



On the firms' component of wage dispersion: Endogenous effort versus search frictions



Adrian Masters

Department of Economics, University at Albany, SUNY, 1400 Washington Ave, Room BA109, Albany, NY 12222, USA

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ABSTRACT

This paper presents an environment in which firms' productive heterogeneity passes through to wage dispersion via sequential search and endogenous worker effort levels. Despite small gains from trade, the model is able account for more than two thirds of the measured firm component of wage dispersion. The implied narrow range of worker utility effectively pins down the lowest wage in the distribution and higher wages simply compensate workers for their extra effort.

1. Introduction

It has been well established that substantial wage dispersion exists among otherwise similar workers [Acemoglu \(2002\)](#), [Lemieux \(2006\)](#) and [Autor et al. \(2008\)](#). Of that wage dispersion, [Abowd and Kramarz \(1999\)](#) attribute about one third to firm effects. This paper investigates the importance of endogenous worker effort for understanding how variations in firm productivity translate into variations in worker pay.

In a competitive market for homogeneous labor the law of one price holds so that, despite firm heterogeneity, workers all get paid alike. [Hornstein et al. \(2011\)](#) (henceforth HKV) show that search frictions alone do not help much in reconciling the theory with the data. Unless the value to unemployment is unrealistically low, a worker will never take a low wage offer when a much higher one could arrive at any moment.

Here, endogenous work effort is introduced to the Diamond-Mortensen-Pissarides (DMP) framework with ex ante productive heterogeneity among firms. High productivity firms offer high wages but require greater effort from their workers. The workers face a simple labor supply-type problem in which they trade off disutility of work for higher pay. The model can generate any degree of wage dispersion while matching empirical labor market flows. The main point is that, what matters to workers is utility rather than the wage per se and, consistent with HKV, the utility dispersion among workers remains small.

To demonstrate the quantitative relevance of the model I simulate it using realistic parameter values. The mean to minimum (*Mm*) ratio, as introduced by HKV, that emerges for wages is 1.25. From the discussion in HKV, the *Mm* ratio from all sources of wage dispersion for observably similar workers should be around 1.8. Given that, from

[Abowd and Kramarz \(1999\)](#), firm heterogeneity should explain about one third of this, the implication is that the model can explain close to all of the firm effect.

Both search frictions and endogenous effort contribute to how firm productive heterogeneity translates into wage dispersion. To demonstrate that the latter plays the larger role, I shut down endogenous effort and the *Mm* wage ratio drops from 1.25 to 1.004.

A growing literature has emerged following HKV that attempts to identify additional sources of wage dispersion from otherwise identical workers. In [Ortego-Marti \(2016\)](#) the basic source of wage dispersion is match-specific productivity as in [Pissarides \(2000\)](#) Chapter 6. He also incorporates skill decay while unemployed. The skill decay makes workers more eager to take jobs which lowers the reservation match productivity at which hiring occurs. This increases wage dispersion even among those with the same realized level of skill. [Burdett et al. \(2011\)](#) use learning-by-doing to obtain a similar effect in the [Burdett and Mortensen \(1998\)](#) on-the-job search framework. Unemployment has an opportunity cost to workers in terms of lost productivity gains which lowers their reservation wages. While such mechanisms can generate large degrees of wage dispersion, they have nothing to do with the firms' component. As such they are complementary rather than substitute theories to my approach.

The remainder of the paper proceeds as follows. [Section 2](#) lays out the theoretical model. A numerical simulation of the model appears in [Section 3](#). [Section 4](#) provides some discussion of the results and [Section 5](#) concludes.

E-mail address: amasters@albany.edu.

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2. Model

2.1. Environment

Time is continuous with an infinite horizon. There is mass one of ex ante identical workers who face death according to a Poisson arrival rate δ . Those who die are replaced by newborns so that δ represents both the birth and death rates. Workers are risk neutral. They get utility from the consumption good which they receive in the form of wages. Workers get disutility, $c(e)$ from working with effort level e . The cost function $c(\cdot)$ is twice differentiable, strictly increasing and strictly convex with $c(0) = c'(0) = 0$ and $\lim_{e \rightarrow \infty} c'(e) = \infty$. While unemployed they receive a flow utility b which represents the value of non-market activities.

There are also a large number of ex ante identical entrepreneur/employers who I call firms. The firms can create vacancies by the following process. At flow cost a , they sample ideas, z , at an arrival rate γ from a continuous distribution G . The support of G is $(0, \bar{z}]$. Once a firm realizes an idea, z , it decides whether or not to pursue it. Pursuing an idea means that the firm acquires physical capital k . The cost of capital is normalized to 1. A firm with idea, z , specific capital stock, k , and matched with a worker who provides effort level e produces $z f(k, e)$ units of the consumption good. Ideas are therefore synonymous with match total factor productivities (TFPs). The function $f(\dots)$ is a standard neoclassical production function that has constant returns to scale, is twice differentiable, increasing in both arguments, concave and satisfies the Inada conditions.¹ In order to keep the individual firm's problem stationary there is no depreciation of capital but jobs are subject to destruction at the rate λ .² There is a common discount rate, r .

While it is standard in the search and matching literature for firms to occupy a single worker, in the current context the assumption is not without consequence. Here, the assumption represents an extreme version of the Lucas (1978) span-of-control restriction in which the second worker is not productive. Recall that the focus of the paper is on how differences in firm productivities pass through to wage dispersion. As real firms have multiple workers, the reported productivity range for firms in Syverson (2004) is an underestimate of the range for specific jobs. In the model, allowing for greater productivity dispersion, would make it easier to generate a given degree of wage dispersion. On the observed wages side, Abowd and Kramarz (1999) measure firm rather than job effects. To the extent that firms are characterized by their particular technology, what they identify is the impact of that technology on the wage. And, this is the focus here. Papers that take firm size more seriously, but do not incorporate endogenous effort, include Kass and Kircher (2015), Hawkins (2013) and Masters (2016).

Consistent with competitive search, there is a continuum of potential trading locations. The rate at which workers and firms encounter each other in any location is governed by a matching function, $M(u, v)$ where u is the unemployment rate among workers associated with that location and v is the number of vacant jobs per worker associated with that location. The matching function has constant returns to scale (c.r.s), is twice differentiable, increasing in both arguments and concave. The Poisson arrival rate at which a worker meets firms is then $M(u, v)/u = M(1, \theta)$ where $\theta \equiv v/u$. It is convenient to define the function $m(\theta) \equiv M(1, \theta)$. Then $m(\cdot)$ inherits concavity from $M(\cdot, \cdot)$ and c.r.s means $m(\theta) \geq \theta m'(\theta)$. In order to prevent the trivial equilibrium (in which no vacancies are created) from being stable and rule out other corner-type solutions, I further

¹ That is $f(0, 0) = f(0, e) = f(k, 0) = 0$ for all $k, e > 0$, $\lim_{k \rightarrow 0} f_k(k, e) = \infty$, $\lim_{e \rightarrow 0} f_e(k, e) = \infty$, $\lim_{k \rightarrow \infty} f_k(k, e) = 0$, $\lim_{e \rightarrow \infty} f_e(k, e) = 0$.

² A common alternative assumption here is that the shock is to the match rather than the job. In the standard DMP model with free-entry, the distinction is moot. Here it is more consequential. Indeed, had I chosen match breakdown, there would be no steady state investment.

assume that $\lim_{\theta \rightarrow 0} m'(\theta) = \infty$ and that $\lim_{\theta \rightarrow \infty} m'(\theta) = 0$.

In a decentralized economy, trading locations are occupied by markets. The set of active markets depend on which aspects of jobs become common knowledge. Here, firms post a wage/effort pair, (w, e) , which workers observe and use to direct their search. Markets are therefore indexed by $\omega = (w, e, \theta) \in \Omega = \mathbb{R}_+^3$. The set of active markets is $\Omega_A \subset \Omega$. The equilibrium concept is competitive search as developed by Moen (1997). I seek steady-state symmetric equilibria. Symmetry, here, means that all firms of the same type enter the same market and all workers enter any active market with the same probability as each other.

2.2. Value functions

Let V_j be the value to a firm of type (z, k) of employing a worker in market $\omega = (w, e, \theta)$. Then

$$(r + \lambda)V_j = z f(k, e) - w + \delta[V_f - V_j] \tag{1}$$

where V_f is the value to the firm of holding a vacancy in market ω . Thus

$$(r + \lambda)V_f = \frac{m(\theta)}{\theta}[V_j - V_f].$$

Solving for V_f implies

$$V_f = V_f(\omega; z, k) \equiv \frac{m(\theta)[z f(k, e) - w]}{(r + \lambda)[(r + \delta + \lambda)\theta + m(\theta)]} \tag{2}$$

For a worker who enters the same market, let V_e be the value of employment. Then

$$(r + \delta)V_e = w - c(e) + \lambda(V_u - V_e) \tag{3}$$

where V_u is the value to looking for work in market ω . So,

$$(r + \delta)V_u = b + m(\theta)(V_e - V_u).$$

Solving for V_u implies

$$V_u = V_u(\omega) \equiv \frac{m(\theta)[w - c(e)] + (\delta + \lambda)b}{(r + \delta)(r + \delta + \lambda + m(\theta))} \tag{4}$$

2.3. Equilibrium

Because workers are identical, for multiple markets to coexist, it must be the case that there exists some $V_u^* > 0$ such that $V_u(\omega) = V_u^*$ for all ω in Ω_A . Given z , and their capital stock, when firms decide which market to enter they take this indifference of workers as a constraint to their problem. Let

$$\omega^*(z, k) \in \arg \max_{\omega} V_f(\omega; z, k) \text{ subject to: } V_u(\omega) = V_u^* \tag{5}$$

Then, let $V_v(z)$ be the value of pursuing an idea type z to a firm. Thus,

$$V_v(z) = \max_k V_f(\omega^*(z, k); z, k) - k. \tag{6}$$

Firms pursue any idea, z , for which $V_v(z) \geq 0$. They drop any idea, z , for which $V_v(z) < 0$.

Definition 1. An equilibrium is a list, $Z_A, k^*(Z_A), \Omega_A, V_u^*$ such that, given V_u^* ,

- (i) Z_A is the set of ideas for which $V_v(z) \geq 0$.
- (ii) $k^*(Z_A)$ is the set of values of k associated with each $z \in Z_A$, such that k solves Problem (6).
- (iii) Ω_A is the set of markets $\omega = \omega^*(z, k^*(z))$ such that $z \in Z_A$.

Then $V_u^* = V_u(\omega^*(z, k^*(z)))$ for every $z \in Z_A$.

As the infimum to the support of G is 0, for any $V_u^* > 0$ there exists some \hat{z} for which $V_v(\hat{z}) = 0$. It is immediate from the envelope theorem and Eq. (2) that (as long as ω remains in the interior of the positive orthant) V_v is strictly increasing in z so that \hat{z} is unique and $Z_A = [\hat{z}, \bar{z}]$. Free-entry to the market for ideas therefore means

$$a = \gamma \int_{\hat{z}}^{\bar{z}} V_v(z) dG(z). \tag{7}$$

For a given value of z , Problems (5) and (6) can be combined into

$$V_v(z) = \max_{k, \omega} V_f(\omega; z, k) - k \text{ subject to: } V_u(\omega) = V_u^*. \tag{8}$$

This demonstrates that the above environment leads to outcomes identical to one in which the firms also advertise their capital stock. Under a posted wage contract, the firm is residual claimant and investment is efficient regardless of the extent to which it becomes common knowledge.³ By eliminating the wage from Eqs. (2) and (4), problem (8) reduces to

$$V_v(z) = \max_{k, e, \theta} \frac{m(\theta)[zf(k, e) - c(e)] + (r + \delta + \lambda)b - [r + \delta + \lambda + m(\theta)]\delta V_u^*}{(r + \lambda)[(r + \delta + \lambda)\theta + m(\theta)]} - k. \tag{9}$$

The first order conditions in k and e are respectively,

$$m(\theta)z f_k(k, e) - (r + \lambda)[m(\theta) + (r + \delta + \lambda)\theta] = 0 \tag{10}$$

$$z f_e(k, e) - c'(e) = 0 \tag{11}$$

Clearly, problem (9), is strictly concave in k and e implying that for any given value of θ there is unique pair $(k(\theta), e(\theta))$ that solve Eqs. (10) and (11). Substituting these functions back into the maximand in problem (9) generates a continuous function of θ which has a maximum on the positive real line. Using Eqs. (10) and (11), it is straightforward to show that $k'(\theta) < 0$ so that by inspection of problem (9), any optimizing value of θ must be finite.

Claim 1. Suppose the functional forms, $m(\theta) = \bar{m}\theta^\eta$, $\bar{m} > 0$, $0 < \eta < 1$; $c(e) = \bar{c}e^\sigma$, $\bar{c} > 0$, $\sigma > 1$; $f(k, e) = k^\alpha e^{1-\alpha}$, $0 < \alpha < 1$. Then for each z such that $V_v(z) \geq 0$, problem (8) has a unique solution.

Proof. See Appendix A.

The value of \hat{z} is obtained from Eq. (7). The value of V_u^* is obtained by setting $V_v(\hat{z}) = 0$. Clearly, if b is large enough, there can be no gains from trade. That is, there is some b_{\max} such that for $b \geq b_{\max}$, $\hat{z} \geq \bar{z}$. In such cases only the trivial equilibrium, which corresponds to autarky, exists. Under the functional form restrictions to be imposed for the numerical analysis and with $b < b_{\max}$, then, there is a unique non-trivial equilibrium. An earlier version of this paper, Masters (2013), shows that this equilibrium also coincides with the Planner's allocation.⁴

Under these functional form restrictions, it is straightforward to show that (given z and V_u^*) e , k and w increase with z and θ decreases with z . So, despite the increased effort required of high wage workers, their jobs are the most sought after. Workers are indifferent across markets because they anticipate a longer search period in the higher wage markets. Workers trade the utility derived from jobs against the time it takes to get one – the unemployment rate is higher in the high wage markets. High wage jobs therefore fill more quickly. The model is consistent with the data in this regard. Holzer et al. (1991) find that for jobs requiring similar skill sets, applicant queue lengths increase with the wage. The model does not, however, make any prediction of a relationship between the wage a worker is hired at and the length of the preceding unemployment spell. This is because the workers are indifferent across markets and can move freely between them. Indeed, they can randomize – the only restriction is that they cannot be fully present in multiple markets.

³ See Masters (2011).

⁴ In that paper $b=0$ but the extension to strictly positive values of b is straightforward.

3. Simulation

3.1. Functional forms

The numerical analysis uses the following functional forms: $m(\theta) = \bar{m}\theta^\eta$, $\bar{m} > 0$, $0 < \eta < 1$; $c(e) = \bar{c}e^\sigma$, $\bar{c} > 0$, $\sigma > 1$; $f(k, e) = k^\alpha e^{1-\alpha}$, $0 < \alpha < 1$.

The distribution $G(\cdot)$ is assumed to be uniform on $(0, \bar{z}]$. Recall that the focus of the paper is on the extent to which the modeled mechanism allows dispersion in firm productivity to pass through to wages. The advantage of using the uniform distribution is that whatever is the threshold, \hat{z} , of productivity above which ideas become viable, the distribution of viable ideas remains uniform. (I provide an example of what happens when the distribution is Pareto in the Robustness section below.)

3.2. External parameters

The basic unit of time is 1 year. So, the common discount rate, r , is set to 0.04. The job destruction rate, λ , is set to 0.2 to reflect the expected life of a job as about 5 years as identified by Cole and Rogerson (1999). The death rate, δ , for workers was set to 0.05. This corresponds to an expected duration of 20 years which would be the average expected remaining participation of workers with 40 year working lives. Blanchard and Diamond (1989) and Petrongolo and Pissarides (2001) discuss matching function parameters. From there I set $\bar{m} = 12$ and $\eta = 0.5$.

The output elasticity of capital, α , is potentially an issue here as much of the focus of the paper is on the intensity of factor (specifically labor) utilization. Micro-data studies such as Syverson (2004) include energy and materials consumption to proxy for variation in factor utilization. But, whatever the true utilization rate is, it should be reflected in payment to factors. Because frictions are small, factor payments are close to their marginal products. Restricting attention to payments to labor and capital in aggregate data should, therefore, internalize the variations in utilization. Following the real business cycle literature, then, $\alpha = 0.33$.

Bils and Cho (1994) provide analysis of factor utilization over the business cycle. They use data from time-and-motion studies in the UK to obtain a Frisch elasticity of effort with respect to the wage of around one third. That is to say, abstracting from any wealth effects, a 3% increase in the hourly wage is required to bring about a 1% increase in labor services per hour from the worker. This implies a value for σ of 4.⁵ All of the externally obtained parameters are summarized in Table 1.

3.3. Normalizations

The parameterized model exhibits certain normalizations that restrict the set of independent adjustments that can be made. The chosen distribution of z has one parameter, \bar{z} . Any change to \bar{z} , however, can be fully corrected for by an adjustment to \bar{c} . To see why, consider problem (9). As long as keeping $zf(k, e) - c(e)$ constant, while Eq. (11) continues to hold, implies that $z f_k(k, e)$ remains un-

⁵ If the instantaneous utility from a wage w and effort level e is $U(w, e) = w - ce^\sigma$, then it is straightforward to show that the Frisch elasticity of effort is

$$e_F = \frac{U_e}{e \left[U_{ee} - \frac{U_{we}^2}{U_{ww}} \right]}$$

Here, $U(w, e) = w - ce^\sigma$. Linearity with respect to the wage reduces the above definition to

$$e_F = \frac{U_e}{e U_{ee}} = \frac{1}{\sigma - 1}.$$

Table 1
External parameter values for the leading example.

\bar{m}	η	σ	α	r	λ	δ
12	0.5	4	0.33	0.04	0.2	0.05

changed, there is no impact of a change in z on any of the variables of interest. After substituting parametric forms we, therefore, impose that, for any z ,

$$zk^\alpha e^{1-\alpha} - \bar{c}e^\sigma = \chi \tag{12}$$

$$(1 - \alpha)zk^\alpha e^{-\alpha} - \sigma\bar{c}e^{\sigma-1} = 0 \tag{13}$$

where χ is an arbitrary constant. Eqs. (12) and (13) can be viewed as a system in \bar{c} and e . Solving for \bar{c} from Eq. (13) yields

$$\bar{c} = \frac{(1 - \alpha)zk^\alpha e^{1-\alpha-\sigma}}{\sigma}.$$

Substitution into Eq. (12) implies

$$zk^\alpha e^{1-\alpha} = \left(\frac{\sigma}{\sigma - 1 + \alpha} \right) \chi$$

which means match output does not change as long as e and \bar{c} are adjusted according to Eqs. (12) and (13). As $z_k^f(k, e) = \alpha zk^{\alpha-1} e^{1-\alpha}$ this does not change either. For any given adjustment, z_0-z_1 in TFP, the implied adjustments to \bar{c} and e are,

$$\frac{\bar{c}_0}{\bar{c}_1} = \left(\frac{z_0}{z_1} \right)^{\frac{\sigma}{1-\alpha}} \quad \text{and} \quad \frac{e_0}{e_1} = \left(\frac{z_1}{z_0} \right)^{\frac{1}{1-\alpha}}.$$

Nothing else changes. For computational purposes I normalize $\hat{z} = 1$ and use Eq. (7) to determine \bar{z} .

A second normalization comes in the choice of a and γ . The only place that either enters the equilibrium conditions is in Eq. (7) where both do and then only their ratio matters. With a lower cost, a , of searching for ideas more entrepreneurs enter so that to generate the same number of jobs their arrival rate of ideas, γ , has to be proportionately lower. The value of γ is set to 1.

A less obvious additional normalization is that the ratio a/γ itself can also be set arbitrarily. This is because it determines the opportunity cost of a vacancy. So, if a doubles, it should be clear that the variables of interest are restored to their former values if output doubles, the cost of effort doubles and the value of non-market activity doubles. The last requirement is easily met by doubling b . Constant returns to scale in production mean output doubles if the production parameters, z and α , remain unchanged but both k and e double. For effort to double requires that \bar{c} shrinks by a factor of 8. Given $\sigma = 4$ the total cost of effort, $\bar{c}e^\sigma$, then doubles as required. Thus some variables (e.g. the aggregate capital stock and the aggregate level of effort) change but the variables of interest (e.g. the unemployment rate and the Mm wage ratio) are fully restored to their initial values. While a normalization like this eliminates a parameter that could be used to match empirical targets, it avoids the thorny issue of what value either a or γ ought to be. The value of a/γ is set to 1.

3.4. Empirical targets

The remaining parameters \bar{c} , and b are determined by consideration of empirical targets. These pertain to the dispersion in TFP, the effective replacement ratio for workers and the unemployment rate.

To the extent that energy and raw materials are not perfect complements with capital and labor, Syverson (2004) overestimates the extent of true TFP dispersion. He finds an inter-quartile (IQ) ratio for TFP of between 1.34 and 1.56. I use the midpoint of this range, 1.45, as a target for the IQ ratio of the observable distribution of TFP. As effort is not measured, the observable distribution includes effort

variation so that the implied true TFP variation is smaller. If I applied the target value to true TFP, the model would generate a larger degree of wage dispersion.

As identified by Sattinger (1985) and highlighted by HKV, the relative value of employment to unemployment is an important determinant of wage dispersion in sequential search models. Hagedorn and Manovskii (2008) argue that what matters is the marginal entrant worker whose flow value of non-market activity should be 95.5% of his marginal product. Shimer (2005), on the other hand, sets that figure at 40%. Attempts to reconcile these numbers have come from both theoretical and empirical analyses. Mortensen and Nagypal (2007) for instance point out that what matters in the DMP framework is the average worker whose value of non-market activity is much lower than that of the marginal worker. Hall (2009) estimates a Frisch system embedded in a generalized DMP framework. What emerges is a “replacement ratio” – the ratio of consumption (i.e. utility) while unemployed to that while employed of around 85%. I adopt this target figure – some robustness analysis is provided below.

In the current model, the relevant replacement ratio, Rr , is the value of non-market activity plus the mean cost of effort divided by the mean wage. So,

$$Rr = \frac{b + \int_{\bar{z}}^z c(e^*(z)) \frac{dG(z)}{1-G(\bar{z})}}{\int_{\bar{z}}^z w^*(z) \frac{dG(z)}{1-G(\bar{z})}}$$

where $e^*(z)$ and $w^*(z)$ are the equilibrium values of the effort level and wage at the firm with TFP z .

The target unemployment rate is 5.5%. This is the typical figure used to characterize the average state of the US labor market for the 20 years or so prior to the financial crisis of 2007.

With two free parameters it is not possible to hit all three of the empirical targets. The values of $\bar{c} = 0.217$ and $b=0.426$ are chosen to obtain an IQ range for observable TFP of 1.45 and a replacement ratio of 0.85. These imply an unemployment rate of 5.93%.⁶

The equilibrium Mm ratio for wages is 1.253. In summarizing the empirical literature on wage dispersion, HKV obtain a range of values for the Mm ratio of wages among similar workers of 1.7–1.9. Thus, at least according to the Mm ratio measure, the model almost fully accounts for the one third of wage dispersion attributable to firm effects as identified by Abowd and Kramarz (1999). The IQ ratio for true TFP is 1.33.

4. Discussion

4.1. Robustness

Here I address a number of issues to do with the reliability of the results. First, I consider the replacement ratio target value. Then, I address the choice of IQ range for observable TPF over the unemployment rate as a target. Lastly, I recalibrate the model using a Pareto distribution for TFP. This permits comparison of other outcomes from the model to the empirical literature.

First, maintaining the IQ ratio for observable TFP at 1.45, I consider what happens when Rr is set to 0.71 (the preferred value of Hall and Milgrom (2008)) and 0.4 (from Shimer (2005)). The results, including a summary of the leading example analyzed above, are included in Table 2.

⁶ An earlier version of the paper, Masters (2013), includes an extension of the model which allows for ex post bargaining with firms posting only their capital stock. This introduces an additional parameter, the bargaining power of the worker. Changes in the bargaining power have almost no effect on the other variables but strongly impact the level of unemployment through the effect on vacancy creation. (The current model corresponds to a bargaining power of 0.5.) Given the proximity we already have, the target value of unemployment would be achieved with a small decrease in the worker's bargaining power.

Table 2
Robustness to changes in replacement ratio with fixed *IQ* ratio for observable TFP.

Target	Parameters		Outcomes			
	<i>b</i>	\bar{c}	<i>u</i> rate %	<i>Mm</i> ratio	true <i>IQ</i>	AMR %
0.85	0.426	0.217	5.93	1.253	1.33	29.8
0.71	0.297	0.212	4.29	1.255	1.33	42.0
0.4	0.004	0.205	2.99	1.261	1.33	61.0

The column headings are largely self-explanatory – true *IQ* is the *IQ* ratio for true TFP, AMR is the average monthly matching rate for unemployed workers.⁷ Lowering *Rr* while maintaining the same degree of dispersion in observable TFP has a significant impact on unemployment but little else. Both *b* and \bar{c} contribute to the cost of working. As such, there is a direct effect that tends to increase wage dispersion. However, holding other parameters constant and lowering both *b* and \bar{c} increases job creation which increases the average worker matching rate. This, then, increases the workers’ value to unemployment and makes them pickier. When workers get pickier, wage dispersion falls. The consequence is that the *Mm* ratio does not change much and the relationship between observable and true TFP dispersion is essentially fixed.

Table 3 examines what happens when the unemployment rate at 5.5% is used as a target instead of the *IQ* ratio for observable TFP.

Because the unemployment rate is held constant across the rows of Table 3, the “*u* rate” column has been replaced by “Obs *IQ*” which reports the *IQ* ratio for observable TFP. The average (monthly) matching rate is 32.3%. Again, rows represent different target values of the replacement ratio. In this case, wage and technology dispersion increase significantly as the replacement ratio is decreased. To maintain the same equilibrium level of unemployment, *b* and \bar{c} move in opposite directions. Without technology dispersion, lowering the replacement rate would necessarily reduce unemployment because it increases the returns to job creation. Here, the increase in the returns to job creation also play out along the extensive margin. So, while the unemployment rate in the lowest productivity market, \hat{z} , decreases significantly from 3.43% to 2.04% and to 0.22%, in the highest productivity market, \bar{z} , the unemployment rate increases from 7.28% to 8.41% and 9.02%. As the productivity distribution is uniform, the measure of firms in each market (i.e. with the same value of *z*) is the same so there are more workers in the high unemployment markets than the low unemployment markets. This is what allows the aggregate unemployment rate to be the same despite very different rates of vacancy creation.

A further potential issue for robustness is the use of the uniform distribution for TFP. Here I look into the use of the Pareto distribution. Thus,

$$G(z) = 1 - \left(\frac{\hat{z}}{z}\right)^\mu$$

The distribution is characterized by two parameters. The scale parameter, $\hat{z} > 0$, which is also the infimum of the support, and the shape parameter, μ .⁸ There are two good reasons to consider this functional form. First, it has a fat right tail similar to what is observed in the data on income and wealth distributions. Second, the conditional distribution for $z > \hat{z}$ is also Pareto with the same shape parameter (see

⁷ The average matching rate here is pinned down by the unemployment rate because

$$u = \frac{\delta + \lambda}{\delta + \lambda + AMR}$$

Given the same separation rate, $\delta + \lambda$, and unemployment rate, the matching rate has to be the same as used in HKV. The figure they use is 43% per month.

⁸ I have assumed in the forgoing that the infimum of the support is 0. As long as the extensive margin is active, so that $\hat{z} > \underline{z}$, this should not affect the results.

Table 3
Robustness to changes in replacement ratio with fixed unemployment rate.

Target	Parameters		Outcomes		
	<i>b</i>	\bar{c}	Obs <i>IQ</i>	<i>Mm</i> ratio	true <i>IQ</i>
0.85	0.710	0.049	1.354	1.182	1.258
0.71	0.042	20.2	1.870	1.782	1.636
0.4	-0.05	2×10^7	2.895	61.41	2.710

Masters (2016)).

Continuing to fix \hat{z} at 1, the shape parameter, μ , is chosen to ensure that Eq. (7) holds. I then use \hat{z} , *b* and \bar{c} to pin down the replacement ratio at 0.85, the *IQ* ratio for observable TFP at 1.45 and the unemployment rate at 5.5%.⁹ Under this parameterization the *Mm* wage ratio is 1.261 the *IQ* ratio for true TFP is 1.320 and the average monthly matching rate is 32.3%. These figures are strikingly similar to the first row of Table 2. This reflects the fact that the “distribution free” characteristic of the *Mm* measure of dispersion (almost) carries over to this environment. Similarly, the *IQ* ratio is close to being distribution free.

Because the *Mm* ratio has not been around for very long, it does not appear in the empirical literature that has sought to measure dispersion in other outcomes. Being closer to measured distributions, the Pareto version of the model helps in this regard. Arai (2003) provides information on the dispersion of both profits and capital-labor ratios across firms from a matched worker-firm panel of Swedish data. The coefficients of variation for each (among male workers) are 3.35 and 4.56. In the model these figures are 3.84 and 1.97. Why might the value of the capital-labor ratio be so far out? While market forces mean that profitability is likely to be similar across industries, the capital-labor ratio is very dependent on the specific technology which Arai (2003) does not control for. To test this I increased the elasticity of output with respect to capital, α , to 0.33 from 0.4 (keeping all other parameters unchanged). Mean profits rise by 14% while the mean capital-labor ratio increases by 52%.

4.2. Source of wage dispersion

Three features of the model contribute to the degree of wage dispersion: the distribution of productivities, variable effort levels, and frictions in the labor market. Here I consider what happens when each of these features is removed.

Clearly, if the distribution $G(\cdot)$ were degenerate there could be no technology dispersion. Without technology dispersion, as long as the cost of effort function is strictly convex, there can be no wage dispersion.

Ideally, to obtain the level of wage dispersion in the version of the model with exogenous effort, it should be recalibrated to achieve the same outcomes with respect to the replacement ratio the *IQ* ratio for observable TFP and unemployment rate. In principle we now have three parameters to achieve this goal. These are \bar{c} , *b* and the exogenous level of effort, \bar{e} . Unfortunately, with exogenous effort all three parameters have identical effects. I, therefore, fixed \bar{c} and *b* at their former values, 0.217 and 0.426 respectively and adjust \bar{e} to achieve a value for *Rr* of 0.85. The value of \bar{e} that emerges is 1.073 which causes unemployment to be 6.19%. Of course, with exogenous effort, the *IQ* ratio for observable TFP equals that of true TFP which is unchanged at 1.33. Despite the higher rate of unemployment, the *Mm* ratio for the wage drops to 1.004. Eliminating endogenous effort all but kills wage

⁹ A third potential benefit of the Pareto distribution is the additional parameter should allow the model to hit all three empirical targets. While this is true of the leading example in which *Rr* = 0.85, the non-linear nature of the model precludes this outcome in general. Using the Pareto distribution is also computationally very intensive.

dispersion.

To shut down labor market frictions, distinct markets indexed by wage/effort pairs are maintained but workers get jobs instantaneously. Consequently workers are indifferent across the jobs themselves rather than simply the markets they occupy. Clearly, with zero unemployment, the model can longer come anywhere near that target. The fact that leisure is never realized does not prevent it from providing utility so the replacement ratio, Rr , is still well defined. I recalibrate the model using \bar{c} and b to set the IQ ratio for observable TFP to 1.45 and Rr to 0.85. The values are $\bar{c} = 0.23$ and $b=0.418$. The Mm ratio for wages is now 1.245. Which confirms the result from HKV that labor market frictions contribute little to wage dispersion. The fact that $1.253 - 1.245 = 0.008$, however, means that the contribution of frictions to the Mm ratio of wages with endogenous effort is about twice its contribution with fixed effort.

The upshot is that ex ante productivity dispersion across firms is the primary source of wage dispersion. Endogenous effort and search frictions both provide a channel by which productive heterogeneity passes through to wages. Endogenous effort amplifies productivity dispersion through its impact on investment. Frictions in the labor market, cause dispersion in worker utility. The model simulation implies that the former channel is the most important. Simply amplifying the dispersion in TFP into the wage does not, however, lead to higher Mm ratios. This is because the Mm ratio is a relative measure of dispersion.

4.3. Comparison to HKV

With much larger Frisch elasticities of effort any degree of wage dispersion can emerge. For instance setting $\sigma = 1.05$ which corresponds to Frisch elasticity of 20, generates (after recalibration to the other targets) an Mm ratio for wages of 1.6 while the IQ ratio for true TFP dispersion drops to 1.014. Reading the technical appendix of HKV, it is reasonable to ask why my model can generate large Mm ratios for the wage. After all, that wages have a larger absolute dispersion than utility or TFP does not mean it shows up in the ratio of two points in the distribution.

Consider a simplified version of my model.¹⁰ Search is random, there is no capital and no worker death but there is a common discount rate r . Except where indicated, the remaining notation is consistent with the preceding analysis. Output is simply $p(z, e)$ and the cost of effort is $c(e)$. The optimal effort level in job type z is

$$e(z) = \arg \max_e p(z, e) - c(e).$$

Let $w(z)$ be the wage paid to a worker in job type z . Then, the reservation utility level (following HKV's technical appendix) is

$$\hat{w} - \hat{c} = \Phi \int_{\hat{z}}^z [w(z) - c(e(z)) - (\hat{w} - \hat{c})] dF(z)$$

where $\hat{w} = w(\hat{z})$, $\hat{c} = c(e(\hat{z}))$, $\Phi = m(\theta)/(r + \lambda)$ and $F(z) = G(z)/(1 - G(\hat{z}))$. As firms do not create vacancies that cannot match, this implies

$$\frac{\hat{w} - \tilde{c}}{\hat{w} - \hat{c}} = \frac{1 + \Phi}{\Phi}$$

where \hat{w} and \tilde{c} are the mean wage and cost of effort under distribution F . As in realistic environments, $\Phi >> 1$, this Mm ratio for utility is close to 1 (1.07 for a 5.5% unemployment rate). For the Mm wage ratio to be large, we therefore require the Mm cost ratio, \tilde{c}/\hat{c} , also to be large.

Now define $Y(z) = p(z, e(z)) - c(e(z)) - (r + \lambda)V_v(z)$. As the match surplus is $Y(z) - rV_u$, $\hat{Y} \equiv Y(\hat{z}) = rV_u = \hat{w} - \hat{c}$. With the bargaining

¹⁰ This version of the model delivers, qualitatively similar results to the general framework but the TFP dispersion required to generate quantitatively meaningful values of the Mm wage ratio, while hitting the other calibration targets, is too large.

power of the firms equal to ϕ we obtain

$$\phi(V_e - V_u) - (1 - \phi)(V_f - V_v) = 0$$

which, after some algebra, implies

$$\frac{\tilde{Y}}{\hat{Y}} = \frac{1 + (1 - \phi)\Phi}{(1 - \phi)\Phi}$$

where \tilde{Y} is $E(Y(z)|z \geq \hat{z})$. This Mm ratio is also quite close to 1 (1.14 for $\phi = 0.5$ and a 5.5% unemployment rate).

Now, assuming $p(z, e) = ze$ and $c(e) = \bar{c}e^\sigma$ it is straightforward to show that $Y(z) = (\sigma - 1)c(e(z)) - (r + \lambda)V_v(z)$. So that

$$\frac{\tilde{Y}}{\hat{Y}} = \frac{(\sigma - 1)\tilde{c} - (r + \lambda)\tilde{V}_v}{(\sigma - 1)\hat{c}} = \frac{\tilde{c}}{\hat{c}} - \frac{(r + \lambda)\tilde{V}_v}{(\sigma - 1)\hat{c}} \tag{14}$$

where $\tilde{V}_v = E(V_v(z)|z \geq \hat{z})$.

Eq. (14) highlights the distinction between this model and those considered by HKV. Allowing free-entry and ex post realization of z , converts this model into essentially that in their technical appendix. Setting $V_v(z) = 0$ (as in HKV) for all z reduces Eq. (14) to $\tilde{Y}/\hat{Y} = \tilde{c}/\hat{c}$. Then as σ approaches 1 from above, it is simple to show that \hat{w}/\hat{w} approaches \tilde{Y}/\hat{Y} from below. This implies an upper bound for \hat{w}/\hat{w} of 1.14. Simply incorporating cost of effort into their framework is not enough to generate the kind of wage dispersion that can emerge from the current model. Rather, here, high productivity firms are the sole proprietors of their technology and receive rents which are reflected in their continuation values, $V_v(z)$, and are therefore excluded from the match surplus. These rents are, in part, obtained from requiring high effort levels from workers in exchange for high wages. The implied high value of \tilde{V}_v , means that with \tilde{Y}/\hat{Y} fixed at 1.14, \tilde{c}/\hat{c} is large and therefore so is \hat{w}/\hat{w} .

4.4. Competing sources of wage dispersion

As mentioned in the introduction, there are other ways to generate wage dispersion among otherwise similar workers. On-the-job search eliminates (or at least reduces) the opportunity cost of taking a low-wage job and in models like [Ortego-Marti \(2016\)](#) or [Burdett et al. \(2011\)](#), employment is valued more highly than is reflected in the difference in income. Of course in reality all three mechanisms may be in play. Indeed, it is interesting to note that the dispersion attributed to on-the-job search by HKV, that obtained by [Ortego-Marti](#) and that obtained here all generate Mm ratios of around 1.2. Combining them may go a long way toward explaining all of the observed dispersion in wages.

A circumstantial yet compelling argument for the role of endogenous effort comes from [Postel-Vinay and Robin \(2002\)](#). They estimate a version of the [Burdett and Mortensen \(1998\)](#) model on a matched panel data set from France. They are able to decompose wage dispersion into firm, worker and search effects. They estimate their model separately for 7 classes of worker differentiated by the level of human capital associated with their jobs. The firm and search effects are significant for all skill levels but the individual worker effects disappear as the complexity of the tasks diminish. At every skill level they find that to fit wage dispersion, the discount rate has to be very large – in the order of 40% annually. With a more realistic, say 5%, discount rate the implied lifetime earnings differential from moving to a higher productivity firm would generate more movement in their model than appears in their data. Essentially, what is happening is that any remaining mis-specification of the model is showing up in the discount rate. As all of the jobs are in the Paris metropolitan area, it seems improbable that the missing component of the model is switching costs. As firms are permitted to match outside offers in the model, firm-specific human capital would show up in the measured wages. Disutility of work positively correlated with the firm effect is, therefore, a likely candidate for the missing component.

5. Conclusion

This paper explores the role of endogenous worker effort in addressing the relationship between firm level TFP dispersion and wage dispersion. By incorporating physical capital investment, it is able to provide a quantitatively important mechanism for understanding the size of firm effects in measured wage dispersion. Here, wage dispersion emerges due to differences in firm productivity and relies on a simple individual labor supply mechanism that permits high productivity firms to offer higher wages in return for greater effort from workers. In equilibrium, the *Mm* ratio of worker utility and the gains from trade remain small. As the ratio is a relative measure of dispersion, however, amplification alone may not bring about a large *Mm* ratio in wages. As sole proprietors of their superior technologies, high productivity firms

receive rents which come partly from the high levels of effort they can require of workers in exchange for high wage jobs. This means that higher productivity firms receive a disproportionately larger share of the net revenue (revenue minus costs incurred). The narrow range of worker utility effectively pins down the lowest wage in the distribution and higher wages simply compensate workers for their extra efforts.

Analytical parsimony has been a motivating factor in restricting analysis to endogenous worker effort within a sequential search framework. The simulation is able to demonstrate the potential for this mechanism to explain a significant share of the firms' component of measured wage dispersion. To enquire further into the quantitative importance of endogenous worker effort a possibility is to incorporate it into a structural model along the lines of [Postel-Vinay and Robin \(2002\)](#).

A.1. Proof of Claim 2

For expositional brevity I solve the dual of (8):

$$V_u^* = \max_{k, \omega} V_u(\omega) \text{ subject to: } V_f(\omega; z, k) - k = V_v(z) \tag{15}$$

That reversing the problem in this way is well known in the directed search literature (see [Rogerson et al. \(2005\)](#)). To see that the result extends to the current environment, consider the Lagrangians of each problem. They are,

$$L_f = V_f(\omega; z, k) - k + \mu_f [V_u(\omega) - V_u^*]$$

and

$$L_w = V_u(\omega) + \mu_w [V_f(\omega; z, k) - k = V_v(z)]$$

where $\mu_i, i = w, f$ are the Lagrange multipliers. After recognizing that $\mu_w = 1/\mu_f$ the first-order conditions are identical.

Substituting for the value functions problem (15) becomes,

$$V_u^* = \max_{k, \omega} \frac{m(\theta)[w - c(e)] + (r + \delta + \lambda)b}{(r + \delta)(r + \delta + \lambda + m(\theta))} \text{ subject to } \frac{m(\theta)[zf(k, e) - w]}{(r + \lambda)[(r + \delta + \lambda)\theta + m(\theta)]} - k = V_v(z). \tag{16}$$

Eliminating the wage leads to

$$V_u^* = \max_{k, e, \theta} \frac{m(\theta)[zf(k, e) - c(e)] - (r + \lambda)[(r + \delta + \lambda)\theta + m(\theta)](k + V_v(z))}{(r + \delta)(r + \delta + \lambda + m(\theta))} \tag{17}$$

The first order conditions for an interior solution with respect to k, e and θ imply

$$m(\theta)[zf_k(k, e) - (r + \lambda)] - (r + \lambda)(r + \delta + \lambda)\theta = 0 \tag{18}$$

$$zf_e(k, e) - c'(e) = 0 \tag{19}$$

$$m'(\theta)[zf(k, e) - c(e) - b] - (r + \lambda)[r + \delta + \lambda + m(\theta) + m'(\theta)(1 - \theta)](k + V_v(z)) = 0. \tag{20}$$

The objective function in Problem (17) is clearly concave in (k, e) which means that for any given θ there exists a unique solution, $(k(\theta), e(\theta))$ to the system of Eqs. (18) and (19). Substituting these into LHS (20) and differentiating with respect to θ yields

$$m''(\theta)[zf(k, e) - c(e) - b] - (r + \lambda)m''(\theta)(1 - \theta)(k + V_v(z)) + m'(\theta)[zf_e(k, e) - c'(e)] \frac{de(\theta)}{d\theta} + \{m'(\theta)zf_k(k, e) - (r + \lambda)[r + \delta + \lambda + m(\theta) + m'(\theta)(1 - \theta)]\} \frac{dk(\theta)}{d\theta}. \tag{21}$$

Using (19) the second line of (21) is zero. Using (18) the contents of the curly brackets becomes

$$-(r + \lambda)(r + \delta + \lambda + m(\theta))/m(\theta).$$

Furthermore, if we impose (20) to be true then the first line of (21) becomes

$$\frac{m''(\theta)(r + \lambda)(r + \delta + \lambda + m(\theta))}{m'(\theta)}(k + V_v(z)).$$

Under this restriction (21) reduces to

$$(r + \lambda)(r + \delta + \lambda + m(\theta)) \left[\frac{m''(\theta)(k + V_v(z))}{m'(\theta)} - \frac{(m(\theta) - \theta m'(\theta))}{m(\theta)} \frac{dk(\theta)}{d\theta} \right]. \tag{22}$$

Then if $m(\theta) \equiv \bar{m}\theta^\eta$, with $\eta \in (0, 1)$ (22) further reduces to

$$-(r + \lambda)(1 - \eta)(r + \delta + \lambda + \bar{m}\theta^n) \left[\frac{(k + V_v(z))}{\theta} + \frac{dk(\theta)}{d\theta} \right]. \quad (23)$$

In general, $\frac{dk(\theta)}{d\theta} < 0$ so (23) cannot be signed. However, if we further impose that $c(e) \equiv \bar{c}e^\sigma$ with $\sigma > 1$ and $f(k, e) \equiv k^\alpha e^{1-\alpha}$ with $\alpha \in (0, 1)$ then Eqs. (18) and (19) can be solved for k and differentiated. This leads to

$$\left[\frac{k}{\theta} + \frac{dk(\theta)}{d\theta} \right] = \frac{\{\bar{m}\theta^n(\sigma - 1)(1 - \alpha) + (r + \delta + \lambda)[\eta(\sigma - 1 + \alpha) + \sigma\alpha]\}k}{\theta[(r + \delta + \lambda)\theta + \bar{m}\theta^n](\sigma - 1)(1 - \alpha)} > 0.$$

So, as $V_v(z) \geq 0$, LHS (20) seen as a function of θ is downward sloping at any point for which Eq. (20) holds. As LHS (20) is continuous in θ , it follows that after taking account of the effect of θ on k and e , the maximand in (17) is quasiconcave in θ . The unique solution to (20) represents a maximum.

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