A theory of minimum wage compliance (or voluntary recognition of unions).

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Abstract

An urn-ball matching model of the labor market is used to develop a theory of minimum wage compliance or voluntary recognition of unions. Workers can direct their search but, in the absence of wage commitment from the firms, they have no basis to do so. The default means of wage formation in one-on-one matches is Nash bargaining. When there are multiple applicants competition drives the workers down to their continuation value. By attracting more applicants, a binding wage floor provides a means for firms to increase matching rates and improve match quality. An otherwise poorly enforced minimum wage acts as a commitment device for the payment of more generous wages.

1 Introduction

Compliance with weakly enforced minimum wage laws and voluntary recognition of unions are examples of choices made by firms that cause them to pay higher wages than they otherwise would. This paper explores the question of why firms make such choices. The answer proposed here is that, by attracting more applicants, higher wages allow firms to fill vacancies more quickly and fill them with more appropriate workers. Firms cannot make a commitment to pay an (*ex ante*) optimal wage and, once applications are received, they offer wages that are too low. To the extent that workers can be made aware, adoption of a minimum wage or recognition of a union can act as a commitment device for the payment of higher wages.

Evidence that the minimum wage law in the USA is weakly enforced was first documented by Ashenfelter and Smith [1979]. More recently, Eckstein *et al* [2006] estimated a structural search-based model of the labor market and back out a measure of non-compliance. While they are 25 years apart and based on different data sets, both studies reveal that between 30% and 40% of those workers who should receive the minimum wage are underpaid. Yet another data source was used by Holtzer *et al* [1991] who look at application rates at jobs paying below, at and above the minimum wage. The rate of non-compliance in their sample (after removing workers in exempt industries) is 25%. Despite this evidence, there has been no theoretical work attempting to understand firms' decisions with respect to compliance.

While unions provide many other services to workers, it is well established that they also provide a wage premium (see Booth [1995]). Some evidence on voluntary recognition of unions in the USA comes from the Federal Mediation and Conciliation Service (FMCS) [2004] p. 18. It reports that of the 1,311 initial contract cases assigned to federal mediators in 2004, 258 were assigned from certification sources other than the National Labor Relations Board (NLRB) "such as voluntary recognitions". How many of these are truly voluntary is not reported but these figures do understate the proportion of non-NLRB certifications. This is because only NLRB certifications are necessarily referred to the FMCS. For the UK, the Central Arbitration Committee [2004] does report voluntary recognitions. At any stage in the formal proceedings, employers can choose to voluntarily recognize unions. Between 2000 and 2004, of 361 applications for recognition, 85 were accepted by employers without a ballot.

As search theory explicitly incorporates the application decision of workers, it provides a natural environment to address the issue of why firms might associate themselves with institutions that lead to paying higher wages than they would otherwise pay.[1] Essentially, if there is a large number of workers and vacancies in a market and, in any period, workers apply to a subset of the vacancies, then the number of applicants at any one vacancy can be described by a discrete probability distribution (that puts positive probability on no applicants at all). The usual approach assumes that firms can commit to wages that become common knowledge. Workers make their job applications on the basis of those "posted" wages. In the absence of any coordination mechanism, it is assumed that homogeneous workers follow symmetric strategies. In equilibrium, workers will be indifferent across applying to each firm and attach a probability of applying to each which depends on the characteristics (including the wage) of the vacancy.

The central theoretical deviation from the literature of this work is that (in the absence of a minimum wage) firms cannot commit to wages.[2] Only the existence of a vacancy becomes common knowledge. In the baseline model with homogeneous firms and workers, I assume that when there is only one applicant the wage is determined by Nash bargaining. Otherwise,

[1] See Rogerson *et al* [2005] for more background to this approach to modelling the labor market.

[2] There is no data set that systematically collects information on the nature of job advertisements. Menzio [2007] suggests that in general there are at best non-contractual indications of the nature of pay and conditions.

competition between workers leads to one worker being hired at random with a wage equal to the workers' continuation value. Julien *et al* [2006] established that while firms retain any of the bargaining power in one-on-one meetings, such a mechanism for wage formation is suboptimal. The point here is to look into when firms might use labor market institutions such as the minimum wage as a commitment device to prevent themselves from paying wages that are too low from an *ex ante* perspective. The key is that by declaring a vacancy as a minimum wage job (or by associating itself with a union) a firm can advertise job openings that incorporate credible wage commitments that workers can use to direct their applications.

In the baseline model, higher wages allow firms to fill vacancies faster. By incorporating match specific heterogeneity, the model is extended to address the issue of whether commitment to a more generous wage structure can also provide a cost effective improvement in match quality. Providing the worker with all the bargaining power in one-on-one meetings isolates the effects of this source of heterogeneity. It is shown that firms will indeed adopt a minimum wage that is low enough.

Most of the prior work on directed search has been focussed on the theoretical development of the framework (see Rogerson *et al* [2005]). An exception to this has been Acemoglu and Shimer [1999] who show that with risk-averse workers, unemployment insurance (UI) can increase output. Essentially, they show that the investment decisions of firms is influenced by the search decisions of workers. Workers will look for more productive jobs if they are insured against long periods of unemployment. Consequently, firms create fewer but more productive jobs in the presence of a UI system than they would in the absence of UI. The net effect for moderate UI coverage is an increase in economic output.

While in principle, the argument put forward in this paper applies to any institution that permits firms to commit to paying higher wages than it otherwise would, for simplicity the analysis is restricted to the example of the wage floor. The paper proceeds as follows. The next section lays out the baseline model with homogeneous workers and firms. The model is analyzed in 3 versions: without minimum wages, with compulsory minimum wages and with voluntary adoption of the minimum wage. Section 3 incorporates match specific heterogeneity. Section 4 concludes.

2 Model

2.1 Basic Environment

The discrete time infinite horizon economy comprises a continuum of *ex ante* homogeneous infinite lived workers and firms. Workers who get jobs are replaced by new entrants to the market so that the mass of unemployed workers is fixed; normalized to 1. Both workers and firms are risk neutral and discount the future at a rate r per period. Workers experience utility from leisure at the rate b per period.

Firms can create as many vacancies as they like but have to pay an advertising cost a per period that the vacancy is held open. The mass of vacancies, v, is controlled by a zero-profit condition. When a firm hires a worker to a vacancy, the match produces $p > b$ units of the perfectly divisible (perishable) consumption good per period. Consumption of one unit of the good provides one unit of utility to firms or workers.

Within any time period, firms post vacancies and then workers simultaneously apply to whichever job they like but they are restricted to one application per period.[3] The main informational restriction is that, as workers apply simultaneously, they do not know precisely how many others have applied for any particular vacancy. Burdett *et al* [2001] show that in fi-

[3]This restriction can be relaxed. Albrecht *et al* [2006] study a directed search environment with multiple applications. As long as workers do not apply to every opening in every period there is a nondegenerate distribution of applicants across vacancies.

nite markets, in the unique symmetric equilibrium of the application game workers will apply with equal probability to each vacancy. Consequently the number applicants who show up at any one firm is binomially distributed according to the population size of workers and vacancies in the market. As the market becomes very large, this distribution converges to Poisson. The appropriate parameter for the Poisson distribution is q, the expected queue length or number of applicants per vacancy. If vacancies are completely indistinguishable, $q = 1/v$. Specifically, for a vacancy with expected queue length, q, the implied probability that it will receive exactly n applications is $q^n e^{-q}/n!$, for $n = 0, 1, 2...$When vacancies differ, due for instance to a deviation from equilibrium behavior, the expected queue lengths adjust so that workers are indifferent across vacancy types supporting their propensity to randomize.

So far, the environment I have described is essentially the large market version of the model of Burdett *et al* [2001] adapted to a labor market context (see Rogerson *et al* [2005]). The point of departure from standard directed search is that I do not assume that firms can commit to posted wages. Instead here, wage formation depends on the realized match configuration. When 2 or more workers apply to the same vacancy, the firm hires one worker, chosen at random, at a wage equal to the workers' (flow) continuation value. The workers are clearly indifferent between employment and continued unemployment at this wage and I assume the worker takes the job. One can think of this wage as emerging from (unmodelled) rounds of Bertrand competition between workers. Where meetings are one-on-one, the firm and worker use generalized Nash bargaining in which the parameter $\beta \in [0, 1]$ represents the bargaining power of the worker.[4]

[4]There is an issue as to whether the firm is simply committing to the Nash bargaining outcome instead of the posted wage. The view put forward here is that when there is a surplus to be divided the Nash bargaining outcome represents a cultural norm. (When there are multiple homogeneous applicants, competition means that there is nothing to bargain over.)

For the workers, the probability that they get to bargain their wage is equal to the number of vacancies multiplied by the probability that any vacancy gets exactly one applicant divided by the number of unemployed workers:

$$qe^{-q}\left(\frac{v}{1}\right) = e^{-q}.$$

Let the asset value to unemployment be V. By assumption, the value to meeting a firm with more than one applicant is V whether the worker gets the job or not. Thus,[5]

$$rV = b + e^{-q}\left(\frac{\hat{w}}{r} - V\right) \tag{1}$$

where, $\hat{w}$ is the bargained wage and the non-bargained wage is rV.

The asset value to holding open a vacancy, V_f, is obtained from:

$$rV_f = -a + qe^{-q}\left(\frac{p-\hat{w}}{r} - V_f\right) + \left[1 - e^{-q} - qe^{-q}\right]\left(\frac{p - rV}{r} - V_f\right). \tag{2}$$

The (flow) match surplus for one-on-one meetings is $p - rV - rV_f$. Nash bargaining leads to the workers getting a share β of this in addition to their continuation value, rV. As long as the surplus is positive,

$$\hat{w} = \beta\left(p - rV_f\right) + (1-\beta)rV. \tag{3}$$

If the match surplus is strictly negative there is no match.

Definition 1 *A zero-profit equilibrium is a mass of vacancies, v^*, such that $q = q^* \equiv (1/v^*)$ solves (2) with $V_f = 0$ where $\hat{w}$ and V are obtained from (1) and (3).*

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[5] End of period valuation means that everything that happens in the ensuing period is discounted. Thus,

$$V = \frac{1}{1+r}\left\{b + e^{-q}\left(V_e\right) + \left(1 - e^{-q}\right)V\right\}$$

where V_e is the value to employment at wage $\hat{w}$. Then

$$V_e = \frac{1}{1+r}\left(\hat{w} + V_e\right)$$

leads to $V_e = \hat{w}/r$ and equation (1) follows.

Solving (1) and (3) for V and $\hat{w}$ indicates that for any q,

$$rV = \frac{\beta e^{-q} p + rb}{\beta e^{-q} + r}, \quad \hat{w} = \frac{\beta p(e^{-q} + r) + (1-\beta)rb}{\beta e^{-q} + r} \tag{4}$$

As $p > b$, one-on-one match surplus is always positive. This also shows that for any given value of q, $\beta = 1$ means $\hat{w} = p$ and $\beta = 0$ means $\hat{w} = rV = b$. Otherwise, $p > \hat{w} > rV > b$. Substituting for V and $\hat{w}$ into (2) and setting $V_f = 0$ yields the following implicit expression for the equilibrium queue length, q^*:

$$a = (p - b)\left[\frac{1 - e^{-q^*}(1 + \beta q^*)}{\beta e^{-q^*} + r}\right] \tag{5}$$

As q varies from 0 to ∞, the expression

$$\left[\frac{1 - e^{-q}(1 + \beta q)}{\beta e^{-q} + r}\right]$$

increases strictly monotonically from 0 to $1/r$. Existence of an equilibrium therefore requires that $ra < p - b$. This is because no matter how tight the market, the firms have to incur the advertising cost for at least one period. Strict monotonicity ensures that whenever the equilibrium exists it is unique.

Clearly, an increase in a or b or a decrease in p causes the equilibrium queue length to increase as firms produce less vacancies. The parameter r here is inversely related to the "thickness" of the market. In thicker markets, the meeting rate is higher so that the extent of discounting between possible meetings is lower. As firms expect to fill their openings more quickly, vacancies become effectively cheaper to create which leads to a decrease in the expected number of workers per vacancy.

2.2 Minimum wage with full enforcement

Let the value to unemployment when all firms comply with a minimum wage, $\bar{w}$, be $\bar{V}$. The minimum wage binds when it exceeds the workers' flow continuation value, $r\bar{V}$. When it does not bind, the market is identical to that without a minimum wage and $\bar{V} = V$ as derived above. The analysis therefore only considers the case in which $\bar{w} > rV$.

There is some question as to how Nash bargaining should be applied in this circumstance. As long as the workers threatpoint is $r\bar{V}$, Nash's Independence of Irrelevant Alternatives axiom means that while $\bar{w}$ lies between $r\bar{V}$ and $\hat{w}$ the minimum wage will not directly influence the outcome of the bargaining.[6] The question is really whether the worker's threatpoint should be $r\bar{V}$ or $\bar{w}$. A minimum wage paying firm has no obligation to hire a worker. Rather, the obligation is that if the worker is hired, the wage to be paid must be at least $\bar{w}$. Because of this, the relevant threatpoint should be $r\bar{V}$ as this is all the worker can base his negotiations on. The worker cannot demand that as a last resort he be hired at $\bar{w}$. Consequently, if $\bar{V}_f$ represents the value to holding open a minimum wage vacancy and $\bar{w} < \beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}$, then $\hat{w} = \beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}$. It is possible, however, that the minimum wage is so high that $\bar{w} > \beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}$. In this case, the minimum wage becomes a relevant alternative as the parties cannot agree (by law) to match at a wage below $\bar{w}$. In general, we have

$$\hat{w} = \max\left\{\beta\left(p - r\bar{V}_f\right) + (1-\beta)r\bar{V}, \bar{w}\right\} \tag{6}$$

When $\hat{w} = \bar{w}$, the minimum wage is *completely* binding otherwise it is termed *partially* binding.[7] Thus, when the minimum wage is partially binding there remains a two point wage distribution with support $\{\bar{w}, \hat{w}\}$. When the minimum wage is completely binding the wage distribution is degenerate at $\bar{w}$.

Given an expected queue length, $\bar{q}$, the number of firms who end up with 2 or more applicants in a given time period is

$$\left(1 - e^{-\bar{q}} - \bar{q}e^{-\bar{q}}\right)\bar{v}$$

[6] See Osborne and Rubinstein [1990] for a complete modern exposition of Nash bargaining.

[7] Notice that had I used $\bar{w}$ as the worker's threatpoint during negotiations, the minimum wage would never completely bind and the environment would be analytically simpler.

where $\bar{v}$ is the mass of vacancies. As $\bar{v} = 1/\bar{q}$, and the number of workers hired by firms with more than one applicant equals the number of firms that get more than one applicant, we have

$$r\bar{V} = b + e^{-\bar{q}} \left(\frac{\hat{w} - r\bar{V}}{r} \right) + \left(\frac{1 - (1+\bar{q})e^{-\bar{q}}}{\bar{q}} \right) \left(\frac{\bar{w} - r\bar{V}}{r} \right) \tag{7}$$

For firms,

$$r\bar{V}_f = -a + \bar{q}e^{-\bar{q}} \left(\frac{p - \hat{w} - r\bar{V}_f}{r} \right) + \left[1 - (1+\bar{q})e^{-\bar{q}}\right] \left(\frac{p - \bar{w} - r\bar{V}_f}{r} \right) \tag{8}$$

Definition 2 *An equilibrium under a fully enforced minimum wage, $\bar{w}$, is a mass of vacancies, $\bar{v}^*$, such that $\bar{q} = \bar{q}^* \equiv 1/\bar{v}^*$ solves (8) with $\bar{V}_f = 0$.*

A characterization of equilibrium is achieved by setting $\bar{V}_f = 0$ in equations (7), (8) and (6) so that $\bar{q}^*$ solves

$$ra = \bar{q}e^{-\bar{q}} (p - \hat{w}) + \left[1 - (1+\bar{q})e^{-\bar{q}}\right] (p - \bar{w}) \tag{9}$$

where

$$\hat{w} = \max\left\{\beta p + (1-\beta)r\bar{V}, \bar{w}\right\}$$

and

$$r^2\bar{q}\bar{V} = r\bar{q}b + \bar{q}e^{-\bar{q}}(\hat{w} - r\bar{V}) + \left[1 - (1+\bar{q})e^{-\bar{q}}\right] \left(\bar{w} - r\bar{V}\right) \tag{10}$$

Lemma 1 *As long as*

$$p - b > \frac{ra}{(1-\beta)} \tag{11}$$

there exists a unique value, $\bar{w}_T \in [r\bar{V}, p]$ of the minimum wage such that

$$\bar{w}_T = \beta p + (1-\beta)r\bar{V}$$

Proof. Setting $\hat{w} = \bar{w}_T$ and substituting into equations (9) and (10) implies,

$$ra = (p - \bar{w}_T)\left(1 - e^{-\bar{q}}\right) \tag{12}$$

$$\bar{w}_T = \frac{\beta p\,[r\bar{q} + 1 - e^{-\bar{q}}] + (1-\beta)r\bar{q}b}{[r\bar{q} + \beta\,(1 - e^{-\bar{q}})]} \tag{13}$$

As $\bar{q}$ increases, equation (13) generates a strictly positive and monotonically decreasing value of $\bar{w}_T$ which approaches $\beta p + (1-\beta)b$ as $\bar{q} \to \infty$. Meanwhile, equation (12) generates a monotonically increasing value of $\bar{w}_T$ which is negative for low values of $\bar{q}$ and approaches $p - ra$ as $\bar{q} \to \infty$. There is at most one value of $\bar{w}_T$. As long as $p - ra > \beta p + (1-\beta)b$, $\bar{w}_T$ exists and it is unique. ■

Claim 1 *Whenever $p - \bar{w} > ra$ there exists a unique equilibrium under a fully enforced minimum wage,*

Proof: See Appendix.

The condition $p - \bar{w} > ra$ is required for firm participation. Without it, firms could not recoup their advertising costs and no vacancies would be created. Under this restriction Claim 1 establishes the existence and uniqueness of equilibrium with a fully enforced minimum wage. The equilibrium can be one of two types: with a partially binding minimum wage or with a completely binding minimum wage. Holding all other parameters fixed, the type of equilibrium that transpires is determined by the magnitude of the minimum wage, $\bar{w}$. Lemma 1, establishes that when $(p-b)\,(1-\beta) < ra$, $\bar{w}_T$ exists. Then, if the minimum wage is set below $\bar{w}_T$, it binds partially. When the minimum wage is set at or above $\bar{w}_T$, it binds completely. When $(p-b)\,(1-\beta) \leq ra$, $\bar{w}_T$ does not exist and equilibrium involves a partially binding minimum wage for every value of $\bar{w}$ between b and $p - ra$.

When $\beta = 0$, $\bar{w}_T = b$ and a minimum wage that binds at all binds completely. Beyond that, it is straightforward to show that, while it continues to exist, $\bar{w}_T$ strictly increases with β. Moreover, from the definition of $\hat{w}$, it should be clear that for $\beta > 0$, $\bar{w}_T > rV$ and (as $p - b > ra$) there is always some range of minimum wages which will only partially bind.

2.3 Voluntary adoption of the minimum wage

The issue considered here is whether a firm might choose to adopt a weakly enforced minimum wage if the associated legal framework imbues sufficient credibility. The US Department of Labor regulation stipulates that workers can be awarded a *maximum* of twice the backpay for up to 2 years if they lodge a successful complaint against their employer. Only when there is "willful" or repeated disregard of the law is there any criminal penalty incurred by the firm.[8]

It is assumed here that when a firm declares itself a minimum wage payer, violation of the law is considered willful and that the penalty for willful violation is sufficiently punitive that no firm adopting the minimum wage will ever violate the law. Throughout this analysis, the value of the minimum wage, $\bar{w}$, remains exogenous to the firms. Firms simply choose whether to adopt the minimum wage or not.

Let ϕ represent the probability with which an individual firm adopts the minimum wage. If Φ represents the probability with which all other firms adopt the minimum wage, an equilibrium in this extended environment is a $\phi^* \in \{0, 1\}$ such that $\phi^* = \Phi$ is each individual firm's optimal adoption choice. Equilibrium is therefore restricted to pure strategy, symmetric Nash. (The possibility of firms using mixed strategy equilibria is considered below.) Under this restriction, then, two types of equilibrium are possible, $\phi^* = 0$ and $\phi^* = 1$. Clearly, the values to being in equilibrium with $\phi^* = 0$ are precisely those that pertain in the equilibrium in the basic environment described in Section 2.1. Similarly, the values to being in equilibrium with $\phi^* = 1$ are precisely those that pertain in equilibrium when the minimum wage is fully enforced (Section 2.2). The issue here is to determine the sets of values of $\bar{w}$ for which either type of equilibrium of the extended environment exists?

Let $\tilde{V}_f$ be the value to creating a minimum wage vacancy ($\phi = 1$) when

[8] See http://www.dol.gov/asp/programs/guide/minwage.htm

all other vacancies are noncompliant (i.e. $\Phi = 0$). Then,

$$r\tilde{V}_f = -a + \tilde{q}e^{-\tilde{q}}\left(\frac{p-\tilde{w}}{r} - \tilde{V}_f\right) + \left(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}\right)\left(\frac{p-\bar{w}}{r} - \tilde{V}_f\right) \tag{14}$$

where

$$\tilde{w} = \max\left\{\beta\left(p - r\tilde{V}_f\right) + (1-\beta)rV, \bar{w}\right\} \tag{15}$$

is the wage paid by the deviant in case of a one-on-one match and $\tilde{q}$ is the expected number of applicants. As workers are fully aware of the characteristics of all vacancies, they apply to the deviant firm in such numbers that makes them indifferent between applying to the minimum wage vacancy and all the other vacancies. The value of $\tilde{q}$ is therefore obtained from

$$rV = b + e^{-\tilde{q}}\left(\frac{\tilde{w} - rV}{r}\right) + \left(\frac{1-(1+\tilde{q})e^{-\tilde{q}}}{\tilde{q}}\right)\left(\frac{\bar{w} - rV}{r}\right) \tag{16}$$

where V has the same value that emerged in the basic model without minimum wages. Noncompliance, $\phi = \Phi = 0$, is an equilibrium if and only if $\tilde{V}_f \leq V_f = 0$.

For full-compliance $\phi = \Phi = 1$ to be an equilibrium, firms should not prefer deviation to noncompliance. Let $\tilde{\tilde{V}}_f$ be the value to noncompliance ($\phi = 0$) when all other vacancies comply (i.e. $\Phi = 1$) with the minimum wage, $\bar{w}$. Then,

$$r\tilde{\tilde{V}}_f = -a + \tilde{\tilde{q}}e^{-\tilde{\tilde{q}}}\left(\frac{p-\tilde{\tilde{w}}}{r} - \tilde{\tilde{V}}_f\right) + \left[1 - e^{-\tilde{\tilde{q}}} - \tilde{\tilde{q}}e^{-\tilde{\tilde{q}}}\right]\left(\frac{p-r\bar{V}}{r} - \tilde{\tilde{V}}_f\right) \tag{17}$$

where

$$\tilde{\tilde{w}} = \beta\left(p - r\tilde{\tilde{V}}_f\right) + (1-\beta)r\bar{V} \tag{18}$$

is calculated using $r\bar{V}$ as the worker's threat point and $\tilde{\tilde{q}}$ is the expected number of applicants at the noncompliant firm. Workers apply to the deviant firm in such numbers that they are indifferent across all vacancies. The value of $\tilde{\tilde{q}}$ is therefore obtained from

$$r\bar{V} = b + e^{-\tilde{\tilde{q}}}\left(\frac{\tilde{\tilde{w}} - r\bar{V}}{r}\right) \tag{19}$$

Compliance, $\phi = \Phi = 1$, is an equilibrium if and only if $\widetilde{\bar{V}}_f \leq \bar{V}_f = 0$.

Claim 2 *Under the parameter restrictions required for existence of equilibria in the basic environment and under minimum wage with full enforcement, either $\phi^* = 0$ or $\phi^* = 1$ type equilibria exist under voluntary compliance. These equilibrium types do not generically coexist.*

Proof: See Appendix

Claim 2 means that a minimum wage that any firms voluntarily adopt will be adopted by all firms. Furthermore, any value of the minimum wage at which firms are indifferent between adoption and non-adoption, the proportion of firms choosing to adopt does not affect the workers' continuation value, i.e. $V = \bar{V}$. This is because for firms to be indifferent between adoption and non-adoption $V_f = \bar{V}_f = 0$ and the associated queue length from offering the minimum wage when every other firm does, is the same as when no other firm offers it. This is why the possibility of mixed strategy equilibria were ignored. By definition, in any mixed strategy equilibrium firms have to be indifferent between adoption and non-adoption of the minimum wage. This means that mixed strategy equilibria only occur at the critical, measure zero set of, parameter values for which both pure strategy equilibria also exist. At those parameter values there is a continuum of mixed strategy equilibria indexed by the proportion of firms adopting the minimum wage.

The foregoing does not prove that the $\phi^* = 1$ type equilibrium ever exists. Claim 3 addresses this question.

Claim 3 *For every $\beta < 1$, there exists a minimum wage, $\bar{w}$, sufficiently close to rV such that the unique equilibrium under voluntary adoption is $\phi^* = 1$*

Proof: See Appendix

The gist of the proof is that, if the value to posting the minimum wage when no one else does is strictly increasing at the point where it just begins to bind, then for some range of values of $\bar{w}$ sufficiently close to rV, $\phi^* = 0$

cannot be an equilibrium. From Claim 2, this implies that over that range, the equilibrium is of type $\phi^* = 1$.

The intuition is clearest in the $\beta = 0$ case. Because, in that case, workers get at least b whether they apply to the minimum wage job or not, every unemployed worker might as well apply. By offering the minimum wage, the deviant firm will fill its job with probability 1 while incurring an infinitesimal increase in the wage. When $\beta > 0$, minimum wage jobs will still attract more workers but to a lesser extent than occurs under $\beta = 0$. This is because, while $\hat{w} > \bar{w}$ workers experience some opportunity cost from applying to minimum wage jobs. They have to trade off the improved outcome when there are multiple applicants with the reduced probability of getting to negotiate their wage. While β is small, $\hat{w}$ is close to rV and the former effect dominates so that queue length increases rapidly with $\bar{w}$. The impact of adoption on the expected queue length continues to make adoption of low enough but binding minimum wages worthwhile to firms as long as $\beta < 1$. Ultimately when $\beta = 1$, the deviant prefers not to implement any binding minimum wage. Here, the increased probability of multiple applicants exactly offsets the increased cost of the wage bill.

2.4 Welfare

As both workers and firms are risk neutral, welfare in the model amounts to output minus costs. However, in order to focus on match formation, there are no separations in this model which means that total output is always growing. Welfare comparisons amount to comparing different economies and the welfare measure is the value to being born in any economy.

Regardless of the method of wage formation or the implied distribution, free-entry of vacancies means that any payments firms receive simply compensate them for vacancy creation costs. Welfare is therefore the utility contribution of a birth minus the associated vacancy cost.

If V_b is the present value of utility generated by a birth then

$$rV_b = b + \frac{(1-e^{-q})}{q}\left(\frac{p}{r} - V_b\right)$$

For each worker there are $v = 1/q$ vacancies which cost a/q to maintain over the expected duration of the worker's job search period. If C_b represents the present value of the costs then

$$rC_b = \frac{a}{q} - \frac{(1-e^{-q})}{q}C_b$$

the flow value of welfare is then $W \equiv rV_b - rC_b$. That is

$$W = \frac{(1-e^{-q})(p-b) - ra}{rq + 1 - e^{-q}} + b$$

It is simple to show that in the absence of a minimum wage, $W = rV$.[9] In the minimum wage with full enforcement, $W = r\bar{V}$.[10] Because of free entry, the firms wash out of the welfare calculation.

The first order condition from maximization of W with respect to q implies that any first-best queue length, q_p, solves

$$(p-b)\left[1 - e^{-q} - qe^{-q}\right] = a(r + e^{-q}) \tag{20}$$

As $W''(q_p)$ is negative, $W(.)$ is quasi-concave meaning that the unique solution to (20) is a global optimum. Under the maintained assumption that $p - b > ra$, q_p always exists.

Comparison of equations (20) and (5) shows that if $\beta = 1$, then $q_p = q^*$. This is what Julien *et al* [2006] refer to as the Mortensen Rule.[11] Of more interest to this paper is whether the minimum wage can achieve optimality for a given value of β. This amounts to asking whether minimum wage policy

[9] Set $V_f = 0$ in equation (2) and add it to equation (1)

[10] Set $V_f = 0$ in equation (8) and add it to equation (7)

[11] This is based on Mortensen [1982]. See also Kultti [1999] who shows how this efficient division of the surplus can arise when firms auction jobs among applicants.

can implement $\bar{q} = q_p$. In order to establish uniqueness of equilibrium with a fully enforced minimum wage, the proof of Claim 1 shows that $\bar{q}$ is continuous and strictly increasing in $\bar{w}$. When $\bar{w} = rV$ $\bar{q} = q^* < q_p$. As $\bar{w}$ approaches $p - ra$, $\bar{q}$ approaches infinity. So, there exists a unique value, $\bar{w}_p$, of $\bar{w}$ at which $\bar{q} = q_p$.

From the foregoing we know that if $\beta = 1$, the optimal minimum wage, $\bar{w}_p = rV$ and that lowering the worker's bargaining power in one-on-one matches away from 1 reduces $\bar{q}$. Then, $\bar{w}_p$ has to rise to keep the expected queue length equal to q_p. Eventually, for low enough values of β, $\bar{w}_p$ will be completely binding. Further lowering β will have no effect on $\bar{w}_p$.

While it has been established that some Pareto improving minimum wage would be adopted by firms, the question remains as to whether firms would voluntarily adopt the optimal minimum wage, $\bar{w}_p$. No general result along these lines has been obtained. When β is close to 1, however, we know that $\bar{w}_p$ is close to rV and continuity implies that firms will adopt that minimum wage when given the chance.

3 Match-specific heterogeneity

The model of the previous section provides a simple example of how firms can benefit from minimum wage adoption. The basic idea is that as long as firms are able to commit to the minimum-wage, the implied improvement in the application rate of workers makes adoption worthwhile. Another possible benefit from a higher application rate is a better match. This section investigates this possibility by incorporating match-specific heterogeneity.[12]

I assume that any encounter between a worker and a firm generates a draw of the match productivity, p, from a continuous distribution F with

[12] Moen [2003] provides a model of directed search with match-specific heterogeneity. In his model, however, firms always meet with a continuum of workers so that the size of the applicant pool does not affect the realized match productivity.

support between $\underline{p}$ and $\bar{p}$. If the variation across matches is attributed to subjective assessments by the firm as to how the worker would fit within the organization, then the realized value of p should be private information to the firm.[13]

To abstract from the role of wage changes on the matching rate I focus on the case where workers have all the bargaining power in one-on-one matches. That is, they get to make a take-it-or-leave-it wage offer to the firm. When the realized queue length at any vacancy exceeds one, I assume the firm gets to hire the most productive worker at the workers' common outside-option value.

Let $G(.|q)$ represent the distribution function of the highest productivity among the workers conditional on 2 or more of them showing up. It is helpful to derive G and some of its properties before continuing with the general analysis of this model. For a given realized queue length, n, the probability that every realized productivity is below p is $F^n(p)$. For given q, n has a Poisson distribution so that contingent on $n \geq 2$, the probability that every realized productivity is below p is

$$G(p|q) = \sum_{n=2}^{\infty} \frac{q^n e^{-q} F^n(p)}{n!(1 - e^{-q} - qe^{-q})}$$

Clearly, as $F^n(p) < F(p)$, $G(p|q) < F(p)$ for all q. From the Taylor series expansion of $e^{qF(p)}$ we have

$$G(p|q) = \frac{e^{qF(p)} - 1 - qF(p)}{e^q - 1 - q} \tag{21}$$

A second important property of $G(.|.)$ is that of first-order stochastic dominance with respect to q. Thus:

[13] The heterogeneity could also emerge from non-pecuniary aspects of the job over which workers have preferences. In that case, the natural assumption is that the worker has the private information. Such an arrangement, with random matching and wage posting, is considered in Masters [1998].

Lemma 2
$$\frac{\partial G(p|q)}{\partial q} < 0. \text{ } for\ p \in (0,1)$$

Proof. From (21) the sign of $\partial G(p|q)/\partial q$, after suppressing the argument in F is the same as the sign of

$$\Gamma(q,F) \equiv \frac{(e^{qF}-1)F}{e^{qF}-1-qF} - \frac{e^q-1}{e^q-1-q}$$

As $\Gamma(q,1)=0$, if for $F<1$, $\partial\Gamma(q,F)/\partial F > 0$,then $\Gamma(q,F)<0$. Now,

$$\frac{\partial\Gamma(q,F)}{\partial F} = \frac{(e^{qF}-1)^2 - q^2F^2e^{qF}}{(e^{qF}-1-qF)^2} = \frac{(e^{qF}-1+qFe^{\frac{qF}{2}})(e^{qF}-1-qFe^{\frac{qF}{2}})}{(e^{qF}-1-qF)^2}$$

the sign of which is the same as the sign of

$$\Phi(q,F) \equiv (e^{qF}-1-qFe^{\frac{qF}{2}})$$

Clearly, $\lim_{F\to 0}\Phi(q,F)=0$ for all q and for $F>0$,

$$\frac{\partial\Phi}{\partial F} = qe^{\frac{qF}{2}}\left(e^{\frac{qF}{2}} - 1 - \frac{qF}{2}\right) > 0$$

So, $\Phi(q,F)>0$ for $F>0$ which means for $F<1$, $\partial\Gamma(q,F)/\partial F>0$ and $\Gamma(q,F)<0$. ∎

With the essential properties of G established, I move to the analysis of the model. For a given expected queue length q, the value to holding a vacancy, V_f is now

$$\begin{aligned} rV_f &= -a + \frac{qe^{-q}}{r}\int_{\hat{w}-rV_f}^{\bar{p}}(y-\hat{w}-rV_f)dF(y) \\ &\quad + \frac{(1-e^{-q}-qe^{-q})}{r}\int_{r(V-V_f)}^{\bar{p}}(y-rV-rV_f)dG(y|q) \qquad (22) \end{aligned}$$

where $\hat{w}$ is the wage chosen by workers in the case of a one-on-one meeting. The first integral term represents the value to the firm from meeting a single worker, the second represents the value to having multiple applicants. The lower limits on each of the integrals represents the lowest match specific productivity at which the specified match type will form. Free-entry of vacancies implies $V_f = 0$ and equation (22) reduces to[14]

$$qe^{-q} \int_{\hat{w}}^{\bar{p}} [1 - F(y)]dy + (1 - e^{-q} - qe^{-q}) \int_{rV}^{\bar{p}} [1 - G(y|q)]\, dy = ra \qquad (23)$$

Consider the wage, $\hat{w}$, that a worker chooses in the case of a one-on-one meeting. The firm will reject the offer if the wage exceeds the worker's realized productivity so

$$\hat{w} = \arg\max_{w} [1 - F(w)]w \qquad (24)$$

As $F(.)$ has finite support, $\hat{w}$ has to exist.

When there are multiple applicants, workers are indifferent between remaining unemployed and getting a job. The value V to being unemployed is therefore given by

$$rV = b + \frac{e^{-q}}{r}[1 - F(\hat{w})] \max\{\hat{w} - rV, 0\} \qquad (25)$$

In this model a *free-entry steady-state equilibrium* is a tuple, $\{V, q, \hat{w}\}$ such that given $\hat{w}$ solves (24), q and V solve (25) and (23).

Necessary conditions for a non-trivial equilibrium are that $\hat{w} > b$ and $ra < \bar{p} - b$. The first is a restriction on the form of $F(.)$ and is a participation constraint on workers. If $\hat{w} < b$, $rV = b$ and workers will not seek employment. The second restriction occurs because firms need to be compensated for their advertising cost, a. Under these restrictions, existence of a non-trivial equilibrium follows from the intermediate value theorem: at $q = 0$, LHS of (23) is 0, as q approaches ∞, LHS of (23) approaches $\bar{p} - b$.

[14] The derivation also utilizes the fact that

$$\begin{aligned} \int_{x^*}^{\bar{x}} [x - x^*]\, dH(x) &= [(x - x^*)H(x)]_{x^*}^{\bar{x}} - \int_{x^*}^{\bar{x}} H(x)dx \\ &= \int_{x^*}^{\bar{x}} [1 - H(x)]dx \end{aligned}$$

where $H(.)$ is any distribution function with finite upper bound to its support, $\bar{x}$.

In the absence of further restrictions on F, multiple equilibria cannot be ruled out. Stochastic dominance of $G(.|q)$ with respect to q, that V is decreasing in q and that $\hat{w} > rV$ imply that for any given value of $\hat{w}$, LHS of (23) is strictly increasing in q. A sufficient condition for uniqueness of $\hat{w}$ and hence of equilibrium is that F has a non-decreasing hazard, $f(y)/(1-F(y))$.

In any equilibrium, let $\tilde{V}_f$ be the value to an individual firm of a one-time option to offer a minimum wage, $\bar{w} > rV$ and to which the firm can commit. As $V_f = 0$, from equation (22) we have

$$r\tilde{V}_f = -a + \frac{\tilde{q}e^{-\tilde{q}}}{r}\int_{\hat{w}}^{\bar{p}}[1-F(y)]dy + \frac{(1-e^{-\tilde{q}}-\tilde{q}e^{-\tilde{q}})}{r}\int_{\bar{w}}^{\bar{p}}(1-G(y|\tilde{q}))dy \quad (26)$$

where $\tilde{q}$, is the expected queue length associated with offering the minimum wage. Workers will adjust their search behavior so as to be indifferent between applying to the firm offering the minimum wage and all other firms so that $\tilde{q}$ is obtained from

$$rV = b + \frac{e^{-\tilde{q}}}{r}(1-F(\hat{w}))(\hat{w}-rV) + \frac{(1-e^{-\tilde{q}}-\tilde{q}e^{-\tilde{q}})}{r\tilde{q}}(1-G(\bar{w}|\tilde{q}))(\bar{w}-rV) \quad (27)$$

The continuation value of the worker and the firm are not affected by this option so, as long as $\hat{w} > \bar{w}$, neither is $\hat{w}$.

Following the analysis of the basic model, we can now ask whether firms would voluntarily adopt the minimum wage.

Claim 4

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} > 0$$

Proof: See Appendix

Thus, for low enough values of the minimum wage, universal non-adoption of the minimum wage cannot be an equilibrium.

The foregoing implies that some firms will adopt a binding minimum wage sufficiently close to rV. Following analysis similar to that of Section 2, it is straightforward to show that when the adoption choice is fully incorporated into this version of the model, similar results transpire. That is, the intersection of the set of parameter values at which all firms adopt the minimum wage with the set of parameter values at which no firms adopt the minimum wage is non-generic in the set of all permissible parameter values. In practical terms this, combined with Claim 4, means that holding all other parameters fixed, there is a critical value of the minimum wage below which all firms will adopt and above which no firms will adopt.

4 Conclusion

This paper provides an analysis of the effects of a wage floor in a model of directed search without wage commitment. This is used to develop a theory of minimum wage compliance (or equivalently, voluntary union recognition). In the baseline model, without match specific productivity, firms will adopt a minimum wage that binds them to pay more than the workers' continuation value. Doing so increases the number of applicants which means that the job is typically filled more quickly. Adding match specific productivity shows that increased numbers of applications also leads to better matches forming. This effect is shown (in isolation) to justify adoption of a binding minimum wage.

The analysis provided here predicts that depending on expected match productivity, either all firms adopt the minimum wage or none do. Clearly, introducing *ex ante* differences between firms in terms of their expected match productivities (as in Moen [1997]) would lead to a situation, as in the data, in which some firms adopt the wage floor and others do not.

5 Appendix

5.1 Proof of Claim 1

Step 1, Existence: Existence of equilibrium follows from the intermediate value theorem. To see why this is so, let

$$\chi(\bar{q}) \equiv \bar{q}e^{-\bar{q}}(p - \hat{w}) + \left[1 - (1 + \bar{q})e^{-\bar{q}}\right](p - \bar{w})$$

which is the RHS of equation (9).

When $\bar{q}$ approaches 0: both $\bar{q}e^{-\bar{q}}$ and $1 - (1 + \bar{q})e^{-\bar{q}}$ approach 0 as well. But, $\hat{w}$ is endogenous. Still, the largest $p - \hat{w}$ can be is p. So $\chi(0) = 0$.

When $\bar{q}$ approaches ∞: $\bar{q}e^{-\bar{q}}$ approaches 0, so $\bar{q}e^{-\bar{q}}(p - \hat{w})$ approaches 0 too. The term $1 - (1 + \bar{q})e^{-\bar{q}}$ approaches 1. So $\lim_{\bar{q}\to\infty} \chi(\bar{q}) = p - \bar{w}$.

The intermediate value theorem also requires that $\chi(.)$ be continuous. Continuity clearly follows as long as $\hat{w}$ is continuous in $\bar{q}$. As the max of continuous functions is continuous,[15] this boils down to showing that $\bar{V}$ is

[15] Let f and g be real valued continuous functions on the real line. Define

$$h(x) \equiv \max\{f(x), g(x)\}$$

Without loss of generality consider a discontinuity in h at $\hat{x}$ such that

$$\lim_{x^- \to \hat{x}} h(x^-) = \lim_{x^- \to \hat{x}} g(x^-) = g(\hat{x})$$

and

$$\lim_{x^+ \to \hat{x}} h(x^+) = \lim_{x^+ \to \hat{x}} f(x^+) = f(\hat{x})$$

where $x^- < \hat{x}$ and $x^+ > \hat{x}$. For h to have a discontinuity requires that $f(\hat{x}) \neq g(\hat{x})$. Suppose $f(\hat{x}) > g(\hat{x})$. So there exists an $\epsilon > 0$ such that $f(\hat{x}) - g(\hat{x}) > \epsilon$. As f and g are continuous

$$\lim_{x^- \to \hat{x}} \left\{f(x^-) - g(x^-)\right\} = f(\hat{x}) - f(\hat{g})$$

so there exists some $\hat{x}^-$ such that for any $x^- > \hat{x}^-$, $f(x^-) - g(x^-) > \epsilon$. But this contradicts the premise that $h(x^-) = g(x^-)$ for all $x^- < \hat{x}$.

continuous in $\bar{q}$. Solving for $r\bar{V}$ yields

$$r\bar{V} = \frac{\bar{q}(\beta e^{-\bar{q}}p + rb) + [1-(1+\bar{q})e^{-\bar{q}}]\,\bar{w}}{\bar{q}(\beta e^{-\bar{q}} + r) + 1 - (1+\bar{q})e^{-\bar{q}}} \tag{28}$$

which is a weighted average of $\bar{w}$ and $(\beta e^{-\bar{q}}p + rb)$ which are continuous in $\bar{q}$ as are the weights, $\bar{q}$ and $1-(1+\bar{q})e^{-\bar{q}}$. The denominator is strictly positive and finite for all $\bar{q} > 0$. L'Hospital's rule reveals that for $\bar{q} = 0$, $r\bar{V} = \frac{\beta p + rb}{\beta + r}$.

So $\chi(\bar{q})$ is continuous, is zero at $\bar{q} = 0$ and approaches $p - \bar{w} > ra$ as $\bar{q} \to \infty$.

Step 2, Uniqueness: We need to demonstrate the monotonicity of $\chi(.)$. Differentiation yields

$$\begin{aligned}
\chi'(\bar{q}) &= (1-\bar{q})e^{-\bar{q}}\,(p-\hat{w}) - \bar{q}e^{-\bar{q}}\frac{d\hat{w}}{d\bar{q}} + \bar{q}e^{-\bar{q}}\,(p-\bar{w}) \\
&= e^{-\bar{q}}\left[(p-\hat{w}) + \bar{q}(\hat{w}-\bar{w}) - \bar{q}\frac{d\hat{w}}{d\bar{q}}\right].
\end{aligned}$$

when $\hat{w} = \bar{w}$, $\chi'(\bar{q}) = e^{-\bar{q}}\,(p-\bar{w}) > 0$. The other possibility is that $\hat{w} = \beta p + (1-\beta)r\bar{V}$. In this case

$$\frac{d\hat{w}}{d\bar{q}} = (1-\beta)r\frac{d\bar{V}}{d\bar{q}}$$

From (28) the numerator of $r\frac{d\bar{V}}{d\bar{q}}$ is

$$\Psi(\bar{q},\bar{w}) \equiv -(p-\bar{w})\beta e^{-\bar{q}}(e^{-\bar{q}}+\bar{q}-1) - r(\bar{w}-b)[1-(1+\bar{q})e^{-\bar{q}}] - re^{-\bar{q}}[\beta p + (1-\beta)b - \bar{w}].$$

As $e^{-\bar{q}} + \bar{q} \geq 1$ for all $\bar{q}$, this means that whenever $\bar{w} \leq \beta p + (1-\beta)b$, $\Psi(\bar{q},\bar{w}) < 0$. But, $r\bar{V} \geq b$ and values of $\bar{w}$ up to $\beta p + (1-\beta)r\bar{V}$ are permissible. Using (13) we obtain

$$\Psi(\bar{q},\bar{w}_T) = \frac{-r\beta(p-b)\,[1-(1+\bar{q})e^{-\bar{q}}]}{[r\bar{q} + \beta(1-e^{-\bar{q}})]\,[\bar{q}(\beta e^{-\bar{q}} + r) + 1 - (1+\bar{q})e^{-\bar{q}}]} < 0$$

Also, it should be clear that for any $\bar{q}$, $\Psi(\bar{q},\bar{w})$ is monotone in $\bar{w}$. This means that for values of $\bar{w} \leq \beta p + (1-\beta)b$, $\Psi < 0$ and for $\bar{w} = \bar{w}_T$, $\Psi < 0$.

So Ψ is negative through out the range $[\beta p + (1-\beta)b, \bar{w}_T]$. When $\bar{w}_T$ does not exist the maximal value of $\bar{w}$ is $p - ra$. The last term in $\Psi(\bar{q}, p - ra)$ is $-re^{-\bar{q}}[ra - (1-\beta)(p-b)] < 0$ from Lemma 1. Then again monotonicity of Ψ with respect to $\bar{w}$ means that it is negative on $[\beta p + (1-\beta)b, p - ra]$.

As $\Psi(\bar{q}, \bar{w}) < 0$, $\frac{d\hat{w}}{d\bar{q}} < 0$ and $\chi'(\bar{q}) > 0$ whenever $\hat{w} = \beta p + (1-\beta)r\bar{V}$.

When $\bar{w} = \bar{w}_T$, $\chi(\bar{q})$ will not be differentiable but it is continuous and both the left and right derivatives are strictly positive so that it is monotonically increasing over $\mathbb{R}_+$.

5.2 Proof of Claim 2

We know from the preceding subsections that there are unique non-trivial equilibria without minimum wages and when the minimum wage is fully enforced as long as $p - ra > b$ and $p - ra > \bar{w}$ respectively. In this subsection we have the additional constraints that $\tilde{V} \leq 0$ and $\tilde{\tilde{V}}_f \leq 0$.

The boundary to the set of parameter values for which $\phi^* = 0$ is an equilibrium is defined by $\tilde{V}_f = V_f = 0$. After substituting these values into the appropriate equations above (respectively (1), (2), (3), (14), (15) and (16)) we get

$$\begin{aligned}
rV &= b + e^{-q}\left(\frac{\hat{w}}{r} - V\right) \\
a &= qe^{-q}\left(\frac{p - \hat{w}}{r}\right) + \left[1 - e^{-q} - qe^{-q}\right]\left(\frac{p - rV}{r}\right) \\
\hat{w} &= \beta p + (1-\beta)rV \\
a &= \tilde{q}e^{-\tilde{q}}\left(\frac{p - \tilde{w}}{r}\right) + \left(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}\right)\left(\frac{p - \bar{w}}{r}\right) \\
\tilde{w} &= \max\left\{\beta p + (1-\beta)rV, \bar{w}\right\} \\
rV &= b + e^{-\tilde{q}}\left(\frac{\tilde{w} - rV}{r}\right) + \left(\frac{1 - (1+\tilde{q})e^{-\tilde{q}}}{\tilde{q}}\right)\left(\frac{\bar{w} - rV}{r}\right)
\end{aligned} \tag{29}$$

The boundary to the set of parameter values for which $\phi^* = 1$ is an equilibrium is defined by $\tilde{\tilde{V}}_f = \bar{V}_f = 0$. After substituting these values into the appropriate equations above (respectively (6), (7), (8), (17), (18) and (19)), we get

$$\begin{aligned}
\hat{w} &= \max\left\{\beta p + (1-\beta) r\bar{V}, \bar{w}\right\} \\
r\bar{V} &= b + e^{-\bar{q}}\left(\frac{\hat{w} - r\bar{V}}{r}\right) + \left(\frac{1-(1+\bar{q})e^{-\bar{q}}}{\bar{q}}\right)\left(\frac{\bar{w} - r\bar{V}}{r}\right) \\
a &= \bar{q}e^{-\bar{q}}\left(\frac{p - \hat{w}}{r}\right) + \left[1-(1+\bar{q})e^{-\bar{q}}\right]\left(\frac{p - \bar{w}}{r}\right) \\
a &= \tilde{\bar{q}}e^{-\tilde{\bar{q}}}\left(\frac{p - \tilde{\bar{w}}}{r}\right) + \left[1 - e^{-\tilde{\bar{q}}} - qe^{-\tilde{\bar{q}}}\right]\left(\frac{p - r\bar{V}}{r}\right) \\
\tilde{\bar{w}} &= \beta p + (1-\beta) r\bar{V} \\
r\bar{V} &= b + e^{-\tilde{\bar{q}}}\left(\frac{\tilde{\bar{w}} - r\bar{V}}{r}\right)
\end{aligned} \tag{30}$$

System (29) is identical (up to a relabelling) to system (30). On the common boundary these equilibria necessarily coexist. Smoothness of the functional forms ensures that the boundary is non-generic (zero measure) in the permissible parameter space. So either they coexist everywhere or where one equilibrium exists the other does not. To show that it is latter, consider a minimum wage very close to the maximum permissible value, $p - ra$. At very high values of the minimum wage, only the $\phi^* = 0$ equilibrium exists.

Existence of $\phi^* = 0$**.** We set $V_f = 0$ and show that as $\bar{w} \to p - ra$, $\tilde{V}_f < 0$ (i.e. there is no incentive to deviate from equilibrium behavior). From (14),

$$r\tilde{V}_f\left(r + 1 - e^{-\tilde{q}}\right) = \tilde{q}e^{-\tilde{q}}\left(p - \tilde{w}\right) + \left(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}\right)\left(p - \bar{w}\right) - ra$$

as $\tilde{w} \geq \bar{w}$,

$$r\tilde{V}_f\left(r + 1 - e^{-\tilde{q}}\right) \leq \left(1 - e^{-\tilde{q}}\right)\left(p - \bar{w}\right) - ra.$$

As $\bar{w}$ approaches $p - ra$, $\tilde{V}_f$ can only remain non-negative if $\tilde{q} \to \infty$. But, from (16), the set of applicants per minimum wage job can only get very large if rV approaches b. From (4), however, $rV > b$ which means $\tilde{q}$ has to remain finite and $\tilde{V}_f < 0$.

Non-existence of $\phi^* = 1$. We set $\bar{V}_f = 0$ and show that $\bar{w} \to p - ra$ implies $\tilde{\tilde{V}}_f > 0$ (i.e. every firm would prefer to deviate from equilibrium behavior). From (8),

$$ra = \bar{q}e^{-\bar{q}}(p - \hat{w}) + \left[1 - (1+\bar{q})e^{-\bar{q}}\right](p - \bar{w}) \leq \left[1 - e^{-\bar{q}}\right](p - \bar{w})$$

As $\bar{w} \to p - ra$, for this inequality to hold, we need $\bar{q} \to \infty$. Which means that from (7), $r\bar{V} \to b$. As $\tilde{\tilde{w}} > r\bar{V}$, from (19) this can only happen if $\tilde{\tilde{q}} \to \infty$ in which case, (17) implies that in the limit,

$$r\tilde{\tilde{V}}_f(1 + r) = -ra + p - r\bar{V} \to p - b - ra > 0.$$

As we have identified a location, $\bar{w}$ close to $p - ra$, where these equilibria do not coexist, the shared boundary to existence can only mean that that when one equilibrium exists the other does not. Moreover, the the signs of $\tilde{V}$ and $\tilde{\tilde{V}}_f$ are well defined over the whole parameter space. Either one is negative or the other is negative or they are both zero.

5.3 Proof of Claim 3

From Claim 2, we simply have to show that for low enough $\bar{w}$, $\phi^* = 0$ is not an equilibrium when $\beta < 1$. This requires that individual firms would find it profitable to adopt the minimum wage when all other firms do not. As firms would clearly be indifferent between adoption and non adoption of the minimum wage that just binds (i.e. $\bar{w} = rV$), this amounts to showing that

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} > 0.$$

Restricting attention to partially binding minimum wages, substituting for $\tilde{w}$ from (15) into (14) and (16) yields the following pair of equations in $\tilde{V}_f$ and $\tilde{q}$.

$$G^1(\tilde{V}_f, \tilde{q}; \bar{w}) \equiv r^2\tilde{V}_f + ra - \tilde{q}e^{-\tilde{q}}(1 - \beta)\left(p - rV - r\tilde{V}_f\right) - (1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})\left(p - \bar{w} - r\tilde{V}_f\right) = 0$$

$$G^2(\tilde{V}_f, \tilde{q}; \bar{w}) \equiv r^2\tilde{q}V - r\tilde{q}b - \tilde{q}e^{-\tilde{q}}\beta\left(p - r\tilde{V}_f - rV\right) \\ - \left(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}\right)(\bar{w} - rV) = 0$$

Using implicit differentiation and Cramer's rule,

$$\frac{d\tilde{V}_f}{d\bar{w}} = \frac{-\begin{vmatrix} G_3^1 & G_2^1 \\ G_3^2 & G_2^2 \end{vmatrix}}{\begin{vmatrix} G_1^1 & G_2^1 \\ G_1^2 & G_2^2 \end{vmatrix}}$$

where G_j^i represents the partial derivative of the ith component of G with respect to the jth argument. Obtaining each of the partial derivatives is straightforward. Once they have been obtained, we can impose $\bar{w} = rV$, which also means $\tilde{V}_f = V_f = 0$, and $\tilde{q} = q^*$. After substituting for rV from (4),

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{(1-\beta)(1 - e^{-q^*} - q^*e^{-q^*})}{r\beta q^*(r + 1 - \beta e^{-q^*})}$$

which is strictly positive while $\beta < 1$ and q^* is finite.

As the preceding analysis was carried out for partially binding minimum wages, it is only valid for $\beta > 0$.When $\beta = 0$, $\hat{w} = rV = b$ and a minimum wage that binds at all binds completely. In that case substituting for $\tilde{w} = \bar{w}$ into (14) and (16) yields

$$\begin{aligned} r\tilde{V}_f &= -a + \frac{(1 - e^{-\tilde{q}})}{r}\left(p - \bar{w} - r\tilde{V}_f\right) \\ rV &= b + \left(\frac{1 - e^{-\tilde{q}}}{\tilde{q}}\right)\left(\frac{\bar{w} - rV}{r}\right) \end{aligned}$$

which imply

$$r\left(r + 1 - e^{-\tilde{q}}\right)\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} = e^{-\tilde{q}}(p - \bar{w})\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} - \left(1 - e^{-\tilde{q}}\right)$$

where

$$\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{1 - e^{-\tilde{q}}}{rV - b}$$

As $rV = b$, deviation to a minimum wage that just binds generates an unbounded queue length of applicants and any firm would choose to adopt the minimum wage.

5.4 Proof of Claim 4

First notice that

$$\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} = \left.\left(\frac{\partial \tilde{V}_f}{\partial \bar{w}} + \frac{\partial \tilde{V}_f}{\partial \tilde{q}}\frac{d\tilde{q}}{d\bar{w}}\right)\right|_{\bar{w}=rV} \tag{31}$$

From (26)

$$r^2 \left.\frac{\partial \tilde{V}_f}{\partial \bar{w}}\right|_{\bar{w}=rV} = -(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(1 - G(\bar{w}|\tilde{q}))$$

and

$$\begin{aligned} r^2 \left.\frac{\partial \tilde{V}_f}{\partial \tilde{q}}\right|_{\bar{w}=rV} &= (1-\tilde{q})e^{-\tilde{q}} \int_{\hat{w}}^{\bar{p}} (y - \hat{w})dF(y) + \tilde{q}e^{-\tilde{q}} \int_{\bar{w}}^{\bar{p}} (y - \bar{w})dG(y|\tilde{q}) \\ &\quad -(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}) \int_{\bar{w}}^{\bar{p}} \frac{\partial G(y|\tilde{q})}{\partial \tilde{q}} dy \end{aligned}$$

From (27),

$$\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(1 - G(\bar{w}|\tilde{q}))}{r^2V - rb - (1-\tilde{q})e^{-\tilde{q}}(1 - F(\hat{w}))(\hat{w} - rV)}$$

and using (25) to substitute for $r^2V - rb$,

$$\left.\frac{d\tilde{q}}{d\bar{w}}\right|_{\bar{w}=rV} = \frac{(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(1 - G(\bar{w}|\tilde{q}))}{\tilde{q}e^{-\tilde{q}}(1 - F(\hat{w}))(\hat{w} - rV)}$$

which is positive. Substituting back into (31) yields

$$\begin{aligned} \left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV} &= \left[\frac{(1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}})(1 - G(\bar{w}|\tilde{q}))}{\tilde{q}e^{-\tilde{q}}(1 - F(\hat{w}))(\hat{w} - rV)}\right] \times \\ &\quad \left\{ \begin{array}{l} -\tilde{q}e^{-\tilde{q}}(1 - F(\hat{w}))(\hat{w} - rV) + (1-\tilde{q})e^{-\tilde{q}} \int_{\hat{w}}^{\bar{p}} (y - \hat{w})dF(y) \\ +\tilde{q}e^{-\tilde{q}} \int_{\bar{w}}^{\bar{p}} (y - \bar{w})dG(y|\tilde{q}) - (1 - e^{-\tilde{q}} - \tilde{q}e^{-\tilde{q}}) \int_{\bar{w}}^{\bar{p}} \frac{\partial G(y|\tilde{q})}{\partial \tilde{q}} dy \end{array} \right\} \end{aligned}$$

The sign of $\left.\frac{d\tilde{V}_f}{d\bar{w}}\right|_{\bar{w}=rV}$ clearly depends on the sign of the contents of the curly brackets. The last term is positive from because $\frac{\partial G(y|\tilde{q})}{\partial \tilde{q}} < 0$ as established above. The first 3 terms can be written as

$$\tilde{q}e^{-\tilde{q}}\left[\int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q}) - (1-F(\hat{w}))(\hat{w}-\bar{w}) - \int_{\hat{w}}^{\bar{p}}(y-\hat{w})dF(y)\right]$$
$$+e^{-\tilde{q}}\int_{\hat{w}}^{\bar{p}}(y-\hat{w})dF(y)$$

in which the contents of the square brackets can be written as

$$\begin{aligned} & \int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q}) - \int_{\hat{w}}^{\bar{p}}(y-\bar{w})dF(y) \\ > \; & \int_{\bar{w}}^{\bar{p}}(y-\bar{w})dG(y|\tilde{q}) - \int_{\bar{w}}^{\bar{p}}(y-\bar{w})dF(y) \\ = \; & \int_{\bar{w}}^{\bar{p}}(F(y) - G(y|\tilde{q}))dy > 0. \end{aligned}$$

the last line comes from integration by parts.

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