemma 2.

Lemma 2 establishes that regardless of worker bargaining power, wages are 'too high' in the Second Best problem. The insight is related to Lemma 5 in Fredriksson and Holmlund (2001) which, with unobserved job search effort, establishes that the optimum implies too little aggregate search (regardless of worker bargaining power). Here the Planner's trade-off is between incomplete insurance and too high wages (and too low vacancy creation rates). Increasing b further would reduce the consumption gap between the employed and unemployed. Unfortunately, increasing b implies a further increase in negotiated wages (where Lemma 1 implies $u(w) = u(b) + c$). The downside is that the higher negotiated wage yields lower match surplus $x = \pi - w$ and hence lower vacancy creation levels (by budget balance). The Planner's trade-off is therefore between too low vacancy creation rates (via too high wages) and incomplete insurance.

Note that as $c \to 0$, both types of optima converge to $(\overline{\phi}, \overline{w})$; the second best distortion arises only when there is a strictly positive disutility to work. This immediately implies that for small values of c, the welfare loss associated with the second best problem is small. Although workers have incomplete insurance, consuming wage $w^{SB} > w^{FB}$ while employed and $b < w^{FB}$ while unemployed, the consumption risk $[w^{SB} - b]$ is small for c small.

However even if c were relatively large, the aggregate welfare loss implied by the Second Best problem appears small for relevant parameter values. In particular, suppose in the first best that optimal frictional unemployment is say 5%. Even if the consumption gap $w^{SB} - b$ is large, consumption variance is small as 95% of the population consume w^{SB}, the other 5% consume b. The first best smooths out this consumption variance, but unless the degree of risk aversion is very high, it seems unlikely that the resulting welfare gain is large. We verify this below using simulations.

As for reasonable parameter values the Second Best problem yields a payoff which is close to the First Best outcome, it follows that a duration dependent UI program cannot yield significant welfare increases.

5.3 The Third Best Problem.

Suppose now that the government does not observe the job hiring process. As explained in the Introduction, this implies vacancy creation subsidies may not be feasible. For example, a firm could report a vacancy and claim the subsidy even though it has no intention of hiring a new worker. Restricting $s = 0$, we now consider the welfare value of a duration dependent UI program $b(.)$.

To see why a duration dependent UI program is optimal, consider the Second Best solution described in Theorem 2, except now the Planner is constrained to set $s = 0$. Clearly with no vacancy subsidies, the government reduces the employment tax $x < \pi - w$ so that firms make positive profit ex-post (and so will invest in vacancies). Unfortunately given the same level of b, positive profit implies workers negotiate even higher wages. As workers extract even more rents from a match then, ceteris paribus, equilibrium job creation rates will fall. Hence the Planner's problem, given $s = 0$, is to maintain vacancy creation rates close to the efficient Second Best level while providing effective insurance against unemployment risk.

With $s = 0$, equilibrium vacancy creation rates are determined by

$$a = \frac{\alpha_f}{r+\delta}[\pi - \overline{w} - x] \tag{17}$$

where

$$\overline{w} = \int_0^\infty \alpha_w e^{-\alpha_w \tau} w^*(\tau) d\tau$$

is the average hiring wage. Increasing vacancy creation rates requires either reducing the employment tax (with a consequent reduction in the level of UI payments) or re-structuring UI payments so that *average* wages are lower.

Significant insight is obtained by reconsidering the risk neutral case. Using (4) implies

$$\begin{aligned}\overline{w} + x \quad &= \quad c + \theta(r+\delta)\int_0^\infty \alpha_w e^{-\alpha_w \tau}\left[\int_{t=\tau}^\infty e^{-r(t-\tau)} b(t)dt\right] d\tau \\ &\quad\; +(1-\theta)(\pi - c) - \theta[\delta\overline{V} - x]\end{aligned}$$

and integration by parts yields

$$\overline{w} + x = c + \theta(r+\delta)\int_0^\infty \Psi(t)b(t)dt + (1-\theta)(\pi - c) - \theta[\delta\overline{V} - x].$$

where

$$\Psi(t) = \frac{\alpha_w}{\alpha_w - r}[e^{-rt} - e^{-\alpha_w t}].$$

Ignoring for the moment the re-entitlement effect (as implied by the $\delta\overline{V} - x$ term), $\Psi(t)$ describes the direct marginal impact of UI payment $b(t)$ on gross labour costs $\overline{w} + x$; i.e. a marginal increase in $b(t)$ directly raises gross labor costs by $\theta(r+\delta)\Psi(t)dt$. Note, $\Psi = 0$ at $t = 0$; as implied by (4), early UI payments do not distort wages - it is the entitlement to continued UI payments which distorts job seeker (reservation) wages. Also note that $\Psi \rightarrow 0$ as $t \rightarrow \infty$; discounting implies that a UI payment in the indefinite future has no distortionary effect on average wages. The largest wage distortion arises where Ψ is a maximum, which occurs at $\tau^* = [\ln \alpha_w - \ln r]/[\alpha_w - r]$. An economy where $r = 5\%$ per annum and $\alpha_w = 4$ [expected duration of unemployment equals 13 weeks] yields $\tau^* = 1.1$ years. As most unemployed workers have unemployment durations less than one year and as payments which are received in less than one year's time are not discounted much, UI payments around the one year mark are the most distortionary. It should also be noted that Ψ falls slowly after this peak (approximately at rate r).

Taking the re–entitlement effect explicitly into account yields the following, more complicated, expression.

Lemma 3. Risk neutral workers and a fair employment tax in a steady state imply gross labor costs

$$\overline{w} + x = c + \theta(r+\delta)\int_{t=0}^{\infty} \widehat{\Psi}(t)b(t)dt + (1-\theta)\frac{r(r+\delta+\alpha_w)[\pi - c]}{r(r+\delta+\alpha_w) + \theta\delta\alpha_w} \quad (18)$$

where

$$\widehat{\Psi}(t) = \Psi(t)\left\{1 - \gamma(t)\right\} \quad (19)$$

and

$$\gamma(t) = (1 - r/\alpha_w)\left[\frac{e^{-rt} - e^{-(r+\alpha_w)t}}{e^{-rt} - e^{-\alpha_w t}}\right]\frac{\theta\delta(r+\alpha_w)}{\theta(r+\alpha_w)\delta + r(r+\alpha_w+(1-\theta)\delta)}.$$

Proof is in the Appendix.

$\widehat{\Psi}(t)$ describes the net impact of $b(t)$ on gross labor costs, $\overline{w} + x$, taking re-entitlement effects into account. It can be shown that $0 \leq \hat{\Psi}(t) \leq \Psi(t)$ for all $t \geq 0$; i.e. taking re-entitlement effects into account reduces the distortion of $b(t)$ on gross labor costs. This occurs because, as previously demonstrated for the risk neutral worker case, the re-entitlement effect implies a transfer from future hiring firms to current hiring firms. Although the current hiring firm may have to compensate the worker for foregone UI payment $b(t)$, the re-entitlement effect implies the firm is able to extract rents from future hiring firms who may also have to compensate the worker for foregone $b(t)$ in the next unemployment spell. That transfer mitigates the cost of $b(t)$ on gross labor costs. Indeed note that $\widehat{\Psi} = \Psi$ either when $\theta = 0$ (the firm receives none of the future rents and so bears the full direct impact of the benefit stream) or $\delta = 0$ (there is no risk of future layoff and therefore no future rents to share).

As with Ψ, note that $\hat{\Psi}(0) = 0$ - early UI payments do not distort gross labor costs, it is the option of future UI payments which are distortionary. $[1 - \gamma(t)]$ describes the extent to which re-entitlement effects mitigate the impact of $b(t)$ on gross labor costs. Assuming $r < \alpha_w$, this term increases monotonically from $1 - \gamma_0$ (when $t = 0$) to $1 - \gamma_0(1 - r/\alpha_w)$ (when $t = \infty$) where $\gamma_0 \in [0, 1]$. For r/α_w small, this variation is small and implies the peak of the function $\widehat{\Psi}$ lies very close to the peak of Ψ. For instance, suppose $r = 5\%$ per annum, $\alpha_w = 4$, and $\delta = 0.25$ [implying an average employment spell equal to 4 years]. At these parameters, then $\theta = 0$ implies $\widehat{\Psi} = \Psi$ and their peaks coincide at 1.11 years, while $\theta = 1$ implies the peak of $\widehat{\Psi}$ shifts to 1.16 years. Hence for relevant parameter values, the re-entitlement effect scales down the distortion implied by $b(.)$ on gross labour costs. The largest distortions occur at durations of approximately one year.

6 Simulations

We now use policy simulations not only to demonstrate formally that decreasing UI payments with duration is indeed welfare increasing (when there are no vacancy creation subsidies), but also to establish some idea of their quantitative importance. We also compare those implications against the standard Nash bargaining approach. In particular, adopting the Cahuc and Lehmann (2000) interpretation - that Nash bargaining represents union wage bargaining with an insider/outsider distortion - we can consider how optimal policy changes in the presence of unions.

For simplicity and realism we consider two-tier UI schemes of the form

$$b(\tau) = \begin{cases} b_0 \text{ for } \tau < T \\ \underline{b} \text{ for } \tau \geq T, \end{cases}$$

where the shrinking pie condition requires both

$$u(b_0), u(\underline{b}) \leq \frac{r}{r+\delta}[u(\pi - x) - c + \delta\overline{V}]. \tag{20}$$

We assume a CRRA utility function, $u(w) = w^{1-\sigma}/(1-\sigma)$, and a Cobb-Douglas matching function $m(\phi) = A\phi^{\eta}$. The parameters used in the leading example are provided in Table 1.

Although this is not a calibration exercise some effort has been made to use parameter values that are consistent with those used in the literature. The value for $r = 4\%$ per annum comes from the business cycle literature (*e.g.* Hansen (1985));[7] σ is a typical value obtained from structural estimation of labor market models (see Lentz (2002)); θ and η are taken from Mortensen and Millard (1997); $\delta = 20\%$ per annum comes from Cole and Rogerson

[7]Note, the simulations are computed in continuous time, but we use a year as the reference unit of time for the discount rates and hazard rates.

π	flow match output	1
c	flow disutility of work	0.2
a	flow advertising cost	10
r	common discount rate	0.04
δ	job destruction rate	0.2
A	Scale parameter on matching function	16
σ	risk aversion parameter (workers)	2
θ	firm's bargaining power	0.5
η	elasticity of matching *w.r.t* vacancies	0.7

Table 1: Parameters for leading example.

(1999) and implies a job lasts on average for 5 years. A is within the range suggested by Blanchard and Diamond (1989). The value of a is chosen to imply an average unemployment spell length of 13 weeks in the Third Best outcome with $T = 0$, which implies $\alpha_w = 4$ for that policy outcome. This value for a is consequently high and reflects the assumption that advertising is the only outlay the firm makes. In reality firms also have to cover any capital expenditures, which here is subsumed into a (e.g. Acemoglu (1997)). An alternative approach (see Albrecht and Vroman (2002)) is to incorporate a flow user cost of capital that the firm pays for the whole lifetime of the job. Using an advertising cost, however, is more consistent with the literature.

The results are reported in Table 2.[8] Each row describes the optimal

[8] The algorithm used to solve for the optimal two-tier benefits works as follows. For every $(b_0, \underline{b})$, $\bar{V}$, x, ϕ we obtain $\underline{w}$ from equation (3) and solve for $w(0)$ from equation (2) using a shooting method. This generates the wage path. With this, holding $(b_0, \underline{b})$, x, and $\bar{V}$ fixed we can solve for ϕ from equation (11). This is used to update the value of $\bar{V}$ using equation (12). When for given $(b_0, \underline{b})$ and x, $\bar{V}$ converges, the budget balancing (equation

best	T	b_0	$\underline{b}$	tax %	V/U	U %	$\bar{w}$	$\overline{w}+x$	Welfare $u^{-1}(W)$
1st	0	-	0.8882	-	0.1719	4.11	0.8882	-	0.7589
2nd	0	-	0.7563	10.9	0.1941	3.79	0.8910	-	0.7563
3rd	0	-	0.7381	3.49	0.1489	4.53	0.8804	0.9153	0.7480
3rd	0.5	0.7576	0.7380	3.63	0.1457	4.59	0.8795	0.9158	0.7483
3rd	1	0.7577	0.7376	3.65	0.1455	4.60	0.8794	0.9159	0.7483
3rd	2	0.7577	0.7367	3.65	0.1455	4.60	0.8794	0.9159	0.7483
Nash	0	-	0.4292	3.29	0.0751	7.11	0.8982	0.9211	0.7217
Nash	0.5	0.4244	0.4429	3.29	0.0751	7.11	0.8981	0.9210	0.7217

Table 2: Results, Example 2

policy outcome given the Planner's problem (First, Second or Third Best scenarios, where T is measured in years). The last two rows summarise the optimal two-tier policy using the standard Nash bargaining approach. In particular, the wage is determined as

$$w^{\text{Nash}} = \arg\max_{w} (\pi - x - w)^{\theta} \left(\frac{u(w) - c + \delta \bar{V}}{r+\delta} - \bar{V} \right)^{1-\theta}$$

i.e., the threatpoint of insiders is the value of being laid-off and, by assumption, firms cannot wage discriminate between insiders and outsiders.

Each row reports the optimal UI schedule $(b_0, \underline{b})$; the budget balancing tax rate (x) as a percentage of π; equilibrium labor market tightness $\phi = V/U$; the unemployment rate U; the average wage paid $\bar{w}$; and gross labor costs $\overline{w}+x$. The final column reports the Utilitarian Welfare measure in units

(13)) value of x is calculated and used to up-date the tax rate. Again the algorithm iterates on x until it converges. The final level of calculation is to maximize the welfare function over choices of $(b_0, \underline{b})$ subject to the shrinking pie condition (20).

of the consumption good.

There are several interesting features. First the welfare gap between the First Best and Second Best problems is small. In the First Best all consume $w^{FB} = 0.89$, in the Second Best the Planner sets $b = 0.76$ and workers negotiate $w^{SB} = 0.89$. Given $b < w^{SB}$, the Planner compensates for incomplete insurance by increasing vacancy creation rates above the First Best level and so lowers the level of unemployment.

The simulations for the Third Best policy demonstrate that lowering UI payments with duration is welfare increasing. The resulting increase in welfare, however, is small as is the suggested optimal rate of decline. Note that gross labour costs, $\overline{w} + x$, are almost identical across policy outcomes. These simulations suggest that once policy implements the 'right' level of gross labor costs; i.e. $\overline{w} + x$ is consistent with efficient vacancy creation rates, then tinkering with the duration profile of UI payments offers little added return.

The alternative Nash bargaining approach implies insiders have a greater threatpoint and so negotiate higher wages. This is not only because insiders are assumed to have full UI entitlement. The constant UI case above, see (9), implies the unemployed worker threatpoint is $u(b)/r$ with strategic bargaining, while the Nash bargaining approach instead assumes threatpoint $\overline{V} > u(b)/r$. The higher threatpoint implies insiders negotiate higher wages. At the optimum, to stop wages being driven too high, the Planner compensates by reducing the level of UI. Comparing the optimal policy outcomes in the Third Best optimum with $T = 0$ (i.e. constant UI), strategic bargaining implies optimal UI $b = 0.74$ and $w = 0.88$, while Nash bargaining implies $b = 0.44$ and $w = 0.90$. Reflecting the greater wage distortion, there is greater consumption risk and higher unemployment with Nash bargaining.

In contrast to the strategic wage bargaining case and consistent with the arguments of Cahuc and Lehmann (2000), the bottom row establishes that increasing UI payments with duration is welfare increasing, though the suggested increase is again small, and the increase in welfare appears insignificant.

Most interestingly, note that the Second Best optimum described in the second row is also consistent with Nash bargaining. Given there is no match surplus in the equilibrium outcome, the Nash bargaining equation yields the same outcome as the strategic bargaining approach (the firm and worker have nothing to bargain over). The essential insight, therefore, is that a co-ordinated policy approach using vacancy creation subsidies yields a much greater increase in welfare than simply varying UI payments with duration.

The simulations therefore suggest there is little welfare improvement by varying UI payments with duration. This can be explained, at least in part, by the re-entitlement effects described above. For example, $r = 0.05$, $\delta = 0.2, \alpha_w = 4$ imply gross wage distortion $\Psi = 0.94$ at one year's duration. Taking re-entitlement effects into account implies a net distortion of only $\widehat{\Psi} = 0.33$; i.e. re-entitlement effects cut the gross distortion by around two thirds.

Suppose instead we increase the discount rate r to 10% per annum, and reduce a to 9 (so that the average unemployment duration remains at around 13 weeks in the optimal Third Best policy with $T = 0$). The direct effect of b at one year's duration is now slightly lower, $\Psi = 0.91$, but the net effect is higher $\widehat{\Psi} = 0.47$ as future re-entitlement effects are discounted more. As UI payments at medium durations now distort gross labour costs more (recall, $\Psi = \widehat{\Psi} = 0$ at zero duration) the return to lowering UI payments with duration is increased. As described by Table 3, increasing r leads to a steeper optimal benefit profile. Nevertheless, the welfare benefit of using a

best	T	b_0	$\underline{b}$	tax %	V/U	U %	$\bar{w}$	$\bar{w}+x$	Welfare
3rd	0	-	0.7192	3.65	0.1344	4.85	0.8710	0.9075	0.7407
3rd	0.5	0.7617	0.7185	3.97	0.1283	5.00	0.8691	0.9088	0.7411
3rd	1	0.7617	0.7164	4.02	0.1274	5.02	0.8688	0.9090	0.7412

Table 3: Results for $r = 0.1$

best	T	b_0	$\underline{b}$	tax %	V/U	U %	$\bar{w}$	$\bar{w}+x$	Welfare
3rd	0	-	0.6537	6.97	0.1256	9.65	0.7679	0.8376	0.6645
3rd	0.5	0.6543	0.6535	6.86	0.1290	9.49	0.7678	0.8364	0.6645
3rd	1	0.6543	0.6535	6.86	0.1289	9.49	0.7678	0.8364	0.6645

Table 4: Results for $\delta = 0.4$

duration dependent UI program remains small. One suspects that generating significant welfare effects would require very high discount rates.

Increasing the job destruction rate implies the converse effect - the worker expects to be laid-off sooner and so future re-entitlement effects are discounted less. The net distortion $\widehat{\Psi}$ is correspondingly small. Table 4 considers optimal policy for job destruction rate $\delta = 40\%$ per annum, so that jobs on average last for only 2.5 years (and increases a to 11 so that average unemployment spells remain at 13 weeks at the optimum). Table 4 demonstrates that optimal UI profiles are even flatter and the welfare gain to using a duration dependent UI scheme is non-existent.

7 Conclusion

This paper has considered optimal UI in an equilibrium matching framework where UI payments distort wages. For reasons analogous to those given in Shavell and Weiss (1979), a duration dependent UI program can increase welfare. UI payments at medium durations, particularly around the one-year mark, raise significantly the option value of remaining unemployed at short unemployment durations. In Shavell and Weiss (1979) this leads to too little job search effort (a quantity distortion) whereas here it leads to too high wages (a price distortion). When job creation subsidies are ruled out by assumption, strategic bargaining implies UI payments should decrease with duration (to reduce the distortion of UI on average hiring wages). In contrast, the typical Nash bargaining approach implies UI payments should increase with duration. In both cases, however, policy simulations suggest that the associated welfare gain is slight.

Perhaps the most interesting result is that large welfare gains are possible (particularly in the Nash bargaining example) if optimal UI policy is co-ordinated with optimal job creation subsidies. This reflects that at the policy optimum, the Planner faces a trade-off between improved insurance and lower equilibrium vacancy creation rates (via higher wages). Job creation subsidies are optimal as they target this distortion directly.

This insight also applies to the optimal UI literature with unobserved job search effort. In that literature, marginally improved unemployment insurance leads to marginally lower aggregate search effort and hence lower vacancy creation rates via the thick market externality. Noting that congestion externalities imply workers are better off with lower aggregate search effort, the Planner's underlying trade-off is again between better insurance and lower equilibrium vacancy creation rates (via lower job search effort). Job creation subsidies remain an appropriate policy instrument. Furthermore once optimal job creation subsidies are implemented, it is not clear that a duration dependent UI program is useful. In the optimal program, the Planner increases b to close the consumption gap between employed and unemployed workers until aggregate job search effort falls to its optimal second best level. But given a convex search effort cost technology, search effort dispersion across unemployed workers is inefficient. From an equilibrium perspective it is not clear that making UI payments duration dependent will significantly increase welfare.

The critical insight, therefore, is that the design of optimal UI cannot be considered in isolation from other relevant policy instruments. Job creation subsidies improve welfare by generating efficient re-employment rates. An important problem for future research is to consider both market failures - unobserved job search effort and strategic wage bargaining - in the same framework. The policy problem is potentially interesting as given optimal job creation subsidies, the level of b has to hit two margins - it has to induce average wages consistent with the second best and also induce second best aggregate search effort.

8 Appendix.

Proof of Claim 1. Consider any agreement w' at any duration $\tau + dt$ with $\tau \geq 0$ and $dt > 0$ but small. A Pareto dominating allocation exists at duration τ if a w exists where

$$\frac{u(w) - c + \delta\overline{V}}{r + \delta} - \frac{1}{1 + rdt}\left[u(b(\tau)dt + \frac{u(w') - c + \delta\overline{V}}{r + \delta}\right] \geq 0, \qquad (21)$$

$$\frac{\pi - x - w}{r+\delta} - \frac{1}{1+rdt}\frac{\pi - x - w'}{r+\delta} \geq 0. \tag{22}$$

Consider

$$w = \frac{w' + rdt[\pi - x]}{1+rdt}$$

which implies that (22) is satisfied with equality. Now consider (21). The restriction (1) implies

$$\begin{aligned} & \frac{u(w) - c + \delta\overline{V}}{r+\delta} - \frac{1}{1+rdt}\left[u(b(\tau)dt + \frac{u(w') - c + \delta\overline{V}}{r+\delta}\right] \\ \geq & \frac{1}{r+\delta}\left[u(w) - \frac{u(w') + rdtu(\pi - x)}{1+rdt}\right] \end{aligned}$$

and given the choice of w above, concavity of u implies that (21) is also satisfied. (1) therefore implies that immediate trade Pareto dominates any agreement at any later date. As it also implies immediate trade dominates never reaching agreement [e.g. set $w = \pi - x$], then the Shrinking Pie condition (1) guarantees that immediate agreement is always jointly efficient.

Proof of Lemma 1.

Consider any candidate solution (b, s, x, ϕ, w) to the Second Best problem, where (b, s, x, ϕ, w) satisfy (14)-(16) and $s < a$. Note that $s < a$ implies $\pi - w - x > 0$ by (15) and (14) then implies $u(w) - c > u(b)$. Together these conditions guarantee the Shrinking Pie condition.

Now consider a policy variation $(db, ds, dx, d\phi, dw)$ which also satisfies (14)-(16). Given we have two degrees of freedom, consider such a policy variation which also implies $d\phi = 0$. In that case, (14)-(16) imply (ds, db, dx, dw) must satisfy

$$u'(w)dw - u'(b)db = A[u''(w)[\pi - w - x]dw - u'(w)[dw + dx]],$$

$$ds = \frac{m}{\phi(r+\delta)}[dw + dx],$$

$$mdx = \delta db + \delta\phi ds,$$

and $d\phi = 0$. Solving these three expressions in terms of ds yields

$$\frac{dw}{ds} = \frac{[r\phi u'(b)/\delta] - [A\phi(r+\delta)u'(w)/m]}{u'(w) + mu'(b)/\delta - Au''(w)[\pi - w - x]},$$

$$u'(b)\frac{db}{ds} = [u'(w) - Au''(w)[\pi - w - x]]\frac{dw}{ds} + A\phi(r+\delta)u'(w)/m.$$

Such a perturbation with $ds > 0$ implies total welfare change

$$\frac{dW}{ds} = \frac{1}{\delta+m}\left[mu'(w)\frac{dw}{ds} + \delta u'(b)\frac{db}{ds}\right].$$

Substituting out $dw/ds, db/ds$ using the above yields

$$\frac{dW}{ds} = \frac{u'(w)u'(b)r(1+m/\delta) + Au'(w)(r+\delta)[u'(b) - u'(w)] - Au'(b)ru''(w)[\pi - w - x]}{(\delta+m)[u'(w) + mu'(b)/\delta - Au''(w)[\pi - w - x].}$$

Now $c > 0$ and $u(w) - c > u(b)$ imply $w > b$. Hence concavity of u implies $u'(b) > u'(w)$. As $s < a$ implies $\pi - w - x > 0$, an increase in s using this policy perturbation is strictly welfare increasing.

As $s > a$ is not feasible [$s > a$, (15) and (14) contradict the Shrinking Pie condition], optimality implies $s = a$ (at which point the Shrinking Pie condition binds) and (14), (15) then imply the Lemma.

Proof of Lemma 3. Let $B(t) = \int_t^\infty e^{-r(s-t)}b(s)ds$. Given workers are risk neutral, then (5) in Section 3.1 implies

$$\overline{V} = \frac{x}{\delta} + \int_0^\infty \alpha_w e^{-(r+\alpha_w)t}\frac{w^*(t) - c + \delta\overline{V}}{r+\delta}dt.$$

where x is the fair employment tax, and (4) implies the equilibrium wage, $w^*(.)$, satisfies

$$w^*(t) - c + \delta\overline{V} = \theta(r+\delta)B(t) + (1-\theta)[\pi - c + \delta\overline{V} - x].$$

Substituting out $w^*(.)$ in the equation for $\overline{V}$ and solving yields

$$\delta\overline{V} - x = \frac{\alpha_w \delta\theta \int_0^\infty e^{-(r+\alpha_w)t} B(t)dt + \frac{\alpha_w \delta(1-\theta)}{(r+\delta)(r+\alpha_w)}[\pi - c]}{1 - \frac{\delta\alpha_w(1-\theta)}{(r+\delta)(r+\alpha_w)}}.$$

The text has established that

$$\overline{w} + x = \theta(r+\delta)\int_0^\infty \alpha_w e^{-\alpha_w t} B(t)dt + (1-\theta)\pi + \theta c + \theta(x - \delta\overline{V}).$$

Substituting out $\overline{V}$ using the above and rearranging yields

$$\begin{aligned}\left[1 - \frac{\delta\alpha_w(1-\theta)}{(r+\delta)(r+\alpha_w)}\right](\overline{w}+x) \quad &= \quad (1-\theta)\pi + \theta c + \theta\int_0^\infty e^{-\alpha_w t}\alpha_w B(t)[r+\delta-\delta\theta e^{-rt}]dt \\ &\quad -\frac{\delta\alpha_w(1-\theta)}{(r+\delta)(r+\alpha_w)})\left[\pi + \theta(r+\delta)\int_0^\infty e^{-\alpha_w t}\alpha_w B(t)dt\right]\end{aligned}$$

As integration by parts implies

$$\int_{t=0}^\infty e^{-\gamma t}B(t)dt = \int_{t=0}^\infty e^{(r-\gamma)t}\left[\int_{s=t}^\infty e^{-rs}b(s)ds\right]dt = \int_{t=0}^\infty \frac{e^{-rt}-e^{-\gamma t}}{\gamma - r}b(t)dt$$

for any $\gamma > 0$ and $\gamma \neq r$, we can use this condition to substitute out all terms involving $B(.)$ in the previous expression, and standard algebra then yields the condition stated in the Lemma.

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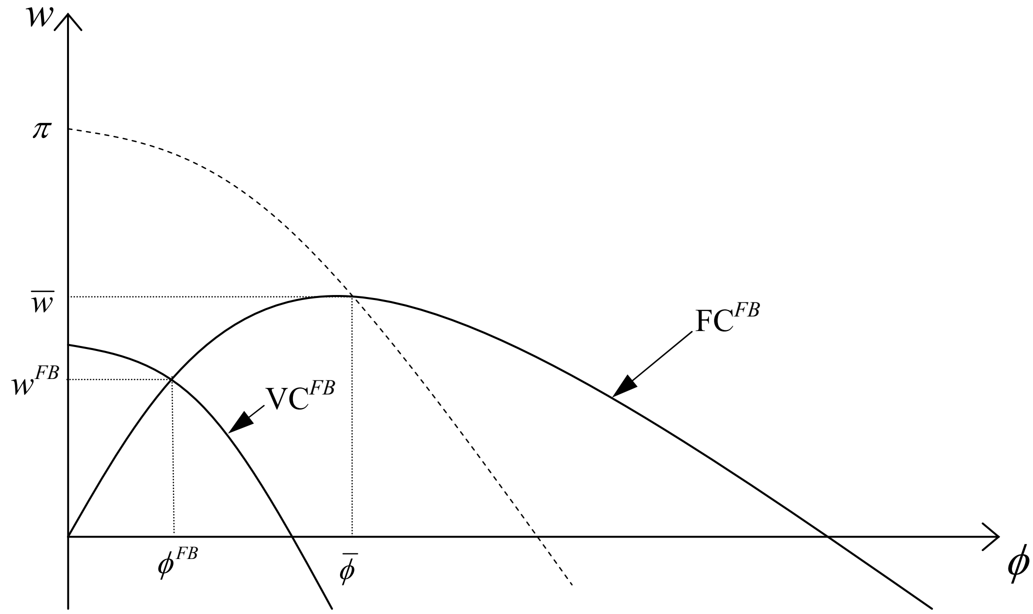

Figure 1

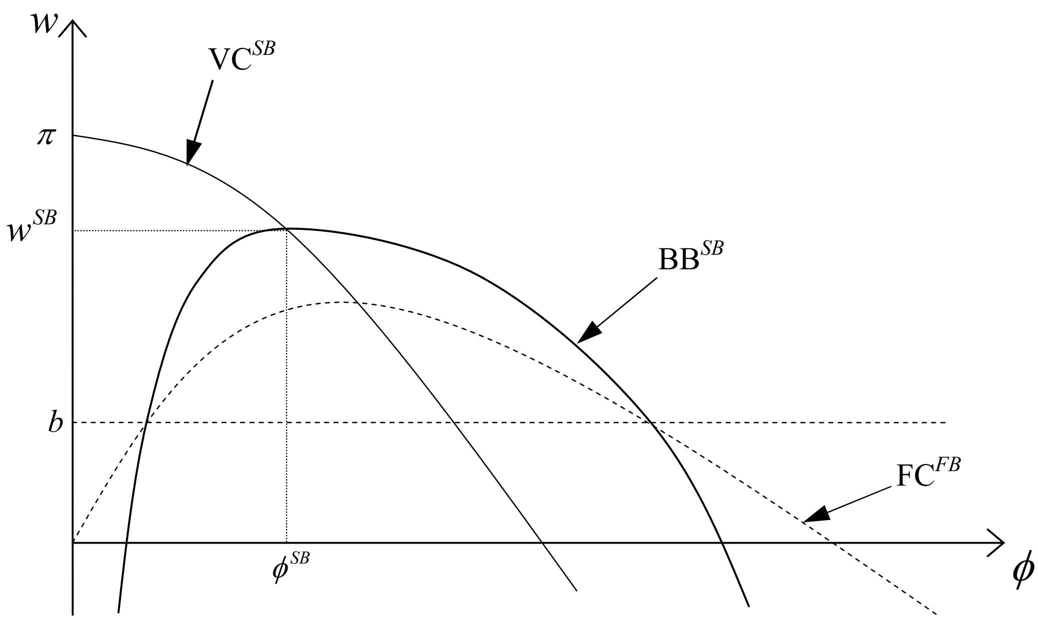

Figure 2