Labor Market Policy in the Presence of a Participation Externality

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Abstract

A participation externality occurs when vacancy creation depends on workforce composition. As marginal workers enter the labor market, they lower the average quality of the workforce. This suppresses vacancy creation but is not internalized by the new entrants. This paper studies how this externality interacts with search externalities and the efficacy of policies at addressing it. These externalities interact because either party may retain an inefficient share of the surplus and workforce composition affects the expected surplus. We show that when chosen optimally, minimum wages and unemployment insurance partially address both externalities, but minimum wages primarily affect participation, while unemployment insurance primarily affects search externalities.

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1 Introduction

A participation externality arises when firms create vacancies based on the composition of the workforce but potential entrants (job seekers who differ in their abilities) do not internalize the impact of their participation on that

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composition. This paper describes this externality, investigates its interactions with previously understood job search externalities, and assesses the efficacy of labor market policies.

In a random job search environment, Pissarides (2000) describes two congestion externalities that determine the extent to which the participants (here workers and firms) are under- or over-rewarded for their contribution to aggregate matching. Without the participation externality assessed here, Hosios (1990) demonstrates that trade is (constrained) efficient when the bargaining power of the firms equals the elasticity of the matching function with respect to vacancies. With the participation externality, this "Hosios Rule" causes the participation and search externalities to be orthogonal. In this situation, Masters (2015) shows that a participation threshold set by a social planner restores constrained efficiency. However, away from the Hosios Rule condition, characterizing the interaction between these externalities becomes analytically challenging. Yet evidence suggests that the Hosios Rule does not hold, which means that this is an empirically important task.

To meet this challenge we therefore present two versions of the model: a static model to provide insight and a dynamic model for quantitative analysis. In both versions, the frictional environment mirrors a key feature faced by low-wage workers. These workers differ by ability, while jobs are ex-ante homogeneous. As a result, workers determine their participation based on their ability which may not meet the minimum threshold as determined by labor market policies or the trading environment.[1]

Analysis of our static environment reveals clear relationships between the participation externality, the two congestion externalities from search, and labor market policies. Under the Hosios Rule, it is straightforward to demonstrate the orthogonality of the participation and search externalities. Away from the Hosios Rule, the externalities tend to counteract each other. If firm bargaining power is too low, the search externalities amplify the impact of the participation externality (and vice-versa), but the impact of the participation externality itself shrinks. The sign of the impact on welfare from changes to the participation threshold depends only on the elasticity of the matching function and not on the bargaining power of the firm per se. This quasi-neutrality result emerges because lower firm bargaining power reduces their ability to recoup their sunk cost of job posting but also raises their equilibrium matching probability.

Potential policy interventions in the one-shot model support raising the

[1]See Syverson (2011) and Song et al. (2019) among others for evidence that better workers have higher wages in similar jobs.

participation threshold. However, it is typically assumed that the government cannot identify individual ability levels. We therefore consider two ubiquitous policies that are not contingent on ability: unemployment insurance (UI) and the minimum wage. Both policies affect the participation threshold and can, therefore, be useful in addressing the participation externality. UI essentially pays people to stay out of the market while a binding minimum wage causes low-ability workers to drop out of the market.

In discouraging entry, both create additional distortions. The minimum wage raises wages for the lowest ability groups that remain in the market which tends to suppress vacancy creation and therefore reduce the welfare gains accrued from excluding the lowest ability workers. Still, where it just binds, welfare is increasing in the minimum wage regardless of worker bargaining power. A UI scheme set to achieve the same participation threshold raises wages by even more than does the minimum wage. As such, it is better at addressing the excess vacancy creation associated with low worker bargaining power. But, if firms have low bargaining power, the minimum wage does the least additional harm as measured by welfare. Indeed, for low enough firm bargaining power, UI may not be welfare enhancing at any level.

To quantitatively evaluate these relationships, the dynamic model endogenizes the workers' continuation values thereby increasing the responsiveness of the match surplus to changes in bargaining power. This heightened responsiveness strengthens the search externalities in relation to the participation externality.

Four key questions emerge in the dynamic model:

- What is the constrained efficient allocation?
- What are the optimal minimum wage and UI policies?
- How do these policies affect participation and the search externalities?
- How do the externalities interact?

Our calibration indicates that worker bargaining power is low relative to the Hosios Rule – the orthogonality result does not apply and the participation and search externalities interact. As expected, the Planner's ability cut-off (at \$7.04) is higher than the cut-off in the laissez-faire economy (estimated at \$6.04) and achieves a welfare gain of 1.54pp. The minimum wage and UI both achieve higher levels of welfare than the laissez- faire economy, but do not achieve the same level as the planner. Although the minimum wage can exclude low ability workers and raise wages for some workers who receive less than the “Hosios wage” it overly inflates wages for many of the marginal entrants. In our quantitative analysis, the optimal minimum wage ($7.15) raises welfare by 0.32pp above the laissez-faire economy. By contrast, UI does not distort wages for marginal entrants, but raises wages for all workers. Our analysis indicates that the optimal UI (an additional $1.53 to a participation threshold of $7.57) raises welfare by 0.86pp.

The remainder of the paper is laid out as follows. Section 2 places this paper within the recent literature on externalities and labor market policy in the Diamond-Mortensen-Pissarides (DMP) framework. Section 3 describes the static model. Section 4 develops the dynamic model. Section 5 discusses the data and the calibration of the dynamic model. The results and decomposition appear in Section 6. Section 7 concludes.

2 Literature

The participation externality studied here emerges in four other papers: Gavrel (2011), Masters (2015), Julien and Mangin (2017) and Mangin and Julien (2021). Both Julien and Mangin (2017) and Mangin and Julien (2021) generalize the Hosios Rule to an environment with a participation decision and an opportunity cost of search. They show that constrained efficiency equates the bargaining power of the firm to the sum of the matching elasticity, a surplus elasticity and a participation elasticity. To focus on the participation externality, Masters (2015) eliminates the opportunity cost of search and shows that implementing the standard Hosios Rule combined with a participation threshold that excludes the same low ability workers as the Social Planner's policy can achieve constrained efficiency. Gavrel (2011) provides a static model and assumes that a fixed division of output rather than the more usual Nash bargaining determines the terms of trade. Consequently, a transfer scheme, akin to unemployment insurance (UI) studied here, raises the ability threshold for participation without affecting wages. It, therefore, implements constrained efficiency under the Hosios Rule. Beyond this, none of these papers look into how labor market policy addresses the participation externality nor do they consider how the participation and search externalities interact.

Hungerbühler and Lehmann (2009) show that the minimum wage can emerge as part of an optimal tax and transfer scheme in a DMP environment when worker bargaining power is low. Their model has ex ante heterogeneous workers and a participation decision. By allowing for complete market segmentation by ability level, however, it abstracts from the participation externality explored here.

Several papers consider the minimum wage in a frictional model. Braun (2019) investigates the relationship between the minimum wage and people's propensity to commit property crime. She provides a model in which workers are ex ante heterogeneous in ability. Her model contains no vacancy creation and as a consequence cannot contain a participation externality. Instead, an externality emerges from the wage setting protocol. Firms do not fully internalize their choice of wage offer on the worker's propensity to commit crime. Wages can be too low in equilibrium, so a minimum wage can raise wages to reduce crime rates. Flinn (2006) considers an environment with vacancy creation, but no participation margin. Lavecchia (2020) and Lee and Saez (2012) both consider a participation decision in models where workers are heterogeneous with respect to skills. The former does not explore the effect of a participation externality, though one could arise in his environment. The latter considers a perfectly competitive environment, and hence does not feature the externalities in our model or their interactions.

The large literature on optimal UI is generally focused on the trade-off between providing consumption smoothing to risk averse workers and moral hazard emerging from the government's inability to observe job search effort. Papers typically consider this trade-off either in a partial equilibrium principle-agent framework (Shavell and Weiss (1979), Hopenhayn and Nicolini (1997)) or in a decentralized market context (Fredriksson and Holmlund (2001), Coles and Masters (2006), Acemoglu and Shimer (1999)). None of these look into the role of UI in addressing the participation externality considered in this paper.

3 Static Model

Throughout this section longer derivations have been moved to Appendix A.

3.1 Environment

The economy exists for one period. A unit mass continuum of workers indexed by their ability level, $p \sim F(.)$ with continuous density, $f(.)$ on $[0, \bar{p}]$ populates the economy. A large mass of firms can each create one vacancy at a cost a. In equilibrium, a free-entry condition determines the level of vacancy creation. Workers and firms are both risk neutral. Whether they look for work or not, the workers who do not get a job receive value from leisure, z. To ensure that gains from trade exist we assume that $\bar{p} > z + a$. A firm matched to a worker of ability p produces p units of output.

All workers start out jobless. Those who wish to seek employment, participate in the labor market. Search is random. Participants meet a vacancy with probability $m(\theta)$ where, θ represents "labor market tightness" defined as the ratio of the mass of vacancies, v, to the mass of participating workers, u. The meeting function, $m(.)$, is strictly increasing and strictly concave with $m(0) = 0$, $m'(0) = 1$ and $\lim_{\theta\to\infty} m(\theta) = 1$. Then, a vacancy meets a job-seeker with probability $m(\theta)/\theta$ which is strictly decreasing in θ. This means that $\eta(\theta)$, the elasticity of $m(\theta)$, is less than one.

3.2 Efficiency

To make the Planner's problem comparable with that of the Policy Maker, the same search frictions present in the decentralized economy also constrain the Planner.[2] The Planner controls the mass of vacancies, v, and a minimum productivity level, $\hat{p}$, required for participation in the matching game. The Planner chooses $\hat{p}$ to ensure that all meetings lead to a match. Thus, the mass of participants, $u = 1-F(\hat{p})$ and the market tightness, $\theta = v/[1-F(\hat{p})]$. The trade-off the Planner faces is that raising $\hat{p}$ improves match quality but reduces their quantity.

The social welfare function is

$$W(\theta, \hat{p}) = zF(\hat{p}) + \int_{\hat{p}}^{\bar{p}} [m(\theta)p + (1 - m(\theta))z]\, dF(p) - [1 - F(\hat{p})]a\theta. \quad (3.1)$$

The Planner's problem is to maximize W over θ and $\hat{p}$. The necessary conditions for an optimum imply

$$m'(\theta_p)\mathbb{E}_{[p\geq\hat{p}_p]}(p - z) = a \quad (3.2)$$

and

$$\frac{m(\theta_p)}{\theta_p}(\hat{p}_p - z) = a \quad (3.3)$$

where the subscript, p, refers to the Planner's solution. That such an interior solution exists follows from the assumption that $\bar{p} > z + a$. Equation 3.2 equates the marginal benefit from creating an additional vacancy to its marginal cost. Equation 3.3 equates the marginal cost in terms of output of raising $\hat{p}$ to the marginal benefit in terms of the saved vacancy costs.

[2] In Section C.1 we solve the model in which the Planner is able to direct workers towards matching locations based on their type. We also consider how a decentralized economy can support that allocation.

3.3 Market Economy

In the market economy, generalized Nash bargaining, in which the bargaining power of the worker is $\beta \in [0, 1]$, determines wages. As is standard in the literature using the DMP framework, the outside option of each participant is their threat point. For the workers that is z and for the firms it is 0. We assume that workers who are indifferent between market entry and sitting it out, choose the latter.

In the laissez-faire economy, firms create vacancies up to the point where they expect to break-even. If a meeting does not become a match, the worker receives z, and the firm must still pay its vacancy creation cost. Consequently, the surplus from a match with a type p worker is $p - z$. This implies that workers of type $p \leq z$ do not generate gains from trade with any employer. As a result, these workers do not participate in the labor market. Anyone else gets $z + \beta(p - z)$ if they match and z if they do not so participation is a strictly dominant strategy. Letting $(\theta^*, \hat{p}^*)$ represent the market economy equilibrium outcome, we have

$$\frac{m(\theta^*)}{\theta^*}(1 - \beta)\mathbb{E}_{[p \geq \hat{p}^*]}(p - z) = a \tag{3.4}$$

and

$$\hat{p}^* = z. \tag{3.5}$$

Comparing Equation 3.2 and Equation 3.4 for a given threshold of ability for participation, $\hat{p}$, the Hosios Rule (i.e. $\beta = 1 - \eta(\theta^*)$) implies efficient vacancy creation. But, this does not resolve the participation externality. Comparing Equation 3.3 and Equation 3.5 shows that the participation threshold in the laissez-faire economy is too low. As a result, even at the Hosios Rule, Equation 3.2 implies that the market tightness is also too low. In the market economy, firms cannot pre-commit to reject low ability workers with $p > z$. The prospect of hiring those workers lowers expected profits and reduces vacancy creation below the optimal level.

3.4 Participation externality

The Hosios Rule cannot induce full constrained efficiency because of a participation externality. When workers choose to participate, they do not take into account the impact of their choices on the average quality of the unemployment pool. But, vacancy creation depends precisely on that average quality – the externality directly impacts firms and thereby indirectly impacts the other workers.

How this externality interacts with the search externalities, which have been more extensively studied (Pissarides, 2000), is of interest here. Search externalities arise on both sides of the market whenever an individual's private return from searching does not accurately compensate the contribution to welfare. Absent a participation externality (e.g. when $F(.)$ is degenerate), the Hosios surplus sharing rule equates social contributions and private returns on both sides of the market.[3] Alternatively, in a directed search equilibrium (e.g. Moen (1997)) participants take the terms of trade as given and the search externalities become pecuniary.

The focus on the participation externality also guides the choice of model structure. Of particular importance is that the model lacks an opportunity cost of search and as a result, the laissez-faire ability cut-off, $\hat{p}^*$, does not depend on the market tightness. By contrast, Julien and Mangin (2017) introduces an explicit search cost which leads to an "output" externality that has its own interactions with the participation and search externalities. Assuming that unsuccessful job seekers and those that sit out of the market both receive z deliberately abstracts from the output externality. Doing so means that at the Hosios Rule the participation and search externalities are orthogonal.

Artificially raising the ability cut-off, $\hat{p}$, above z further elucidates the impact of the participation externality on welfare in the market equilibrium. Using the same welfare measure as in the Planner's problem (Equation 3.1), the following gives the equilibrium market tightness, θ^*, given $\hat{p}$:

$$\frac{m(\theta^*)}{\theta^*}(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) = a. \tag{3.6}$$

Then,

$$\frac{dW}{d\hat{p}} = \frac{\partial W}{\partial \hat{p}} + \frac{\partial W}{\partial \theta}\frac{d\theta^*}{d\hat{p}}. \tag{3.7}$$

The first term, $\frac{\partial W}{\partial \hat{p}}$, captures the participation externality in isolation while the second term captures its interaction with the search externalities. From Equation 3.1,

$$\frac{\partial W}{\partial \hat{p}} = f(\hat{p})\left[a\theta^* - m(\theta^*)(\hat{p}-z)\right]. \tag{3.8}$$

[3] This happens because firms create vacancies whenever the average expected return exceeds the cost, a. The Planner, meanwhile, creates vacancies based on the marginal expected social return. The Hosios Rule equates those expected returns.

Rewriting this expression using Equation 3.6 yields

$$\frac{\partial W}{\partial \hat{p}} = f(\hat{p})m(\theta^*)\left[(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) - (\hat{p}-z)\right]. \tag{3.9}$$

When $\hat{p} = z$ this is clearly positive for all values of β, but as $\hat{p}$ rises towards $\bar{p}$ it becomes negative. An alternative view is that a hold-up problem generates the participation externality. Firms cannot pre-contract with their future employees, so they cannot recoup any incurred vacancy costs. But, the Planner fully internalizes those costs. Thus, raising $\hat{p}$ above z without affecting vacancy creation precludes firms from meeting with those workers whose ability does not justify the vacancy creation cost. Of course, raising $\hat{p}$ too high can begin to preclude them from meeting with higher ability workers that the Planner would have the firms hire.

The term that captures the interaction of the participation and search externalities has two components. The first is the direct effect of changes in the market tightness on welfare,

$$\begin{aligned}\frac{\partial W}{\partial \theta} &= m'(\theta^*)\int_{\hat{p}}^{\bar{p}}(p-z)dF(p) - [1-F(\hat{p})]\,a \\ &= \frac{m'(\theta^*)\,[1-F(\hat{p})]\,a\theta^*}{m(\theta^*)(1-\beta)} - [1-F(\hat{p})]\,a \\ &= \frac{[1-F(\hat{p})]\,a}{(1-\beta)}\,[\beta-(1-\eta(\theta^*))]\,. \end{aligned} \tag{3.10}$$

This term is clearly positive for values of β above the Hosios Rule and negative below it. This evidences the orthogonality of the participation and search externalities at the Hosios Rule. The second component in the interaction term, derived in Section A.1, is,

$$\frac{d\theta^*}{d\hat{p}} = \frac{(1-\beta)m(\theta^*)f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{[1-F(\hat{p})]\,a(1-\eta(\theta^*))} > 0. \tag{3.11}$$

As the Policy Maker excludes more and more of the lower ability workers, the average ability of those that remain in the labor force increases. Free entry of vacancies means that the market tightness in the remaining market also increases.

The upshot is that the search externalities exacerbate (resp. ameliorate) the participation externality if worker bargaining power, β, is too high (resp. low) relative to the Hosios Rule value. As stated above, that firms cannot pre-commit to reject low ability entrants means that market tightness is too low even at the Hosios Rule. Increasing β above the Hosios Rule value further reduces vacancy creation and puts more pressure on a government to somehow raise the participation threshold. From Equation 3.9, however, increasing β reduces the direct value of raising $\hat{p}$.

Bringing all of the above together (see Section A.2),

$$\frac{dW}{d\hat{p}} = \left(\frac{\beta f(\hat{p}) m(\theta^*)}{1 - \eta(\theta^*)} \right) \left[\eta(\theta^*) \mathbb{E}_{[p \geq \hat{p}]}(p - z) - (\hat{p} - z) \right] \tag{3.12}$$

which collapses to Equation 3.9 at the Hosios Rule. Equation 3.12 shows that changes in β do not directly affect the sign of the impact of $\hat{p}$ on welfare. The direct and indirect effects exactly cancel out. Raising β reduces the firms' ability to recoup their sunk cost, a, but also raises their equilibrium matching probability. Ultimately, what matters for the sign of the effect is the elasticity of the matching function. It is immediate from Equation 3.12 that when $\hat{p} = z$, welfare is increasing in $\hat{p}$ for all (interior) values of β – a direct intervention to raise the participation threshold for workers is, on aggregate, always beneficial. In Section A.3 we show that an optimal value of $\hat{p}$ exists. Moreover, from Equation 3.12 that optimal value solves

$$\eta(\theta^*) \mathbb{E}_{[p \geq \hat{p}]}(p - z) = (\hat{p} - z)$$

and implements the Planner's solution at the Hosios Rule.

3.5 Policy initiatives

3.5.1 The Minimum Wage

In the market economy the Policy Maker cannot directly observe worker ability and, so, cannot directly manipulate $\hat{p}$. As wages are observable, however, the Policy Maker can use them as proxy for ability. That is, a minimum wage could act as an imperfect means to address the participation externality. The search externalities provide further possible justification for a minimum wage. It is well known in the search and matching literature that if homogenous workers have low bargaining power (i.e. $\beta < 1 - \eta$) a minimum wage set to the "Hosios wage", that would emerge under the Hosios Rule, implements constrained efficiency.[4]

[4]Here too, it is straightforward to show that a minimum wage made contingent on worker ability, $\bar{w}(p)$, implements constrained efficiency if $\beta < 1 - \eta(\theta_p)$ and

$$\bar{w}(p) = \left\{ \begin{array}{c} \hat{p}_p \text{ for } p \leq \hat{p}_p \\ b + (1 - \eta(\theta_p))(p - b) \text{ for } p > \hat{p}_p \end{array} \right.$$

As a minimum wage set below z has no impact, the Policy Maker only considers $\bar{w} \geq z$. Matches between a worker with $p \leq \bar{w}$ and any firm yield no gains from trade and these workers leave the workforce. When a minimum wage is below the worker's ability but higher than the bargained wage, $\beta p + (1-\beta)z$, pairwise Pareto optimality implies that the firm hires the worker at the minimum wage. If the minimum wage lies below $\beta p + (1-\beta)z$, Nash's Independence of Irrelevant Alternatives axiom implies that the worker gets hired at the bargained wage as if there were no floor (Muthoo, 1999). So, contingent on getting hired, a type p worker gets paid

$$\max\{\bar{w}, \beta p + (1-\beta)z\}.$$

Let $\tilde{p}$ be the highest ability level that receives the minimum wage so that $\bar{w} = \beta\tilde{p} + (1-\beta)z$. Then,

$$\tilde{p} = \frac{\bar{w} - (1-\beta)z}{\beta}. \tag{3.13}$$

Whenever $\bar{w} > z$, the market equilibrium is characterized by

$$\hat{p}^* = \bar{w}$$

and

$$\frac{m(\theta^*)}{\theta^*}\left\{\frac{F(\tilde{p}) - F(\bar{w})}{1 - F(\bar{w})}\mathbb{E}_{p\in[\bar{w},\tilde{p}]}(p - \bar{w}) + \frac{1 - F(\tilde{p})}{1 - F(\bar{w})}(1-\beta)\mathbb{E}_{[p\geq\tilde{p}]}(p - z)\right\} = a.$$

These imply that the participation threshold is effectively exogenous and θ^* is characterized by,

$$\frac{m(\theta^*)}{\theta^*\left(1 - F(\bar{w})\right)}\left\{\int_{\bar{w}}^{\tilde{p}}(p - \bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p - z)dF(p)\right\} = a. \tag{3.14}$$

Meanwhile, the welfare measure is the same as in Equation 3.1 with $\hat{p}$ replaced by $\bar{w}$. We have

$$\frac{dW}{d\bar{w}} = \frac{\partial W}{\partial \bar{w}} + \frac{\partial W}{\partial \theta}\frac{d\theta^*}{d\bar{w}} \tag{3.15}$$

where the two terms on the RHS capture the direct and indirect effects respectively. The goal is to assess the extent to which the minimum wage can address the participation externality as compared to direct manipulation of $\hat{p}$.

Mirroring Equation 3.8 we have,

$$\frac{\partial W}{\partial \bar{w}} = f(\bar{w})\left[a\theta^* - m(\theta^*)(\bar{w} - z)\right]. \tag{3.16}$$

The direct impact of the minimum wage captures the extent to which it effectively excludes lower ability workers and is identical to the direct impact of raising $\hat{p}$. As such, for values of $\bar{w}$ close enough to z this is positive. As $\frac{m(\theta)}{\theta}$ cannot exceed 1, however, high values of $\bar{w}$ can reduce welfare by excluding too many workers.

From Equation 3.10 we have

$$\frac{\partial W}{\partial \theta} = m'(\theta^*)\int_{\bar{w}}^{\bar{p}}(p - z)dF(p) - \left[1 - F(\bar{w})\right]a. \tag{3.17}$$

And, from Equation 3.14,

$$\frac{d\theta^*}{d\bar{w}} = \frac{f(\bar{w})a\theta^* - m(\theta^*)\left[F(\tilde{p}) - F(\bar{w})\right]}{a\left[1 - \eta(\theta^*)\right]\left[1 - F(\bar{w})\right]} \tag{3.18}$$

which is derived in Section A.4. It is not possible to sign this expression in general. A binding minimum wage mimics $\hat{p}$ in that it increases the average quality of the labor force which puts upward pressure on θ^*. However, it also raises the wages of those with productivities between $\bar{w}$ and $\tilde{p}$ which tends to suppress vacancy creation. The negative term disappears when $\bar{w} = z$.

Combining all of the above by substitution into Equation 3.15 (see Section A.5) leads to,

$$\begin{aligned}\frac{dW}{d\bar{w}} = \frac{\beta f(\bar{w})m(\theta^*)}{1 - \eta(\theta^*)}&\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p - z) - (\bar{w} - z)\right] \\ &+ \frac{\beta f(\bar{w})m(\theta^*)\eta(\theta^*)}{\left[1 - \eta(\theta^*)\right]\left[1 - F(\bar{w})\right]}\int_{\bar{w}}^{\tilde{p}}(\tilde{p} - p)dF(p) \\ &+ \frac{f(\bar{w})m(\theta^*)(\bar{w} - z)}{1 - \eta(\theta^*)}\left[\beta - (1 - \eta(\theta^*))\right] \\ &+ \frac{m(\theta^*)\left[F(\tilde{p}) - F(\bar{w})\right]\left[a - m'(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p - z)\right]}{a\left(1 - \eta(\theta^*)\right)}.\end{aligned} \tag{3.19}$$

The first term in Equation 3.19 is identical to Equation 3.12. The subsequent terms therefore capture the impact of changing the minimum wage vis-à-vis direct manipulation of $\hat{p}$. When the minimum wage just binds, (i.e. when $\bar{w} = z$) only the first term remains. This tells us that (locally) adjusting $\bar{w}$ and $\hat{p}$ only differ in their second order effects. In particular we see that a just-binding minimum wage is beneficial to welfare for all values of β.

When $\bar{w} > z$, the last three terms of Equation 3.19 matter. The second term is always positive and reflects the fact that workers with abilities in the range $[\bar{w}, \tilde{p}]$ have their wages raised to $\bar{w}$ – they get $\beta(\tilde{p} - p)$ more than they receive by raising $\hat{p}$ independently. The third term in Equation 3.19 pertains to the marginal worker whose ability is just at $\bar{w}$. As $\bar{w}$ increases that worker moves out of the labor force and loses the income $\bar{w} - z$. Of course, raising $\hat{p}$ causes the marginal worker to lose income too, but because $\beta < 1$, not as much.[5] The extent to which this excess loss of income, $(1-\beta)(\bar{w}-z)$, is a good or bad thing for aggregate welfare depends, according to the Hosios Rule, on whether β exceeds $1 - \eta(\theta^*)$ or not. The final term of Equation 3.19 comes from the indirect effect of $\bar{w}$ on welfare and is zero whenever $\theta^* = \theta_p$. Because $m(.)$ is strictly concave, this term is negative when $\theta^* < \theta_p$. Compared to simply raising $\hat{p}$, a minimum wage raises some wages and therefore suppresses vacancy creation. So, if market tightness is already too low, the welfare contribution from this term is negative.

As welfare is increasing at $\bar{w} = z$ and converges to z for high values of $\bar{w}$, similar analysis to that for adjusting $\hat{p}$ implies that the optimal minimum wage exceeds z. Because of the wage distortions caused by the minimum wage, even at the Hosios Rule, the optimal minimum wage does not (generically) implement the Planner's solution.

3.5.2 Unemployment insurance (UI)

Rather than price them out of the market with a minimum wage, the Policy Maker might attempt to pay low ability workers to sit out of the market (i.e. make a disability payment). By assumption, the Policy Maker cannot observe job search behavior. As a result, even the high ability workers would sign up for such payments and forgo them if they get a job. Such a policy effectively becomes UI in that everyone who fails to match receives the payment.[6] Around the world, real UI schemes incorporate features designed to ensure that payments only go to actual job seekers. Incorporating such features goes beyond the scope of this paper.

[5] With a binding minimum wage, the marginal worker earns $\bar{w} = \hat{p}$. If the Policy Maker could directly set $\hat{p} > b$, the marginal worker would earn $w = \beta\hat{p} + (1 - \beta)b$.

[6] Of course, unemployment insurance is something of a misnomer here. With risk-neutral workers the policy provides no "insurance" value. The focus here is on how the payments interact with the participation externality. Introducing risk-aversion would only serve to obscure that interaction. We stick with the insurance moniker because it reflects closely related policies of the same name.

The scheme pays b at the end of the period to anyone who did not get a job. To avoid introducing a fiscal externality coming from taxation that is contingent on employment, workers fund UI payments through a lump-sum tax, τ. They receive these payments on top of the value of their leisure, z.[7]

The equilibrium conditions Equation 3.4 and Equation 3.5 become

$$\frac{m(\theta^*)}{\theta^*}(1-\beta)\mathbb{E}_{[p\geq\hat{p}^*]}(p-z-b)=a$$

and

$$\hat{p}^* = z + b. \tag{3.20}$$

Clearly, $\hat{p}^*$ moves one-to-one with b and becomes the de facto policy instrument, $\hat{p}$. Substituting b out of the equilibrium conditions yields the following characterization of equilibrium market tightness, θ^*:

$$m(\theta^*)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})\,dF(p) = a\theta^*(1-F(\hat{p})). \tag{3.21}$$

This differs from Equation 3.6 because, now as b (i.e. $\hat{p}$) increases, it raises wages by $(1-\beta)b$ at every worker ability level and reduces the expected return to vacancy creation. Direct manipulation of $\hat{p}$, by comparison, simply excludes low ability workers and does not raise wages.

As the UI payments are transfers and workers are risk-neutral, introducing UI does not change the welfare measure, Equation 3.1. So, to measure the impact on welfare of a change in b we need

$$\frac{dW}{db} = \frac{\partial W}{\partial \hat{p}} + \frac{\partial W}{\partial \theta}\frac{d\theta^*}{d\hat{p}}. \tag{3.22}$$

Using Equation 3.21,

$$\frac{\partial W}{\partial \hat{p}} = f(\hat{p})m(\theta^*)\left[(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})-(\hat{p}-z)\right]. \tag{3.23}$$

Comparing this to Equation 3.9 shows that when $b=0$, the direct impact of raising it is identical to that of simply raising $\hat{p}$ without UI. But for higher values of b, $\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) < \mathbb{E}_{[p\geq\hat{p}]}(p-z)$. By raising wages, the positive impact of raising $\hat{p}$ through UI payments peters out more quickly than from raising it directly. UI, therefore, imperfectly targets the participation externality.

[7] The value of z that emerges from the data may well contain some employment contingent payments. What matters here is the extent to which increasing those payments can address the participation externality.

Now,

$$\frac{\partial W}{\partial \theta} = m'(\theta)\int_{\hat{p}}^{\bar{p}}(p-z)dF(p) - (1-F(\hat{p}))a$$
$$= m'(\theta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p) - (1-F(\hat{p}))a + (1-F(\hat{p}))m'(\theta)(\hat{p}-z)$$
$$= \left(\frac{1-F(\hat{p})}{1-\beta}\right)\left\{\left[\eta-(1-\beta)\right]a + (1-\beta)m'(\theta)(\hat{p}-z)\right\} \tag{3.24}$$

where the final expression uses Equation 3.21. Again the difference between the impact of raising UI payments and direct manipulation of the participation threshold stems from the way UI payments increase wages. When $b = 0$, comparison of Equation 3.10 and Equation 3.24 shows that raising UI or directly raising the participation threshold yields identical results. When UI payments are strictly positive, the additional positive term, $(1-\beta)m'(\theta)(\hat{p}-z)$, means that an increase in θ has a more positive (or less negative) impact on welfare than occurs when $\hat{p}$ is simply raised above z. This occurs because workers are better off when they receive higher wages.

So, how does UI affect market tightness?

$$\frac{d\theta^*}{d\hat{p}} = \frac{a\theta f(\hat{p}) - m(\theta)(1-\beta)(1-F(\hat{p}))}{m'(\theta)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p) - a(1-F(\hat{p}))}$$
$$= \frac{(1-\beta)m(\theta^*)\left[f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (1-F(\hat{p}))\right]}{(1-\eta(\theta^*))a(1-F(\hat{p})).} \tag{3.25}$$

This is the same as Equation 3.11 except for the negative $(1-F(\hat{p}))$ in the numerator. This negative pressure on vacancy creation from increases in b, again, stems from the increase in wages at every ability level which depresses vacancy creation. The net impact of an increase in UI payments on labor market tightness can be negative even when $b = 0$.[8] The extent to which this is a good or bad thing depends as usual on the workers' bargaining power.

[8] The term $f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})-(1-F(\hat{p}))$ is negative if $\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p)$, sometimes called the "surplus function" is log-concave (see Bagnoli and Bergstrom (2005)). If the density of a distribution is log-concave so is the surplus function. Examples of such distributions include uniform and normal. In the calibration of the dynamic version of the model we use a log-normal distribution. The density of that distribution is not log-concave but its cdf is. It exhibits a mixed surplus function in that parts can be log-concave and parts are log-convex.

Substituting the above analysis into Equation 3.22 (see Section A.6) yields

$$\begin{aligned}\frac{dW}{db} &= \left(\frac{\beta f(\hat{p})m(\theta)}{1-\eta(\theta)}\right)\left[\eta(\theta)\mathbb{E}_{[p\geq\hat{p}]}(p-z)-b\right] \\ &+ \left(\frac{m(\theta)}{(1-\eta(\theta))a}\right)\left\{m'(\theta)b(1-\beta)\left[f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})-(1-F(\hat{p}))\right]\right. \\ &\qquad - f(\hat{p})(1-\beta)b(1-\eta(\theta))a \\ &\qquad \left. -(1-F(\hat{p})\left[\beta-(1-\eta)\right]a\right\}. \qquad (3.26)\end{aligned}$$

As $b = \hat{p} - z$ the first line is identical to Equation 3.12. The second term, therefore captures the impact of changing the UI payment vis-à-vis direct manipulation of $\hat{p}$. When $b = 0$ the contents of the curly braces reduces to the third term only. This is zero at the Hosios Rule and negative (resp. positive) when β is high (resp. low). Increasing UI payments has a second order effect on welfare at the Hosios rule only. More generally, an increase in UI payments increases worker continuation values, reduces the size of the surplus, raises wages and reduces ex post profits for all levels of ability. This puts downward pressure on vacancy creation. But, when bargaining power is low ($\beta < 1 - \eta$) laissez-faire vacancy creation is too high so increasing b from zero can be more beneficial for welfare than increasing $\hat{p}$ in isolation.

For strictly positive values of b, the second term in the curly braces in Equation 3.26 is strictly negative. The sign of the first term is generally ambiguous depending on the nature of the distribution of abilities (see footnote 8). The impact of further increases in UI payments on welfare relative to what increasing $\hat{p}$ does is therefore difficult to characterize in any general way. Section 6 quantitatively explores this further.

Section A.7 shows the existence of an optimal value of b. For high enough values of β, welfare may be decreasing in b at $b = 0$ and the optimal value is negative. Under that scenario, wages are lower than under laissez-faire which counteracts the effect of the high β and increases vacancy creation. The cost of $b < 0$ is that it introduces some workers with ability below z into the market. The lower is $f(z)$ the lower is that cost.

Because of the wage distortions caused by the UI payments, in isolation, they cannot (generically) implement the Planner's solution even at the Hosios Rule.

3.5.3 Comparing the efficacy of UI and the minimum wage

Section 6 quantifies the extent to which each policy addresses the participation externality. Here we compare how they work. Both policies are readily implemented ways to raise the participation threshold, $\hat{p}$, which is too low in laissez-faire. Both, however, have side-effects. Under UI, a type p worker's wage is $\beta p + (1-\beta)(z+b)$. Increases in b therefore increase wages at every ability level by $(1-\beta)b$. Meanwhile with a minimum wage, a type p worker's wage is $\max\{\bar{w}, \beta p + (1-\beta)z\}$.

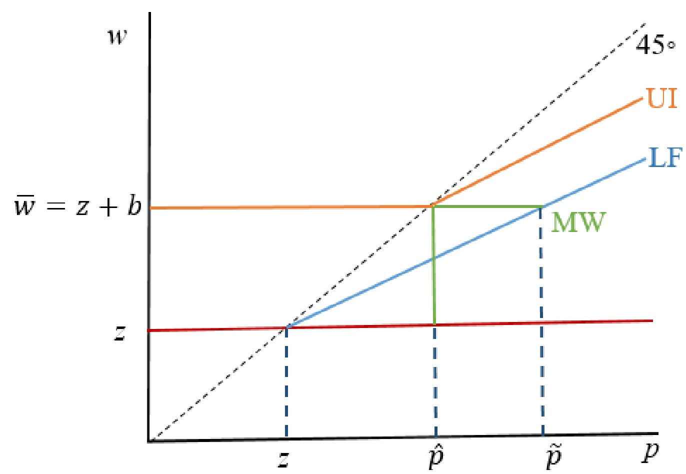

Figure 3.1: Comparing policy effectiveness.

Figure 3.1 depicts the impact of both policies on worker (before tax) incomes by ability level. In the laissez-faire economy, the participation threshold is z, non-employed workers all receive z and the light-blue line marked "LF"represents the wages of the employed. For comparability, both policies imply the same participation threshold, $\hat{p}$. With a minium wage, $\bar{w} = \hat{p}$, in place the non-employed receive z. The green line marked "MW"represents the incomes of the employed with abilities in the range $[\hat{p}, \tilde{p}]$. Employed workers with abilities above $\tilde{p}$ get the laissez-faire wage. With UI payment, $b = \hat{p} - z$, in place, the non-employed receive $z + b$. the upward sloping portion of the orange line marked UI represents the wages of the employed. The upshot from Figure 3.1 is that for the same participation threshold, workers wages are higher for every ability level with UI than with a minimum wage in place. This means that firms create fewer vacancies because they face lower expected profits under the UI policy.

From this, the minimum wage comes out as the least disruptive way to raise $\hat{p}$ and, therefore, more directly able to target the participation externality. However, the impact on overall efficiency depends on how each policy also interacts with the search externalities. By raising wages by more than the minimum wage at every ability level, UI is better at addressing the excess vacancy creation associated with low worker bargaining power. But, if $\beta > 1-\eta$, the minimum wage does the least additional harm as measured by welfare.

While both of these policies can generate efficiency gains over laissez-faire, they are not necessarily Pareto improving. In Section C.2 we provide an analysis of who are likely winners and losers under these policies.

3.5.4 Combining Policies

It is immediately apparent from Figure 3.1 that if $\bar{w} < z + b$ there is no benefit from combining the policies – optimizing over both would be the same as optimizing over b alone. Under the proviso that $\bar{w} \geq z + b$ the Policy Maker's optimization problem becomes:

$$\max_{b,\bar{w},\theta} zF(\bar{w}) + \int_{\bar{w}}^{\bar{p}} [m(\theta)p + (1-m(\theta))z]\, dF(p) - [1-F(\bar{w})]a\theta$$

subject to the free-entry condition:

$$\frac{m(\theta^*)}{\theta^*\,(1-F(\bar{w}))}\left\{\int_{\bar{w}}^{\tilde{p}} (p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}} (p-z-b)dF(p)\right\} = a.$$

Notice that b does not enter the objective function at all. Let b_c be the value of b that solves the free entry condition with $\theta^* = \theta_p$ and $\bar{w} = \hat{p}_p$. At $b = b_c$, the choice of $\bar{w}$ is identical to the Planner's problem and the combined policy implements constrained efficiency.

The question then comes down to what range of the parameter space is consistent with $\hat{p}_p > z + b_c$. Letting μ represent the Lagrange multiplier on the constraint, Section A.8 shows that

$$\mu = \frac{m(\theta)}{a\,[1-\eta(\theta)]}\left[\int_{\bar{w}}^{\tilde{p}} (p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}} (p-z-b)dF(p) - \eta(\theta)\int_{\bar{w}}^{\bar{p}} (p-z)dF(p)\right]. \quad (3.27)$$

Meanwhile, the first order condition for b becomes

$$\mu \frac{m(\theta)(1-\beta)[1-F(\tilde{p})]}{\theta[1-F(\bar{w})]} = 0.$$

Ignoring the possibility that $\tilde{p} = \bar{p}$ because our quantitative experiment uses $\bar{p} = \infty$, a solution for b_c requires $\mu = 0$. More generally, the slope of welfare with respect to b has the same sign as μ and optimal b occurs when the free-entry condition does not bind.

Now it is immediate from Equation 3.27 that μ is decreasing in b. Evaluating μ when $b + z = \bar{w}$ yields

$$\mu = -\frac{m(\theta)[1-F(\bar{w})]}{a\,[1-\eta(\theta)]} \left\{ [\beta - (1-\eta(\theta))]\, \mathbb{E}_{[p \geq \bar{w}]}(p - \bar{w}) + \eta(\theta)(\bar{w} - z) \right\}. \tag{3.28}$$

If $\beta > 1 - \eta(\theta)$ then μ is negative when $b + z = \bar{w}$. Lowering b below this value makes μ less negative and can eventually cause a change in sign. That is, welfare is decreasing in b for values close to $\bar{w} - z$ but could be increasing in b for lower values.

The upshot is that as long as a finite value of b_c exists, $\beta > 1 - \eta(\theta)$ implies $\bar{w} > z + b_c$. Moreover, the combination of a minimum wage, $\bar{w} = \hat{p}_p$ and $b = b_c$ implements constrained optimality. Equation 3.28 demonstrates that the same result is true for $\beta < 1 - \eta(\theta)$ as long as $1 - \eta(\theta) - \beta$ is not too large.

Operationally, we can set $\bar{w} = \hat{p}_p$ and then use b to set wages. As when $\beta > 1 - \eta(\theta)$ wages are too high, the implication is that b_c would be negative. The minimum wage addresses the participation externality and the inverted UI (the unemployed pay the rest of the population) addresses the search externalities. When $\beta < 1 - \eta(\theta)$ but sufficiently close that $\hat{p}_p > z + b_c$ many of the wages are too low in equilibrium and optimal benefits would be positive. Once β gets so low that $\hat{p}_p > z + b_c$, the combined policy cannot implement constrained optimalilty because UI effectively becomes the sole policy instrument.

3.5.5 Implementation of constrained efficiency with vacancy subsidies

The question here is whether introducing a vacancy subsidy, s, (also paid for by lump-sum taxation on all workers) combined with either UI or a minimum wage can implement constrained efficiency.

With UI the equilibrium conditions become

$$\frac{m(\theta^*)}{\theta^*}(1-\beta)\mathbb{E}_{[p\geq\hat{p}^*]}(p-z-b) = a-s$$

and

$$\hat{p}^* = z+b.$$

Eliminating b means the Policy Maker's problem becomes

$$\max_{\hat{p},s,\theta}\left\{zF(\hat{p}) + \int_{\hat{p}}^{\bar{p}} [m(\theta)p + (1-m(\theta))z]\, dF(p) - [1-F(\hat{p})]a\theta\right\}$$
$$\text{s.t.} \quad m(\theta)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-\hat{p})dF(p) - [1-F(\hat{p})]\theta(a-s) = 0.$$

It is immediate from this that the Policy Maker can choose s so that the constraint does not bind, and the problem reduces to that of the Social Planner.

Similarly, with the minimum wage, the Policy Maker's problem is now

$$\max_{\bar{w},s,\theta}\left\{zF(\hat{p}) + \int_{\bar{w}}^{\bar{p}} [m(\theta)p + (1-m(\theta))z]\, dF(p) - [1-F(\bar{w})]a\theta\right\}$$
$$\text{s.t.} \quad m(\theta)\left\{\int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) + (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right\} - [1-F(\hat{p})]\theta(a-s) = 0.$$

Again, with a free choice of s, the constraint does not bind.

With either set of policy instruments, the Policy Maker can implement constrained efficiency. The only caveat here is that the government has to raise taxes to pay for s (and b in the case of UI). If $z < \tau$ workers who do not participate in the market, or who do not get a job, could end up with negative utility. Were we to require that $\tau \leq z$, full constrained efficiency may not be achievable.

3.5.6 Further discussion

The emphasis on simplicity rules out a number of potential generalizations. One natural generalization, endogenous search intensity, would yield ambiguous predictions. Because they have higher returns to employment, higher ability workers would search harder, but would not internalize the impact of their intensity choice on the other market participants. By contrast, low ability workers would limit their search, which could reduce the potential impact of their presence on congestion externalities. But, with a low matching rate, they would remain in the market for a long time and exacerbate the congestion externality experienced by other participants. A binding minimum wage would exclude the lowest ability workers but increase the search intensity of the marginal worker with p in $[\bar{w}, \tilde{p}]$. UI excludes the lowest ability workers and reduces the returns to search at every remaining ability level. How these changes affect welfare depends on the functional form of the search cost function and the bargaining power of the workers vis-à-vis the Hosios Rule.

Another possibility is that workers are additionally heterogeneous in their values of non-market activities, z. This is a concern that is relevant to low-income markets and to policy makers. In that environment, low ability workers with high z would never enter the market, distributing the consequences of the participation externality more diffusely. The policies we consider would not change that. An earned-income tax credit (EITC) could induce some quite high ability workers with even higher z into the market but might also bring in more low ability workers. A minimum wage coupled with EITC could prevent the latter outcome.

4 Dynamic Model

Ascertaining the quantitative importance of the participation externality requires a dynamic version of the model. The analytical results here largely mirror those of the static model so much of the work has been moved to Appendix B.

4.1 Environment

The dynamic model is cast in continuous time so all of the utility sources, p, z, a etc. become flows. The function $m(.)$ now represents a Poisson meeting rate[9], λ denotes an exogenous job destruction rate, and firms and workers face a common discount rate, r. Workers live forever.

[9]More specifically, $m(.)$ is strictly concave and strictly increasing on $\mathbb{R}_+$ with $\lim_{\theta\to\infty} m(\theta) = \infty$. And, $m(\theta)/\theta$ is strictly decreasing with $\lim_{\theta\to\infty} m(\theta)/\theta = 0$. To avoid a corner solution, $\lim_{\theta\to 0} m'(\theta) = \infty$.

4.2 Efficiency

Risk-neutrality implies that welfare, W, amounts to discounted transitional benefits minus costs. Thus,

$$W = \int_0^\infty \left(z\left[F(\hat{p}_t) + u_t\right] + \left[1 - F(\hat{p}_t) - u_t\right] \mathbb{E}_{[p \geq \hat{p}_t]}\left(p\right) - \theta_t u_t a \right) e^{-rt} dt \quad (4.1)$$

where the subscript t, signifies time. The variable, $\hat{p}_t$, is the cut-off level of ability below which a worker does not participate in the labor market.[10] The first term in the parentheses sums the welfare obtained from non-market activities by excluded workers and job seekers. The second, sums output across matched workers. The final term is the cost of maintaining the vacancies.

The Planner chooses a path for the population share of job seekers, u_t, the labor market tightness, θ_t, and the ability exclusion threshold, $\hat{p}_t$, to maximize welfare. The dynamics of unemployment constrain the Planner,

$$\dot{u}_t = \lambda\left(1 - F(\hat{p}_t) - u_t\right) - m(\theta_t)u_t \quad (4.2)$$

where the dot over a variable indicates its rate of change with respect to time.

From the first order conditions for θ and $\hat{p}$, we derive the following dynamic counterparts to Equation 3.2 and Equation 3.3 (see Section B.1),

$$\frac{m'(\theta_p)\mathbb{E}_{[p \geq \hat{p}_p]}(p - z)}{r + \lambda + m(\theta_p) - \theta_p m'(\theta_p)} = a \quad (4.3)$$

$$\frac{m'(\theta_p)\left[\left(m(\theta_p) + \lambda\right)\left(\hat{p}_p - z\right) + \lambda \mathbb{E}_{[p \geq \hat{p}_p]}(p - \hat{p}_p)\right]}{\lambda(m(\theta_p) + \lambda)} = a. \quad (4.4)$$

Notice that whenever $z > 0$ the Planner excludes at least those workers whose ability is less than z. They cause congestion to the other workers in the matching process and, if employed, reduce total welfare. So, the constrained efficient value for $\hat{p}$ cannot be less than z.

In Section B.2 we show that as long as $m(\theta)\eta(\theta) > r$ the Planner always chooses a value for $\hat{p}_p$ strictly larger than z. We call $m(\theta)\eta(\theta) > r$ the "Thick-Market" condition. Whenever $m(\theta)\eta(\theta) \leq r$, the Planner sets $\hat{p} = z$. This

[10] Technically, the definition of welfare here requires that the value of $\hat{p}_t$ increase over time so that the Planner can terminate matches as they become unviable. Because the Planner is subject to matching frictions, were $\hat{p}_t$ to decrease over time, the distribution of current match productivities would temporarily differ from a simple truncation of $F(.)$. However, the focus here is on steady states and as the economy converges to steady state, the distinction between increasing and decreasing paths for $\hat{p}_t$ disappears.

highlights a notable difference between the dynamic model and the static one. In the latter $\hat{p}_p$ is larger than z for any non-trivial equilibrium. This would also be true in the dynamic model in the limit as r approaches zero. In general, as long as r is small enough, the long-term benefit associated with improved selectivity of match formation outweighs the short-term loss of output associated with excluding some workers with $p > z$ from matching.

4.3 Market economy

In the dynamic market economy, generalized Nash bargaining determines wages where the worker's outside option is to continue to look for work. All dynamic market analysis is carried out for steady states.

Again, workers indifferent between market entry and sitting it out, choose the latter. Workers with ability levels below $\hat{p}$ choose not to participate. For any worker with $p \geq \hat{p}$ we have,

$$rV_u(p) = m(\theta)\left(V_e(p) - V_u(p)\right) + z \tag{4.5}$$

$$rV_e(p) = w(p) + \lambda\left(V_u(p) - V_e(p)\right) \tag{4.6}$$

where $V_u(p)$ is their value to unemployment and $V_e(p)$ is their value to employment. Then, given free entry of vacancies, the value of hiring a type p worker to the firm, $V_f(p)$, is given by

$$(r+\lambda)\,V_f(p) = p - w(p). \tag{4.7}$$

Eliminating the value functions reveals that the wage is a weighted average of worker ability and the effective flow income from non-employment:

$$w(p) = \frac{\beta\left(r+\lambda+m(\theta)\right)p + (1-\beta)(r+\lambda)z}{r+\lambda+\beta m(\theta)}. \tag{4.8}$$

In the laissez-faire environment, $\hat{p} = z$. This means that any worker of ability $p > z$ creates match surplus. But, as long as the Thick-Market condition holds, the Planner would exclude some workers whose productivity is above z. Those workers' failure to internalize the impact of their participation on vacancy creation leads to a participation externality.

With free-entry, the vacancy creation condition is,

$$\frac{m(\theta)}{\theta\left(1-F(z)\right)}\int_z^{\bar{p}} V_f(p)dF(p) = a. \tag{4.9}$$

Definition 1 *A free-entry market equilibrium is a list* $\{V_u(p), V_e(p), V_f(p), w(p), \theta^*, \hat{p}^*\}$ *such that:*

- $\hat{p}^* = z$.
- *Given* θ^**, the value functions emerge from optimal search and matching.*
- *For* $p \geq z$*,* $w(p)$ *is given by Equation 4.8.*
- θ^* *solves the free-entry condition, Equation 4.9.*

Straightforward algebra yields the following characterization of θ^*:

$$\frac{m(\theta^*)(1-\beta)\mathbb{E}_{[p \geq z]}(p-z)}{\theta^*(r+\lambda+\beta m(\theta^*))} = a. \tag{4.10}$$

This corresponds to Equation 3.4 in the static model and equates the firms' marginal private value to marginal cost of vacancy creation. In Section B.3 we show that the market equilibrium exists and is unique.

4.4 Dynamic model policy analysis

Of interest here again is the role of the participation externality, how it interacts with the search externalities and how to address it using either a minimum wage or UI. By its nature the dynamic model is more complex than the static one and results here are less amenable to succinct interpretation. Here we provide a synopsis of the results and remand the proofs to Appendix B.

4.4.1 Direct control of $\hat{p}$

In this scenario, the ability threshold for market entry, $\hat{p}$, is exogenous and the free entry condition yields a unique equilibrium market tightness, θ^*,

$$\frac{m(\theta^*)(1-\beta)\mathbb{E}_{[p \geq \hat{p}]}(p-z)}{\theta^*(r+\lambda+\beta m(\theta^*))} = a. \tag{4.11}$$

The Policy Maker can adjust $\hat{p}$ to impact welfare, still obtained from Equation 4.1, subject to the law of motion for unemployment and the free entry condition, Equation 4.11.[11]

[11] A possible cause for concern here is that condition Equation 4.11 was derived in steady-state. The only source of dynamics in the economy, however, comes from the fact that the measure of unemployment, u, is not a jump variable and u does not appear in Equation 4.11 – θ^* simply jumps to its new steady-state value whenever $\hat{p}$ (or any other policy variable) is changed.

In Section B.4 we show that welfare is increasing in $\hat{p}$ at $\hat{p} = z$ whenever

$$(1-\eta)\left[r+\lambda+m(\theta^*)\right]\left[\eta(\theta^*)m(\theta^*)-r\right] \\ +m(\theta^*)\left[\beta-(1-\eta(\theta^*))\right]\left(\left[r+\lambda+m(\theta^*)\right]\eta(\theta^*)-r\right)>0.$$

This is the General Thick-Market condition. It collapses to the basic Thick-Market condition at the Hosios Rule. If $\beta > 1-\eta(\theta^*)$ and $\eta(\theta^*)m(\theta^*) > r$ the second term is positive so this requirement is less stringent than the basic Thick-Market condition. The opposite is true if $\beta < 1-\eta(\theta^*)$.

Whenever the Generalized Thick-Market condition holds, we show in Section B.6 that there exists an optimal value of $\hat{p} > z$. Recall that in the static model we showed that if η is invariant to θ, optimal $\hat{p}$ is invariant to β. Here we are unable to confirm this result analytically, but we do find this to be true in all of our numerical simulations. Comparison of Equation 4.3 and Equation 4.11 reveals that at the Hosios Rule, optimal $\hat{p}$ implements the Planner's solution.

4.4.2 The minimum wage

Again here, $\hat{p} = \bar{w}$. And again, we let $\tilde{p}$ be the ability level above which workers receive their bargained wage. For workers with $p > \tilde{p}$, the laissez-faire value functions, Equation 4.5, Equation 4.6 and Equation 4.7 still apply. For workers with $p \in [\bar{w}, \tilde{p}]$, they are Equation 4.5, Equation 4.6 and Equation 4.7 with $w(p)$ replaced by $\bar{w}$.

Definition 2 *A free-entry market equilibrium, with a binding minimum wage, $\bar{w}$, is a list $\{V_u(p), V_e(p), V_f(p), w(p), \theta^*, \tilde{p}\}$ such that given θ^* and $\tilde{p}$, the value functions emerge from optimal search and matching and Equation 4.8 gives $w(p)$ for $p \geq \tilde{p}$. Then, $\tilde{p}$ solves $w(\tilde{p}) = \bar{w}$ and θ^* solves the free-entry condition, Equation 4.9.*

Straightforward algebra yields the following characterization of equilibrium in terms of θ^* and $\tilde{p}$:

$$a = \frac{m(\theta^*)}{\theta^*\left(1-F(\bar{w})\right)}\left[\int_{\bar{w}}^{\tilde{p}}\frac{p-\bar{w}}{r+\lambda}dF(p)+(1-\beta)\int_{\tilde{p}}^{\bar{p}}\frac{p-z}{r+\lambda+\beta m(\theta^*)}dF(p)\right] \tag{4.12}$$

$$\tilde{p} = \frac{\left[r+\lambda+\beta m(\theta^*)\right]\bar{w}-(1-\beta)(r+\lambda)z}{\beta\left(r+\lambda+m(\theta^*)\right)}. \tag{4.13}$$

These equations correspond respectively to Equation 3.14 and Equation 3.13 in the static model analysis. The uniqueness of $\tilde{p}$ follows because $w(p)$ is strictly increasing in p. In Section B.3 we show that with $\bar{w} < \bar{p}$, equilibrium exists and it is unique. In Section B.5 we also show that equilibrium values of θ^* and $\tilde{p}$ both decrease with worker bargaining power, β. We show further, in Section B.7, that whenever the Generalized Thick-Market condition holds, welfare is increasing in the minimum wage at $\bar{w} = z$ and that an optimal strictly binding minimum wage exists.

4.4.3 Unemployment Insurance

We abstract from the complexities of real-world UI systems to focus on how an indefinite stream, b, paid to all non-employed workers interacts with the participation and search externalities. We assume that the Policy Maker finances UI payments through a lump-sum tax, τ, assessed on all workers.

Following the logic from the laissez-faire market economy above, for any $p \geq \hat{p}$ we now have,

$$\begin{aligned} rV_u(p) &= m(\theta)\left(V_e(p) - V_u(p)\right) + z + b - \tau \\ rV_e(p) &= w(p) - \tau + \lambda\left(V_u(p) - V_e(p)\right) \end{aligned}$$

and

$$(r+\lambda)\, V_f(p) = p - w(p).$$

Nash bargaining implies

$$w(p) = \frac{\beta\left(r+\lambda+m(\theta)\right)p + (1-\beta)(r+\lambda)(z+b)}{r+\lambda+\beta m(\theta)}.$$

Here $\hat{p} = z + b$. The free-entry condition is identical to Equation 4.9 and the definition of equilibrium is identical to Definition 1. Equilibrium market tightness is now characterized by

$$a = \frac{m(\theta^*)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{\theta^*\left(r+\lambda+\beta m(\theta^*)\right)} \tag{4.14}$$

which corresponds to Equation 3.21 from the static model analysis. Existence and uniqueness of the equilibrium is demonstrated in Section B.3. We also show in Section B.8 that at the Hosios Rule, welfare is increasing in b at $b = 0$ whenever the basic Thick-Market condition holds. As we found in the static model, however, at $b = 0$,

$$\left.\frac{d\theta^*}{d\hat{p}}\right|_{(4.14)} \neq \left.\frac{d\theta^*}{d\hat{p}}\right|_{(4.11)}.$$

So, away from the Hosios Rule, the Generalized Thick-Market condition does not apply to UI payments.

5 Calibration

In this section, we discuss the calibration of the dynamic model presented in Section 4. We first introduce our data and sample selection criteria. Then we discuss externally calibrated parameters. Last, we explain our identification strategy using key labor market moments and show the model fit.

5.1 Data

We use monthly Current Population Survey (CPS) data from January 2012 to December 2014[12] and restrict the sample to include only states where the minimum wage was \$7.25 in 2012, individuals who did not obtain a high school diploma or equivalent, and those who were between 25 and 54 years of age throughout the period under consideration. We drop anyone who reports ever being an unemployed new worker, unable to work, or retired.

5.2 Model Calibration

We follow a standard approach in the search and matching literature and preset a selection of parameters with estimated values from other papers with closely related models. We discuss these preset parameters as well as our functional form assumptions in Section 5.2.1. We estimate the parameters associated with key distinctions in the model by targeting the empirical specifications described in Section 5.2.2.

5.2.1 Externally Calibrated Parameters and Functional Forms

We start the calibration by making functional form assumptions that follow much of the related literature. We assume that the distribution of abilities, $F(.)$ is log-normal with parameters μ and σ. We also assume that the aggregate matching function is Cobb-Douglas with an elasticity of matching with respect to vacancies of η so that $m(\theta) = \bar{m}\theta^{\eta}$.

Following our functional form assumptions, we have 10 parameters to calibrate. We externally calibrate parameters with direct empirical interpretations $(\bar{w}, \lambda)$ and then we preset a selection of parameters to common values in the literature $(r, \eta, \bar{m})$. We use simulated method of moments to calibrate the remaining parameters $(\beta, \mu, \sigma, a, z)$ in Section 5.2.2. Throughout our calibration we assume that the period length is one month, though we focus on the steady state and thus do not discretize the model.

[12] This is a window of time in which employment was reasonably stable, but many individual states had not yet raised their minimum wages above the \$7.25 federal level.

We set $\bar{w}$ to be the federal minimum wage from 2012 to 2014, \$7.25. In our empirical analysis, we restrict our sample to states in which the federal minimum wage is binding as discussed in Section 5.1. The separation rate, λ, directly translates into the flow rate from employment to unemployment. Thus, we target this flow rate in our empirical sample, which is 0.0329. Following the real business cycle literature, we set the discount rate, r, to be consistent with 4% per year. We follow the results in Blanchard et al. (1990) and Petrongolo and Pissarides (2001) and set the elasticity of the matching function with respect to market tightness, η, to be 0.5. Because there is a one-to-one relationship between the advertising cost, a, and $\bar{m}$ in the current model, we choose to normalize $\bar{m}$ to unity. We present the values for our parameters in Table 5.2.

5.2.2 Calibrated Parameters

In this section, we discuss the moments that we use to estimate the remaining parameters of the model. We also describe which moments are most closely associated with each parameter, though our parameters are jointly estimated.

We exploit the participation threshold to identify the parameters of the productivity process, μ and σ. In the absence of a binding minimum wage, non-participation is exclusively determined by the productivity distribution. What distinguishes μ from σ is that they have opposite effects on the measure of non-participants. An increase in σ further skews the distribution, leading to fewer participants, while an increase in μ shifts the distribution to the right, resulting in fewer non-participants. By contrast, increases in either μ or σ cause the average wage and the standard deviation of wages, moments we also target, to increase. Incorporating these three moments allow us to separately identify the productivity parameters. In the CPS, we find a non-participation rate of 20.6%, and a mean and standard deviation of the log-wage distribution of 2.45 and 0.32, respectively.

Our remaining challenge is to separately identify worker bargaining power, β, from non-labor market utility, z. From the wage equation (Equation 4.8), it is clear that an increase in β or z affects wages. However, an increase in β causes a non-linear shift in wages across the wage distribution, while an increase in z causes an upward shift of all wages by the same amount. This intuition is most easily seen by considering the derivative of wages with respect to each parameter in our static model, where wages above the minimum wage are $w(p) = \beta p + (1-\beta)z$. For β, this exercise yields

$$\frac{\partial w}{\partial \beta} = p - z \tag{5.1}$$

which is larger for high ability workers ($p >> z$) than marginal workers ($p \approxeq z$). This leads to the non-linearity we exploit for identification. By contrast, the derivative of wages with respect to z yields

$$\frac{\partial w}{\partial z} = (1-\beta) \tag{5.2}$$

which is an identical increase for all workers. This difference allows us to identify each parameter. From Equation 4.13, an increase in either z and β cause a decrease in the measure of workers receiving the minimum wage. They differ in their effect on the dispersion of wages: an increase in β results in an increase in the standard deviation of wages, by causing wages to more closely reflect the skewness of the productivity distribution. By contrast, an increase in z leads to a uniform upward shift of the wage distribution, *reducing* the standard deviation of wages. In concert with targeting the wage distribution, targeting the mass of workers at the minimum wage allows us to separately identify these parameters. In the data, the share of workers employed at the minimum wage is 3.32%, which we identify as workers who report earning $7.25 in the CPS.

Finally, we follow the standard convention in the search literature to calibrate the cost of vacancy creation, a. We target the unemployment rate which, from the free entry condition (Equation 4.9), increases with a. While the national average unemployment rate for the whole workforce during this time period was 7.3%, we restrict our sample to strongly attached but lower-educated workers. This results in an unemployment rate of 10.4%.

We present our calibration results in Table 5.1. The model nearly precisely matches all the targeted moments.

Table 5.1: Model Fit.

Moment	Data	Model
Unemp. Rate	0.10	0.10
Non-Part. Rate	0.21	0.21
$E[ln(w)]$	2.45	2.45
$SD[ln(w)]$	0.32	0.34
$P(w=\bar{w})$	0.03	0.03

We present the parameters that achieve this fit in Table 5.2. The first five rows are those that we estimate as described above, while the remaining are externally calibrated. In our model, average hourly productivity is \$13.74, meaning that our estimate of leisure utility, $z = 6.04$ is 44% of average productivity. This is close to Shimer (2005), but lower than Hagedorn and Manovskii (2008a). Like other recent work (see Hagedorn and Manovskii (2008b), Mitman and Rabinovich (2015) and Mitman and Rabinovich (2019) among others), we find that $\beta < 1 - \eta$, which lends credence to the validity of our calibration strategy. Our estimated vacancy creation cost of \$169.75 is high relative to previous estimates, but it is worth noting both that our sample yields a higher unemployment rate than the population average and that we normalize the matching scale parameter $\bar{m} = 1$, while many related papers separately estimate both and arrive at smaller values. Were we to impose, say, $\bar{m} = 0.1$ the calibrated value of a would be lower but the results would be unchanged.

Table 5.2: Model parameters.

Parameter	Comment	Value
β	Barg. Power	0.30
μ	Mean of Prod. Dist	2.39
σ	Var. of Prod. Dist	0.50
z	Non-Market Util.	6.04
a	Vacancy Cost	169.75
η	Matching Elast.	0.50
r	Discount Rate	0.0033
λ	Sep. Rate	0.03
$\bar{m}$	Matching Fun. Norm.	1.00
$\bar{w}$	Min. Wage	7.25

Notes: We calibrate the first five parameters using the moments described above. We preset the bottom five using values from related papers in the manner described in Section 5.2.1.

5.2.3 Fit and non-targeted validation

It is no surprise that our model precisely matches our moments. Here, we show that the model is also capable of matching several non-targeted moments. In Table 5.3, we compare the modal wage in our model to the data, the minimum wage elasticity, and the job-finding rate. We come close to matching both the modal wage (\$10.34 in our model and \$10.30 in the data) and the monthly job-finding rate (0.28 in our model and 0.34 in the data), but slightly overestimate the elasticity of employment with respect to the minimum wage (-0.76 in our model vs. -0.69 in the data), when we compare our results to the long-run estimates of Keil et al. (2001). We select this target because our model contains few of the short-run frictions that might limit a firm's employment response. This last target is contentious with estimates ranging from slightly positive to highly negative. We note that our sample is low-skilled workers whom the minimum wage is more likely to affect, and that this group tends to experience the largest employment effects (Neumark and Wascher, 2006).

Table 5.3: Non-targeted moments.

Moment	Data	Model	Comment
Min. wage elasticity	-0.69	-0.76	From Keil et al. (2001)
Modal wage	10.30	10.34	CPS, 2012-2014
Monthly job-finding rate	0.34	0.28	From Shimer (2004)

6 Results

We now use the calibrated economy to consider the impact of minimum wage and unemployment policies on the participation and search externalities. We first compare welfare and labor market outcomes between a laissez-faire benchmark economy and optimal policy economies. We also consider two additional benchmark economies: our economy as calibrated in Section 5 and an economy in which a Social Planner is able to directly control the participation decision of workers. Next, we use our results in Section 4 to quantify the gains by implementing an optimal minimum wage or optimal unemployment insurance. We then explore the channels through which each policy is able to simultaneously affect the participation and search externalities.

6.1 Optimal Policies in the Dynamic Model

We start by quantifying the gains from implementing an optimal minimum wage or unemployment insurance. In our experiments, a Policy Maker in the decentralized economy is able to directly control the minimum wage, $\bar{w}$, or the increase in unemployment utility from z to $z + b$. They are still bound, however, by the decentralized free entry condition, and can only indirectly change θ. Because our calibration indicates that $\beta < 1 - \eta$, we do not implement these policies in combination and note that the resulting policies would be identical to the optimal UI experiment. Last, we calculate the optimal vacancy subsidy that implements the Planner's allocation under minimum wage and UI regimes. We compare these economies to a laissez-faire economy, our calibrated baseline, and to a Social Planner's economy.

We focus on three measures to describe the impact of each policy. First, we assess the effect on the non-participation rate. We calculate this as the share of workers with ability below the cutoff in each economy, $F(\hat{p})$, where $\hat{p}$ is $\bar{w}$ for the calibrated and minimum wage economies, $z + b$ for the optimal UI economy, and $\hat{p}_p$ for the Planner's economy. In addition, under each policy, we calculate the unemployment rate as the steady-state share of participating workers who are not employed. Finally, we quantify the gains achieved from switching policies by calculating welfare using Equation 4.1, where we set the participation threshold to $\hat{p}$ described above. We assume that each economy starts from the laissez-faire level of unemployment and calculate present discounted welfare along the transition to steady state. We compare the welfare gains due to each policy to the calibrated economy.

We present the results in Table 6.1. The "Calibrated economy" is the baseline model calibrated as described in Section 5. The "Planner's optimum" refers to an economy in which a Social Planner selects the optimal participation and labor market tightness according to Equation 4.3 and Equation 4.4. The "Optimal min. wage" economy refers to one in which a Policy Maker optimally sets the minimum wage as described in Section 4.4.2, leaving UI at its baseline value (i.e., $b = 0$). "Optimal UI" refers to an economy in which the minimum wage does not bind and the Policy Maker offers additional UI to workers using a policy determined by solving the problem in Section 4.4.3. The final line calculates the vacancy subsidy required to implement the Planner's allocation in the decentralized economy when the Policy Maker sets participation thresholds so that they equal the Planner's threshold. Under a minimum wage, this means that $\bar{w} = \hat{p}_p$, and under a UI expansion this means that $b = \hat{p}_p - z$. A negative subsidy indicates that implementing the Planner's allocation requires a lump sum tax on vacancies. In each economy, the "Ability Threshold" column refers to the dollar value of $\hat{p}$. $P(w = \bar{w})$ denotes the mass of workers earning the minimum wage. We select the "laissez-faire" economy as a benchmark, because we implement our UI expansion without a minimum wage, making this the appropriate counterfactual.

Our results indicate that both the minimum wage and UI yield welfare improvements over their absence. Our optimal minimum wage is \$7.15, which indicates that the 2012 minimum wage of \$7.25 comes close to implementing the welfare maximizing minimum wage. Setting the minimum

Table 6.1: Outcome comparison with baseline parameters.

	Unemp. Rate	Non-Part. Rate	θ	Ability Threshold ($)	$P(w = \bar{w})$	Welfare Ratio (% of Laissez-faire)
Laissez-faire	0.109	0.117	0.070	6.036	0.000	100.000
Calibrated economy	0.104	0.206	0.079	7.250	0.033	100.314
Planner's optimum	0.151	0.190	0.033	7.043	0.000	101.542
Optimal min. wage	0.105	0.198	0.078	7.149	0.030	100.317
Optimal UI	0.114	0.231	0.064	7.565	0.000	100.862
	Opt. min. wage plus vac. subs.	Opt. UI plus vac. subs.				
Vacancy Subsidy	-168.481	-126.105	Results otherwise identical to Planner's optimum			

wage to $7.15 yields a small welfare gain of 0.003pp, relative to the calibrated economy, but a moderate welfare gain of 0.317pp relative to the laissez-faire economy. The fifth row shows that if the Policy Maker instead funded additional UI payments of $1.52 ($7.56 - $6.04), welfare would increase by 0.548pp relative to the calibrated economy, and yield an 0.862pp welfare gain over the laissez-faire benchmark. With both the participation threshold and vacancy creation under her control, the Planner is able to increase welfare by 1.228pp relative to our calibrated model and 1.542pp relative to the laissez-faire economy. The final row shows that the Policy Maker would need to impose a $168.48 lump-sum tax on vacancies to achieve the Planner's optimum in concert with a minimum wage, and set this tax to $126.11 when using UI as a policy instrument instead.

All three policies respond to the low β relative to the Hosios value. While a near doubling of the cost to create a vacancy appears large, the Planner's labor market tightness is less than half the value under the decentralized economy. This is because β is low relative to its Hosios value and an increase in a artificially reduces the surplus retained by vacant firms, reducing vacancy creation to its optimal level. By contrast, neither the minimum wage nor UI in isolation can achieve the same allocation as the Planner. Notice that the ability thresholds for both policies are above the Planner's value. As they both raise wages along with the participation threshold, these policies are able to simultaneously address the search and participation externalities. As long as the threshold is below the Planner's optimal level increasing it is beneficial on both fronts. Increasing the threshold beyond the Planner's level then sets up a trade-off between these goals. As, for a given value of $\hat{p}$, UI raises wages more than does the minimum wage those forces balance out a higher value than for the minimum wage.

An important subtlety that we will explore in the next section is how the minimum wage and UI achieve these welfare gains. Compared with the laissez-faire economy, each alternative unsurprisingly reduces participation, varying from non-participation rate of 19% under the Planner to 23.1% under Optimal UI. The remaining effects on the labor market are surprisingly disperse across the different policies. The minimum wage policies under either the calibrated economy or our optimal minimum wage reduce the unemployment rate, while increasing θ. However, both the UI policy and the Planner's optimum result in increases in the unemployment rate and a reduction in θ, exactly the opposite of the minimum wage policies. This reflects an important distinction between the policies: while the minimum wage distorts vacancy creation, it does so by changing a subset of negotiated wages; by contrast, UI distorts wages across the wage distribution, leading to potentially more bite on vacancy creation.

6.2 Understanding the Externalities

This section shows how the search and participation externalities guide the optimal choice of a minimum wage or unemployment insurance. Either policy faces three potential sources of inefficiency. First, the model contains the standard search externalities caused by assigning inappropriate shares of the surplus to the firm and the worker. Second, the impact that average worker quality has on vacancy creation causes a participation externality. Last, the policies distort wage formation which directly impacts vacancy creation. We first decompose the effect of the minimum wage among these channels and their interaction, before doing the same for UI. Throughout our decomposition, we quantify the effect of each policy on the externalities by using the change in welfare as we use restrictions to isolate each externality.

6.2.1 The Participation and Search Externalities under the Minimum Wage

In the calibrated model, the minimum wage blunts the impact of both the participation and search externalities on welfare. It screens out low-productivity workers, reducing the participation externality, and increases the share of the surplus received by some workers, artificially helping to address the imbalance in the search externalities caused by $\beta < 1 - \eta$. This means that while the minimum wage may directly address the participation externality, the optimal minimum wage may not coincide with the optimal level of participation. This tension highlights both the challenge faced by a Policy Maker in the decentralized economy and the advantage offered by the minimum wage.

The challenge is that the Policy Maker is unable to directly control vacancy creation and participation. By contrast, the Social Planner may operate both levers of policy. Unable to control vacancies, the decentralized Policy Maker who wishes to use the minimum wage is bound by the free entry condition:

$$a = \frac{m(\theta)}{\theta\,(1 - F(\bar{w}))} \left[\int_{\bar{w}}^{\tilde{p}} \frac{p - \bar{w}}{r + \lambda} dF(p) + (1 - \beta) \int_{\tilde{p}}^{\bar{p}} \frac{p - z}{r + \lambda + \beta m(\theta)} dF(p) \right]$$

which does not generically set θ to coincide with $\frac{\partial \mathcal{H}}{\partial \theta_t} = 0$, the welfare maximizing value from the Planner's problem.[13] Further inspection of this expression yields two key insights: First, average worker quality increases as the minimum wage increases, reducing the participation externality. Second, even under the Hosios Condition, θ is not set optimally by the free entry condition. As a result, policy implementation distorts vacancy creation any time a policy affects wages, which we demonstrate in the following:

$$\int_{\bar{w}}^{\tilde{p}} \frac{p - \bar{w}}{r + \lambda} dF(p) < (1 - \beta) \int_{\bar{w}}^{\tilde{p}} \frac{p - z}{r + \lambda + \beta m(\theta)} dF(p), \forall \tilde{p} > \bar{w}$$

The right hand side shows the surplus accrued by firms when matched with workers with productivity $p \in [\bar{w}, \tilde{p}]$ when all wages are Nash bargained. The left hand side shows the surplus accrued by firms hiring workers on whom the minimum wage binds. Clearly, unless the policy does not affect wages, i.e. $\bar{w} = \tilde{p}$, the minimum wage will distort vacancy creation and affect the search externalities. Balancing this additional margin of impact along with the impact of excluding additional workers from the labor market determines the optimal minimum wage.

Assessing the resolution to this trade-off is integral to decomposing the channels by which the minimum wage operates on welfare. We do this by considering two counterfactual policy experiments that we compare to the outcome of our optimal minimum wage exercise (Table 6.1). For each policy,

[13] Even if it were possible, the inability to control the participation margin would prevent the economy from generically achieving the first best.

we consider an environment in which the Policy Maker is able to directly specify the participation threshold, $\hat{p}$, above which wages are subject to unconstrained Nash bargaining. First, we allow the Policy Maker to optimize over the participation threshold, which she does by solving the problem described in Section 4.4.1. Next, we set the participation threshold equal to the optimal minimum wage, $\hat{p} = \bar{w}^*$. We then use both the laissez-faire and the optimal minimum wage economies as benchmarks to understand how the minimum wage affects each externality.

Comparing these economies allow us to determine the extent to which the minimum wage contributes to welfare by reducing each of the participation and search externalities. First, the change in welfare garnered by moving from the laissez-faire economy to an optimal minimum wage economy yields the cumulative effect of the minimum wage. Second, comparing the laissez-faire economy to the economy in which $\hat{p} = \bar{w}^*$ yields the effect that the optimal minimum wage has on the participation externality. The logic behind this conclusion is that we are removing the impact that the minimum wage has on vacancy creation, outside the impact on participation. The effect of the minimum wage on the search externalities introduces an additional complication. We can calculate the *net* effect that the minimum wage has on welfare from helping balance out the search externalities by comparing the change in welfare moving from the $\hat{p} = \bar{w}^*$ participation economy to our optimal minimum wage economy. However, this understates the impact of the minimum wage on the search externalities.

While quantifying the net effect of the minimum wage on welfare relative to the participation policy is a straightforward calculation, changing the minimum wage affects the participation externality in the process. This interaction is critical to understanding the impact of the minimum wage in our environment. First, moving from an optimal participation policy to a minimum wage dampens the reduction in the participation externality. We can quantify the amount by comparing the $\hat{p}^*$ economy to the $\hat{p} = \bar{w}^*$ economy. The difference in welfare is the reduction in the participation externality forgone by the Policy Maker to further reduce the net negative effect of the search externalities. And as a result, the calculated effect of the minimum wage on search externalities should include this interaction. We report this decomposition in Table 6.2.

This table shows the value of the minimum wage, both in improving the average quality of worker in the market and in distorting the free entry condition so that θ approaches its optimal level. Comparing the third and first rows shows that by increasing the participation threshold, the minimum wage leads to a 0.31pp increase in welfare. The difference between the fourth

Table 6.2: Decomposition of the effect of the minimum wage.

	Unemp. Rate	Non-Part. Rate	θ	Ability Threshold ($)	Welfare Ratio (% of Laissez-faire)
Laissez-faire	0.109	0.117	0.070	6.036	100.000
Optimal Participation	0.105	0.196	0.078	7.125	100.310
Min. Wage Participation ($\hat{p} = \bar{w}^*$)	0.104	0.198	0.078	7.149	100.310
Optimal Min. Wage	0.105	0.198	0.078	7.149	100.317

and third rows shows that in addition to reducing the participation externality, the minimum wage also reduces the cost of the imbalance between the search externalities by 0.007pp. This reduction comes at very little expense from the participation externality: a policy that sets $\hat{p}$ optimally increases welfare less than 0.0005pp more than setting the participation threshold to $\bar{w}$, revealed by the difference between the third and second rows.

This table also highlights the key tension faced by the Policy Maker: above the optimal level of participation, raising the minimum wage trades-off exacerbating the participation externality with balancing out the search externalities. While this interaction is relatively inconsequential in this experiment, our optimal UI experiment will show that this interaction is an important consideration.

6.2.2 The Participation and Search Externalities under Unemployment Insurance

We now turn our attention to unemployment insurance. We first outline the key differences between the minimum wage and UI. Then we consider a similar set of counterfactuals to the ones in the previous section. We decompose the change in welfare due to the imposition of UI described in Section 4.4.3 into the share caused by a reduction in the participation externality and the share caused by reducing the net cost of search externalities.

Unemployment insurance differs from the minimum wage by affecting the flow utility of unemployment for workers of all productivity levels. This directly changes the participation decision of workers, but also affects vacancy creation:

$$a = \frac{m(\theta)}{\theta\,(1 - F(\hat{p}))}(1-\beta)\int_{\hat{p}}^{\bar{p}} \frac{p - (z+b)}{r + \lambda + \beta m(\theta)} dF(p)$$

where b denotes the amount of UI and $\hat{p} = z+b$ is the participation threshold. This expression encodes information about how UI affects the externalities. First, in our baseline model used to find optimal UI, $\hat{p} = z + b$. This means that increasing UI directly improves the pool of participating workers. Second, it decreases the surplus of every match, effectively transferring a larger share of welfare to workers. This means that UI is able to address the imbalance in the search externalities by reducing vacancy creation when the Hosios Condition is not satisfied and $1 - \beta > \eta$.

We use these insights to decompose the effect of UI. As with the minimum wage, we consider two counterfactual economies in which the Policy Maker can directly target the participation threshold. We first restrict the participation threshold to equal that emerging under optimal UI. We allow $\hat{p} = z + b$, but assume that this has no bearing on worker leisure utility. As a result, the participation threshold changes, but the change does not affect wages above the threshold. Next, we first allow the Policy Maker to optimize over the participation threshold. We compare these economies against the laissez-faire and our optimal UI economies.

Like our minimum wage decomposition, this allows us to tease out the impact of UI on the participation and search externalities. As before, moving from the laissez-faire economy to the optimal UI economies tallies the cumulative effect of the policy. Taking the difference between the laissez-faire economy and the $\hat{p} = z + b$ shows the share of this effect caused by a reduction in the participation externality. Then as before, the difference between the $\hat{p} = z + b$ economy and the optimal participation economy is the reduction in the participation externality forgone to reduce the net cost of search externalities. Finally, this difference added to the difference between the $\hat{p} = z + b$ and the optimal UI economies is the change in welfare accrued from reducing the net cost of search externalities. We show this decomposition in Table 6.3.

What this table shows is that unlike the minimum wage, UI operates primarily by reducing the net cost of search externalities. Of the cumulative 0.862pp increase in welfare, relative to the laissez-faire economy, reducing the participation externality only explains about a third (0.252pp, difference between “laissez-faire”and “UI Participation”) of the overall effect. By contrast, UI reduces the net cost of the search externalities enough to cause a 0.61pp increase in welfare (difference between “Optimal UI”and “UI Participation”). The interaction between the search and participation externalities introduced by the changes in UI mutes the overall magnitude of this effect. Increasing UI above the optimal participation threshold aggravated the participation externality and led to a 0.058pp decline in welfare, a siz-

Table 6.3: Decomposition of the effect of unemployment insurance.

	Unemp. Rate	Non-Part. Rate	θ	Ability Threshold ($)	Welfare Ratio (% of Laissez-faire)
Laissez-faire	0.109	0.117	0.070	6.036	100.000
Optimal Participation	0.105	0.196	0.078	7.125	100.310
UI Participation ($\hat{p} = z + b^*$)	0.102	0.231	0.082	7.565	100.252
Optimal UI	0.114	0.231	0.064	7.565	100.862

able impact relative to the minimum wage policy. In the absence of this interaction, the overall effect of UI on the net cost of search externalities would be 0.668pp.

7 Conclusion

In this paper, we explore the participation externality and the role of policy in ameliorating it. In random search models with homogeneous workers, the only source of inefficiency is that which arises from the search externalities. In these environments, imposing the Hosios Rule implements constrained efficiency. In our model, heterogeneity in worker ability leads to a participation externality, in which market entry by low-ability workers depresses vacancy creation and leads a decentralized economy to diverge from the optimum even under the Hosios Rule.

We show that the participation externality has a quantitatively important effect and interacts with other sources of inefficiency. When a Policy Maker optimally implements either UI expansions or the minimum wage, they can partially address the participation externality and do so in concert with addressing the search externalities. UI operates primarily by reducing the net cost of search externalities, while the minimum wage more directly targets the participation externality. However, both do so imprecisely and do not achieve the same welfare gains as a Social Planner could have done. Yet, they still achieve sizable gains over the laissez-faire economy.

Our findings show that the participation externality is a worthy consideration for a policymaker. While our findings do not support a sizable increase in the minimum wage or the generosity of UI, they do indicate that the presence is welfare improving. A richer environment with additional sources of heterogeneity may find even more support for these policies. At the end of the day, however, it is up to policy makers to assess whether the efficiency gains from the imposition of an appropriate policy are worth the social costs associated with employment loss for the lowest ability participants.

A Appendix A: Static model proofs and derivations

A.1 Equation 3.11

From Equation 3.6 we have

$$\begin{aligned}
\frac{d\theta^*}{d\hat{p}} &= \frac{f(\hat{p})\left[\frac{m(\theta^*)}{\theta^*}(1-\beta)(\hat{p}-z)-a\right]}{\left(\frac{m'(\theta^*)\theta^*-m(\theta^*)}{\theta^{*2}}\right)(1-\beta)\int_{\hat{p}}^{\bar{p}}(p-z)dF(p)} \\
&= \frac{\left[\frac{m(\theta^*)}{\theta^*}(1-\beta)(\hat{p}-z)-a\right]}{\left(\frac{m'(\theta^*)\theta^*-m(\theta^*)}{\theta^{*2}}\right)\left(\frac{(1-F(\hat{p}))a\theta^*}{m(\theta^*)}\right)} \\
&= \left(\frac{f(\hat{p})}{1-F(\hat{p})}\right)\left(\frac{a\theta^*-(1-\beta)m(\theta^*)(\hat{p}-z)}{a(1-\eta(\theta^*))}\right)
\end{aligned}$$

Using Equation 3.6 in the numerator this becomes

$$\begin{aligned}
\frac{d\theta^*}{d\hat{p}} &= \left(\frac{f(\hat{p})(1-\beta)m(\theta^*)}{1-F(\hat{p})}\right)\left(\frac{\mathbb{E}_{[p\geq\hat{p}]}(p-z)-(\hat{p}-z)}{a(1-\eta(\theta^*)}\right) \\
&= \frac{(1-\beta)m(\theta^*)f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{[1-F(\hat{p})]\,a(1-\eta(\theta^*))} > 0
\end{aligned}$$

A.2 Equation 3.12

Substituting from Equation 3.9, Equation 3.10 and Equation 3.11 into Equation 3.7 yields

$$\begin{aligned}
\frac{dW}{d\hat{p}} &= \left(\frac{f(\hat{p})}{(1-\beta)(1-\eta(\theta^*))}\right)\left\{\begin{array}{c} [a\theta^*-m(\theta^*)(\hat{p}-z)]\,(1-\beta)(1-\eta(\theta^*))+ \\ [\beta-(1-\eta(\theta^*))]\,[a\theta^*-(1-\beta)m(\theta^*)(\hat{p}-z)] \end{array}\right\} \\
&= \left(\frac{\beta f(\hat{p})}{(1-\beta)(1-\eta(\theta^*))}\right)[a\theta^*\eta(\theta^*)-(1-\beta)m(\theta^*)(\hat{p}-z)] \\
&= \left(\frac{\beta f(\hat{p})m(\theta^*)}{1-\eta(\theta^*)}\right)\left[\eta(\theta^*)\mathbb{E}_{[p\geq\hat{p}]}(p-z)-(\hat{p}-z)\right].
\end{aligned}$$

A.3 Existence of optimal $\hat{p}$

From Equation 3.1 as $\hat{p}$ approaches $\bar{p}$ welfare converges to z. Using Equation 3.6 when $\hat{p} = z$ we have

$$W(\theta^*, z) = z + \beta m(\theta^*) \int_z^{\bar{p}} [p - z]\, dF(p) > z.$$

Let $\theta^*(\hat{p})$ solve Equation 3.6 for given $\hat{p}$. The implicit function theorem tells us that this is continuous in $\hat{p}$. As welfare is increasing in $\hat{p}$ at z, continuity of $W(\theta^*(\hat{p}), \hat{p})$ in $\hat{p}$ implies that there exists some maximal value $\hat{p}_M$ such that for any $\hat{p} > \hat{p}_M$, $W(\theta^*(\hat{p}), \hat{p}) < W(\theta^*(z), z)$. The extreme value theorem then implies that there exists an optimal value of $\hat{p}$ in $[z, \hat{p}_M]$.

Unfortunately, when $F(.)$ has infinite support, $\hat{p}_M$ can be infinite and so then we can only really say the optimal $\hat{p}$ exists on the positive *extended* real line. For any practical purpose, however, as long as we truncate the distribution at any finite $\bar{p}$, $\hat{p}_M$ is finite.

A.4 Equation 3.18

Let

$$\Gamma(\bar{w}) \equiv \frac{1}{(1 - F(\bar{w}))} \left\{ \int_{\bar{w}}^{\tilde{p}} (p - \bar{w}) dF(p) + (1 - \beta) \int_{\tilde{p}}^{\bar{p}} (p - z) dF(p) \right\}$$

Then

$$\frac{d\theta^*}{d\bar{w}} = \frac{-\frac{m(\theta)}{\theta} \frac{d\Gamma(\bar{w})}{d\bar{w}}}{\frac{m'(\theta)\theta - m}{\theta^2} \Gamma(\bar{w})} = \frac{\theta \frac{d\Gamma(\bar{w})}{d\bar{w}}}{[1 - \eta(\theta)]\, \Gamma(\bar{w})}. \tag{A.1}$$

And,

$$\frac{d\Gamma(\bar{w})}{d\bar{w}} = \frac{1}{(1 - F(\bar{w}))^2} \left[\begin{array}{c} \left\{ \frac{1}{\beta}(\tilde{p} - w) f(\tilde{p}) - (F(\tilde{p}) - F(\bar{w})) - \frac{1-\beta}{\beta}(\tilde{p} - z) f(\tilde{p}) \right\} (1 - F(\bar{w})) \\ + f(\bar{w}) \left\{ \int_{\bar{w}}^{\tilde{p}} (p - \bar{w}) dF(p) + (1 - \beta) \int_{\tilde{p}}^{\bar{p}} (p - z) dF(p) \right\} \end{array} \right]$$

Then using Equation 3.13 and substituting back into Equation A.1 we then obtain

$$\begin{aligned} \frac{d\theta^*}{d\bar{w}} = \frac{\theta^*}{1 - \eta(\theta^*)} &\left[\frac{f(\bar{w})}{1 - F(\bar{w})} - \frac{F(\tilde{p}) - F(\bar{w})}{\int_{\bar{w}}^{\tilde{p}} (p - \bar{w}) dF(p) + (1 - \beta) \int_{\tilde{p}}^{\bar{p}} (p - z) dF(p)} \right] \\ &= \frac{f(\bar{w}) a \theta^* - m(\theta^*) [F(\tilde{p}) - F(\bar{w})]}{a [1 - \eta(\theta^*)] [1 - F(\bar{w})]}. \end{aligned}$$

where the final line uses Equation 3.14

A.5 Equation 3.19

Substituting from Equation 3.16, Equation 3.17 and Equation 3.18 into Equation 3.15 and simplifying yields

$$\begin{aligned}\frac{dW}{d\bar{w}} &= f(\bar{w})\left[a\theta - m(\theta)(\bar{w}-z)\right] \\ &\quad + \frac{\left[m'(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - a\right]\left[f(\bar{w})a\theta - m(\theta)(F(\tilde{p}) - F(\bar{w}))\right]}{a\left(1-\eta(\theta^*)\right)}\end{aligned}$$

so,

$$\begin{aligned}\frac{dW}{d\bar{w}} = \frac{f(\bar{w})}{1-\eta(\theta^*)}\left\{m(\theta^*)\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - (\bar{w}-z)\right] - \eta(\theta^*)\left[a\theta^* - m(\theta^*)(\bar{w}-z)\right]\right\} \\ + \frac{m(\theta^*)\left[F(\tilde{p}) - F(\bar{w})\right]\left[a - m'(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z)\right]}{a\left(1-\eta(\theta^*)\right)}.\end{aligned}$$

Now let

$$\Psi \equiv m(\theta^*)\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - (\bar{w}-z)\right] - \eta(\theta^*)\left[a\theta^* - m(\theta^*)(\bar{w}-z)\right]$$

which are the contents of the curly braces. This can be rewritten as

$$\begin{aligned}\Psi = \beta m(\theta^*)\left[\eta(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - (\bar{w}-z)\right] + \eta(\theta^*)\left[(1-\beta)m(\theta^*)\mathbb{E}_{p\geq\bar{w}}(p-z) - a\theta^*\right] \\ + m(\theta^*)(\bar{w}-z)\left[\beta - (1-\eta(\theta^*))\right].\end{aligned}$$

Using Equation 3.14 the second term of this expression becomes

$$\begin{aligned}&\frac{\eta(\theta^*)m(\theta^*)}{1-F(\bar{w})}\left[(1-\beta)\int_{\bar{w}}^{\bar{p}}(p-z)dF(p) - \int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p) - (1-\beta)\int_{\tilde{p}}^{\bar{p}}(p-z)dF(p)\right] \\ =\ &\frac{\eta(\theta^*)m(\theta^*)}{1-F(\bar{w})}\left[(1-\beta)\int_{\bar{w}}^{\tilde{p}}(p-z)dF(p) - \int_{\bar{w}}^{\tilde{p}}(p-\bar{w})dF(p)\right] \\ =\ &\frac{\beta\eta(\theta^*)m(\theta^*)}{1-F(\bar{w})}\int_{\bar{w}}^{\tilde{p}}(\tilde{p}-p)dF(p)\end{aligned}$$

where the final equality uses the definition of $\tilde{p}$, Equation 3.13. Substituting this back into Ψ and then substituting Ψ back into $\frac{dW}{d\bar{w}}$ yields the desired representation, Equation 3.19.

A.6 Equation 3.26

Substituting from Equation 3.23, Equation 3.24 and Equation 3.23 into Equation 3.22 yields

$$\frac{dW}{db} = f(\hat{p})m(\theta^*)\left[(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (\hat{p}-z)\right] + \frac{m(\theta^*)\left[(\eta-(1-\beta))\,a + (1-\beta)m'(\theta)(\hat{p}-z)\right]\left[f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p}) - (1-F(\hat{p}))\right]}{(1-\eta(\theta^*))a}$$

Bringing this over the common denominator of $(1-\eta(\theta^*))a$, dropping arguments, setting $(\hat{p}-z) = b$ and separating out the terms yields a numerator of

$$fm(1-\beta)\mathbf{E}_p(1-\eta)a - fmb(1-\eta)a + m\left[\eta-(1-\beta)\right]af\mathbf{E}_p - m\left[\eta-(1-\beta)\right]a(1-F) + mm'b(1-\beta)m'b\left[f\mathbf{E}_p - (1-F)\right].$$

Here $\mathbf{E}_p \equiv \mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})$. The first 3 terms can be written as

$$fma\left[\eta\beta\mathbf{E}_p - b(1-\eta)\right]$$

Let $E_z \equiv \mathbb{E}_{[p\geq\hat{p}]}(p-z)$ then $E_z = E_p - b$ and

$$\eta\beta E_p - b(1-\eta) = \beta\left[\eta E_z - b\right] - (1-\beta)(1-\eta)b$$

Substitution back into the original expression yields the desired representation.

A.7 Existence of optimal b.

At some point b can get so large that financing it costs more than the total output of the economy. This occurs when

$$b\left(F(z+b) + [1-m(\theta)]\,[1-F(z+b)]\right) > zF(z+b) + \int_{z+b}^{\bar{p}}\left[m(\theta)p + (1-m(\theta))z\right]dF(p),$$

which puts an upper bound b_M on b. As equilibrium welfare is continuous in b, when it is also increasing at $b = 0$, the theorem of the maximum implies that an optimal value of b exists between 0 and b_M.

A.8 Equation 3.27

The lagrangian for the combined policy problem is

$$zF(\bar{w}) + \int_{\bar{w}}^{\bar{p}} [m(\theta)p + (1 - m(\theta))z] \, dF(p) - [1 - F(\bar{w})]a\theta +$$
$$\mu \left[a - \frac{m(\theta^*)}{\theta^* (1 - F(\bar{w}))} \left\{ \int_{\bar{w}}^{\tilde{p}} (p - \bar{w}) dF(p) + (1 - \beta) \int_{\tilde{p}}^{\bar{p}} (p - z - b) dF(p) \right\} \right]$$

The first order condition for θ, which has to hold at all times, boils down to,

$$m'(\theta) \int_{\bar{w}}^{\bar{p}} (p - z) dF(p) - [1 - F(\bar{w})]a + \mu \frac{(1 - \eta(\theta))\, a}{\theta} = 0.$$

So,

$$\mu = -\frac{\theta}{a\,(1 - \eta(\theta))} \left[m'(\theta) \int_{\bar{w}}^{\bar{p}} (p - z) dF(p) - [1 - F(\bar{w})]a \right]$$

Equation 3.27 follows from replacing $[1 - F(\bar{w})]a$ from the free-entry condition.

B Appendix B: Dynamic model proofs and derivations

B.1 Derivation of Equation 4.3 and Equation 4.4

The implied Hamiltonian is,

$$\mathcal{H} = [1 - F(\hat{p}_t) - u_t] \, \mathbb{E}_{[p \geq \hat{p}_t]} (p) + [F(\hat{p}_t) + u_t] \, z - \theta_t u_t a + \mu_t \left[\lambda \left(1 - F(\hat{p}_t) - u_t\right) - m(\theta_t) u_t \right] \quad \text{(B.1)}$$

where μ is a co-state variable. The necessary conditions for an optimum are

$$\frac{\partial \mathcal{H}}{\partial \theta_t} = 0, \quad \frac{\partial \mathcal{H}}{\partial u_t} = r\mu_t - \dot{\mu}_t, \quad \frac{\partial \mathcal{H}}{\partial \hat{p}_t} = 0, \quad \frac{\partial \mathcal{H}}{\partial \mu_t} = \dot{u}_t.$$

In a steady state, $\dot{\mu}_t = \dot{u}_t = 0$, $\theta_t = \theta$, $\hat{p}_t = \hat{p}$, $u_t = u$ for all t. And, after some simplification, the necessary conditions respectively become,

$$\begin{aligned}
a + \mu m'(\theta) &= 0 \\
\mathbb{E}_{[p \geq \hat{p}]} (p - z) + \theta a + \mu \left(r + \lambda + m(\theta)\right) &= 0 \\
\frac{(1 - F(\hat{p}) - u)}{1 - F(\hat{p})} \mathbb{E}_{[p \geq \hat{p}]} (p - \hat{p}) - \mathbb{E}_{[p \geq \hat{p}]} (p - z) - \mu\lambda &= 0 \\
(\lambda + m(\theta))\, u - \lambda \left(1 - F(\hat{p})\right) &= 0.
\end{aligned}$$

Equation 4.3 and Equation 4.4 follow after eliminating u and μ.

B.2 Thick-Market condition

From the derivation of Equation 4.4, welfare is increasing in $\hat{p}$ if and only if LHS of Equation 4.4 is negative. Fixing $\hat{p} = z$ in Equation 4.3 yields the efficient value of θ under that restriction. Then, using Equation 4.3 to substitute out a in Equation 4.4, LHS of Equation 4.4 becomes

$$\frac{\left(\mathbb{E}\left[p|p \geq z\right] - z\right)\lambda m'(\theta)(r - \theta m'(\theta))}{\left(m(\theta) + \lambda\right)\left(r + \lambda + m(\theta) - \theta m'(\theta)\right)}.$$

The result follows because $m'(\theta)\theta = m(\theta)\eta(\theta)$.

B.3 Existence and uniqueness of equilibrium

We start with the binding minimum wage economy as it is the most complex.

B.3.1 Equilibrium with binding minimum wage

Define the RHS of Equation 4.12 as Ψ. We need to look at what happens to Ψ as θ approaches both 0 and ∞. Then, we will consider what happens between the two extremes.

As $\theta \to 0$: $\lim_{\theta \to 0} \tilde{p}(\theta, \bar{w}) = \frac{\bar{w} - (1-\beta)z}{\beta}$ which is finite so under the assumptions made on the matching function $\lim_{\theta \to 0} \Psi(\theta, \bar{w}) = \infty$.

As $\theta \to \infty$: $\lim_{\theta \to \infty} \tilde{p}(\theta, \bar{w}) = \bar{w}$ so $\lim_{\theta \to \infty} \Psi(\theta, \bar{w}) = 0$.

As $a > 0$, the previous results, along with the fact that $\Psi(., \bar{w})$ is continuous, imply existence of equilibrium. Uniqueness follows from the monotonicity of Ψ with respect to θ. Now,

$$\frac{d\Psi}{d\theta} = \frac{\partial \Psi}{\partial \theta} + \frac{\partial \Psi}{\partial \tilde{p}} \frac{\partial \tilde{p}}{\partial \theta}.$$

But, given $\bar{w}$, $\tilde{p}(\theta, \bar{w})$ is the bilaterally efficient value of productivity above which firms negotiate wages with workers rather than pay the minimum wage. So, from the envelope theorem, $\frac{\partial \Psi}{\partial \tilde{p}} = 0$. Then,

$$\frac{\partial \Psi}{\partial \theta} = -\frac{a(1-\eta(\theta))}{\theta} - \frac{\beta(1-\beta)m'(\theta)m(\theta)}{\theta\left[1 - F(\bar{w})\right]\left[r + \lambda + \beta m(\theta)\right]^2} \int_{\tilde{p}}^{\bar{p}} \left[p - z\right] dF(p) < 0. \tag{B.2}$$

B.3.2 Laissez-faire equilibrium

This is a special case of the binding minimum wage equilibrium in which $\hat{p} = z$.

B.3.3 Equilibrium with UI payments

This is a special case of the binding minimum wage equilibrium in which $\bar{w}$ is zero and z is replaced by $z + b$.

B.3.4 Equilibrium with direct control of $\hat{p}$

This is a special case of the binding minimum wage equilibrium in which $\tilde{p} = \bar{w} = \hat{p}$

B.4 General Thick-Market condition

The Hamiltonian associated with choosing $\hat{p}$ remains Equation B.1. But θ^* is now obtained from Equation 4.11. The steady state necessary conditions for an optimal participation threshold of ability are

$$\frac{\partial \mathcal{H}}{\partial u} = r\mu, \quad \frac{d\mathcal{H}}{d\hat{p}} \equiv \frac{\partial \mathcal{H}}{\partial \hat{p}} + \frac{\partial \mathcal{H}}{\partial \theta}\frac{d\theta^*}{d\hat{p}} = 0, \quad \frac{\partial \mathcal{H}}{\partial \mu} = 0.$$

The LHS of second condition represents how welfare depends on $\hat{p}$. The first term is the direct effect and the second is the indirect (or general equilibrium) effect. We are interested in evaluating $\frac{d\mathcal{H}}{d\hat{p}}\Big|_{\hat{p}=z}$. First, notice that the first and third conditions hold for all $\hat{p}$ and θ. So, from the derivation of Equation 4.3 and Equation 4.4 we obtain

$$u = \frac{(1 - F(z))\lambda}{(\lambda + m(\theta^*))} \quad \text{and} \quad \mu = -\frac{\mathbb{E}_{[p\geq z]}(p - z) + a\theta^*}{(r + \lambda + m(\theta^*))}.$$

Now,

$$\frac{\partial \mathcal{H}}{\partial \hat{p}}\bigg|_{\hat{p}=z} = \frac{-f(z)}{1 - F(z)} \left\{ \mathbb{E}_{[p\geq z]}[p - z]u + (1 - F(z))\, \mu\lambda \right\}.$$

Substituting for u and μ yields,

$$\frac{\partial \mathcal{H}}{\partial \hat{p}}\bigg|_{\hat{p}=z} = \frac{f(z)\lambda \left[a\theta^*(\lambda + m(\theta^*)) - r\mathbb{E}_{[p\geq z]}(p - z)\right]}{(\lambda + m(\theta^*))(r + \lambda + m(\theta^*))}.$$

Then, using Equation 4.11 we obtain

$$\frac{\partial \mathcal{H}}{\partial \hat{p}}\bigg|_{\hat{p}=z} = \frac{f(z)\lambda\mathbb{E}_{[p\geq z]}(p - z)\left[m(\theta^*)(1 - \beta) - r\right]}{(\lambda + m(\theta^*))(r + \lambda + \beta m(\theta^*))}.$$

Next,

$$\left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\hat{p}=z} = -u(a + \mu m(\theta^*)).$$

Substituting for u and μ, and using Equation 4.11 we obtain

$$\left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\hat{p}=z} = \frac{a(1-F(z))\lambda\left[\beta-(1-\eta)\right]}{(\lambda+m(\theta^*))(1-\beta)}.$$

To obtain $\frac{d\theta^*}{d\hat{p}}$, let

$$\Gamma(\theta,\hat{p}) \equiv m(\theta)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) - a\theta\left(r+\lambda+\beta m(\theta)\right)$$

so that from the equilibrium condition, Equation 4.11, $\Gamma(\theta^*,\hat{p})=0$. Then

$$\left.\frac{d\theta^*}{d\hat{p}}\right|_{(4.11)} = \frac{-\frac{\partial\Gamma}{\partial\hat{p}}}{\frac{\partial\Gamma}{\partial\theta}}.$$

Now

$$\frac{\partial\Gamma}{\partial\hat{p}} = \frac{m(\theta^*)(1-\beta)f(\hat{p})\mathbb{E}_{[p\geq\hat{p}]}(p-\hat{p})}{1-F(\hat{p})}$$

and

$$\frac{\partial\Gamma}{\partial\theta} = m'(\theta^*)(1-\beta)\mathbb{E}_{[p\geq\hat{p}]}(p-z) - a\left(r+\lambda+\beta\left[m(\theta^*)+\theta^* m'(\theta^*)\right]\right).$$

Using Equation 4.11, we obtain

$$\begin{aligned}\frac{\partial\Gamma}{\partial\theta} &= \frac{m'(\theta^*)a\theta^*\left(r+\lambda+\beta m(\theta^*)\right)}{m(\theta^*)} - a\left(r+\lambda+\beta\left[m(\theta^*)+\theta^* m'(\theta^*)\right]\right)\\ &= -a\left[(1-\eta(\theta^*))(r+\lambda)+\beta m(\theta^*)\right].\end{aligned}$$

So,

$$\left.\frac{d\theta^*}{d\hat{p}}\right|_{\hat{p}=z} = \frac{m(\theta^*)(1-\beta)f(z)\mathbb{E}_{[p\geq z]}(p-z)}{a(1-F(z))\left[(1-\eta(\theta^*))\left(r+\lambda\right)+\beta m(\theta^*)\right]}. \tag{B.3a}$$

Substituting all of these back into $\frac{d\mathcal{H}}{d\hat{p}}$ yields

$$\begin{aligned}\left.\frac{d\mathcal{H}}{d\hat{p}}\right|_{\hat{p}=z} &= \frac{f(z)\lambda\mathbb{E}_{[p\geq z]}(p-z)}{\lambda+m(\theta^*)}\times\\ &\quad\left[\frac{m(\theta^*)(1-\beta)-r}{r+\lambda+\beta m(\theta^*)} + \frac{m(\theta^*)\left[\beta-(1-\eta)\right]}{(1-\eta(\theta^*))\left(r+\lambda\right)+\beta m(\theta^*)}\right]\end{aligned}$$

After bringing the contents of the square brackets over a positive common denominator, the numerator becomes

$$\begin{aligned}&(1-\eta)\left[r+\lambda+m(\theta^*)\right]\left[\eta(\theta^*)m(\theta^*)-r\right]\\ &\qquad+m(\theta^*)\left[\beta-(1-\eta(\theta^*))\right]\left(\left[r+\lambda+m(\theta^*)\right]\eta(\theta^*)-r\right).\end{aligned}$$

B.5 Dependence of $\tilde{p}$ and θ^* on β

First, let

$$X \equiv \int_{\bar{w}}^{\tilde{p}} (p-w)dF(p) \quad \text{and} \quad Y \equiv \int_{\tilde{p}}^{\bar{p}} (p-z)dF(p).$$

Then, we can rewrite Equation 4.12 and Equation 4.13 as

$$\begin{aligned} \mathbb{\Gamma} &\equiv m(\theta^*)\left[\frac{X}{r+\lambda}+(1-\beta)\frac{Y}{r+\lambda+\beta m(\theta^*)}\right]-a\theta^*[1-F(\bar{w})]=0 \\ \Phi &\equiv \beta\left(r+\lambda+m(\theta^*)\right)\tilde{p}-\left(r+\lambda+\beta m(\theta^*)\right)\bar{w}+(1-\beta)(r+\lambda)z=0. \end{aligned}$$

Taking the total derivative we have

$$\begin{pmatrix} \frac{\partial \mathbb{\Gamma}}{\partial \theta^*} & \frac{\partial \mathbb{\Gamma}}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \tilde{p}} \end{pmatrix}\begin{pmatrix} d\theta^* \\ d\tilde{p} \end{pmatrix} = -d\beta \begin{pmatrix} \frac{\partial \mathbb{\Gamma}}{\partial \beta} \\ \frac{\partial \Phi}{\partial \beta} \end{pmatrix}.$$

After dropping the argument in m we obtain

$$\begin{aligned} \frac{\partial \mathbb{\Gamma}}{\partial \theta^*} &= m'\left[\frac{X}{r+\lambda}+(1-\beta)\frac{Y}{r+\lambda+\beta m}\right]-\frac{\beta(1-\beta)m'mY}{(r+\lambda+\beta m)^2}-a[1-F(\bar{w})] \\ &= \left(\frac{\theta^* m'-m}{\theta^*}\right)\left[\frac{X}{r+\lambda}+(1-\beta)\frac{Y}{r+\lambda+\beta m}\right]-\frac{\beta(1-\beta)m'mY}{(r+\lambda+\beta m)^2} \\ &= \left(\frac{\theta^* m'-m}{\theta^*}\right)\left(\frac{X}{r+\lambda}\right)+\frac{(1-\beta)Y\left[(\theta^* m'-m)(r+\lambda)-\beta m^2\right]}{\theta^*(r+\lambda+\beta m)^2}<0 \end{aligned}$$

where the second line uses Equation 4.12. Next,

$$\frac{\partial \mathbb{\Gamma}}{\partial \tilde{p}} = \frac{(\tilde{p}-\bar{w})f(\tilde{p})}{r+\lambda}-\frac{(1-\beta)(\tilde{p}-z)f(\tilde{p})}{r+\lambda+\beta m}$$

which is 0 from Equation 4.13.

$$\frac{\partial \Phi}{\partial \theta^*} = \beta(\tilde{p}-\bar{w})m'(\theta^*)$$

and

$$\frac{\partial \Phi}{\partial \tilde{p}} = \beta(r+\lambda+\beta m(\theta^*)).$$

These tell us that the determinant of the Jacobean above is negative.

$$\begin{aligned}\frac{\partial \Gamma}{\partial \beta} &= -\frac{mY}{r+\lambda+\beta m} - \frac{(1-\beta)m^2Y}{(r+\lambda+\beta m)^2} \\ &= -\frac{mY\,(r+\lambda+m)}{(r+\lambda+\beta m)^2} < 0\end{aligned}$$

and

$$\frac{\partial \Phi}{\partial \beta} = (r+\lambda+m)\,\tilde{p} - m\bar{w} - (r+\lambda)z = \frac{(r+\lambda)(\bar{w}-z)}{\beta} > 0.$$

Cramer's rule tells us that

$$\frac{d\theta^*}{d\beta} = \frac{-\begin{vmatrix} \frac{\partial \Gamma}{\partial \beta} & \frac{\partial \Gamma}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \beta} & \frac{\partial \Phi}{\partial \tilde{p}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Gamma}{\partial \theta^*} & \frac{\partial \Gamma}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \tilde{p}} \end{vmatrix}} = \frac{-\begin{vmatrix} - & 0 \\ + & + \end{vmatrix}}{\begin{vmatrix} - & 0 \\ + & + \end{vmatrix}} < 0.$$

And,

$$\frac{d\tilde{p}}{d\beta} = \frac{-\begin{vmatrix} \frac{\partial \Gamma}{\partial \theta^*} & \frac{\partial \Gamma}{\partial \beta} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \beta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Gamma}{\partial \theta^*} & \frac{\partial \Gamma}{\partial \tilde{p}} \\ \frac{\partial \Phi}{\partial \theta^*} & \frac{\partial \Phi}{\partial \tilde{p}} \end{vmatrix}}.$$

We know that the denominator is negative and the numerator boils down to

$$\frac{(1-\beta)Y(r+\lambda)(\bar{w}-z)\,(m-\theta^* m')}{\beta\theta^*\,(r+\lambda+\beta m)} > 0.$$

B.6 Existence of optimal steady state $\hat{p}$

From Equation 4.1 with $\hat{p}_t = \hat{p}$ for all t, as $\hat{p}$ approaches $\bar{p}$, steady state welfare converges to z. Meanwhile, welfare evaluated at $\hat{p} = z$,

$$W_z = z\,[F(z) + u_z] + [1 - F(z) - u_z]\,\mathbb{E}_{[p \geq z]}\,[p] - a\theta u_z$$

where u_z is the steady state measure of unemployed workers:

$$u_z = \frac{\lambda(1-F(z))}{\lambda + m(\theta)}.$$

From Equation 4.12,

$$a\theta = \frac{m(\theta)(1-\beta)\mathbb{E}_{[p\geq z]}[p-z]}{(r+\lambda+\beta m(\theta))}.$$

These imply

$$W_z = z + \frac{m(\theta)\left[r+\beta\left(\lambda+m(\theta)\right)\right][1-F(z)]\mathbb{E}_{[p\geq z]}[p-z]}{(r+\lambda+\beta m(\theta))\left(\lambda+m(\theta)\right)} > z.$$

Continuity of W in $\hat{p}$ means that there must be a $\hat{p}_m$ such that for all $\hat{p} > \hat{p}_m$, welfare is below W_z. As welfare is increasing in $\hat{p}$ at $\hat{p} = z$, the extreme value theorem then tells us that there exists an optimal value of $\hat{p}$ in $(z, \hat{p}_m)$.

B.7 Welfare analysis for minimum wage

B.7.1 Existence of optimal $\bar{w}$

Follows from existence of optimal $\hat{p}$. Simply replace $\hat{p}$ with $\bar{w}$.

B.7.2 Derivation of equations used to obtain optimal $\bar{w}$.

The relevant Hamiltonian for the optimization problem is identical to that for the Planner (Equation B.1) with $\hat{p}$ replaced by $\bar{w}$. The necessary conditions for an optimum are then Equation 4.12 along with

$$\frac{\partial \mathcal{H}}{\partial u_t} = r\mu_t - \dot{\mu}_t, \quad \frac{\partial \mathcal{H}}{\partial \bar{w}_t} + \frac{\partial \mathcal{H}}{\partial \theta_t}\left.\frac{d\theta_t}{d\bar{w}_t}\right|_{(4.12)} = 0, \quad \frac{\partial \mathcal{H}}{\partial \mu_t} = \dot{u}_t.$$

From these after imposing steady state we obtain

$$\mathbb{E}_{[p\geq \bar{w}]}\left[p\right] + \theta^* a - z + \mu\left(r+\lambda+m(\theta^*)\right) = 0 \tag{B.4}$$

$$\frac{-f(\bar{w})}{1-F(\bar{w})}\left\{\left[1-F(\bar{w})-u\right]\bar{w} + \mathbb{E}_{[p\geq \bar{w}]}\left[p\right]u - \left(1-F(\bar{w})\right)\left(z-\mu\lambda\right)\right\}$$
$$- u\left(a+\mu m'(\theta^*)\right)\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.12)} = 0 \tag{B.5}$$

$$\left(\lambda+m(\theta^*)\right)u - \lambda\left(1-F(\bar{w})\right) = 0 \tag{B.6}$$

where μ is the co-state variable on Equation 4.2. To obtain $\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.12)}$ define the RHS of Equation 4.12) as $\Psi(\theta^*, \bar{w})$ then,

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.12)} = \frac{-\left[\frac{\partial\Psi}{\partial\bar{w}} + \frac{\partial\Psi}{\partial\tilde{p}}\frac{\partial\tilde{p}}{\partial\bar{w}}\right]}{\frac{\partial\Psi}{\partial\theta^*} + \frac{\partial\Psi}{\partial\tilde{p}}\frac{\partial\tilde{p}}{\partial\theta^*}} = -\frac{\frac{\partial\Psi}{\partial\bar{w}}}{\frac{\partial\Psi}{\partial\theta^*}} \tag{B.7}$$

where the final equality follows from the envelope theorem because $\tilde{p}$ is the efficient ability level above which wages are negotiated (i.e. $\frac{\partial\Psi}{\partial\tilde{p}} = 0$).

Now, let $\bar{f} \equiv f(\bar{w})$, $\bar{F} \equiv F(\bar{w})$, $\tilde{F} \equiv F(\tilde{p})$, $\bar{\mathbf{E}} \equiv \mathbb{E}_{[p \geq \bar{w}]}[p]$ and $\tilde{\mathbf{E}} \equiv \mathbb{E}_{[p \geq \tilde{p}]}[p]$. Then, after dropping arguments in the matching function and its derivative, Equation B.4 implies

$$\mu = -\frac{\bar{\mathbf{E}} - z + \theta^* a}{r + \lambda + m}.$$

And, Equation B.6 implies

$$u = \frac{(1 - \bar{F})\lambda}{m + \lambda}.$$

Substituting for u and μ into Equation B.5 yields,

$$\bar{f}\left\{(r + \lambda + m)m\bar{w} + r\lambda\bar{\mathbf{E}} - (\lambda + m)\left[\lambda\theta^* a + (r + m)z\right]\right\} + (1 - \bar{F})\lambda\left\{\left[\bar{\mathbf{E}} - z\right]m' - a(r + \lambda + m - \theta^* m')\right\}\frac{\frac{\partial\Psi}{\partial\bar{w}}}{\frac{\partial\Psi}{\partial\theta^*}} = 0 \quad \text{(B.8)}$$

From Equation 4.12,

$$\frac{\partial\Psi}{\partial\bar{w}} = \frac{(r + \lambda)\bar{f}\theta^* a - m(\tilde{F} - \bar{F})}{(r + \lambda)\theta^*(1 - \bar{F})}$$

and $\frac{\partial\Psi}{\partial\theta^*}$ is obtained from Equation B.2 above.

Then

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.12)} = \frac{(r + \lambda)\bar{f}\theta^* a - m(\tilde{F} - \bar{F})}{(r + \lambda)\theta^*(1 - \bar{F})\left\{\frac{a(1-\eta(\theta^*))}{\theta^*} + \frac{\beta(1-\beta)m'(\theta)m(\theta)}{\theta[1-F(\bar{w})][r+\lambda+\beta m(\theta)]^2}\int_{\tilde{p}}^{\bar{p}}[p - z]\,dF(p)\right\}} \quad \text{(B.9)}$$

To obtain the optimal minimum wage we numerically substitute this into Equation B.8.

B.7.3 General Thick-Market condition applies to minimum wage

As the Hamiltonian for the welfare effects of the minimum wage is identical to that of the Planner (and therefore that for direct $\hat{p}$ adjustments), we know that

$$\left.\frac{\partial\mathcal{H}}{\partial\bar{w}}\right|_{\bar{w}=z} = \left.\frac{\partial\mathcal{H}}{\partial\hat{p}}\right|_{\hat{p}=z}$$

and that

$$\left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\bar{w}=z} = \left.\frac{\partial \mathcal{H}}{\partial \theta}\right|_{\hat{p}=z}.$$

Using Equation 4.12 when $\bar{w} = z$ from Equation B.9 we obtain

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{(4.12)} = \frac{m(\theta^*)(1-\beta)f(z)\mathbb{E}_{[p\geq z]}(p-z)}{a\left[1-F(z)\right]\left[(1-\eta(\theta^*))(r+\lambda)+\beta m(\theta^*)\right]}. \tag{B.10}$$

Now from Equation B.3a and Equation B.10 it is also clear that,

$$\left.\frac{d\theta^*}{d\bar{w}}\right|_{\substack{Equation\ 4.12\\ \bar{w}=z}} = \left.\frac{d\theta^*}{d\hat{p}}\right|_{\substack{Equation\ 4.11\\ \hat{p}=z}}.$$

B.8 Derivation of equations used to obtain optimal b.

Because b is a transfer the relevant Hamiltonian for the optimization problem is identical to that for the Planner (Equation B.1) with $\hat{p}$ replaced by $z+b$. The necessary conditions for an optimum are then equilibrium condition, Equation 4.12, along with

$$\frac{\partial \mathcal{H}}{\partial u_t} = r\mu_t - \dot{\mu}_t, \quad \frac{\partial \mathcal{H}}{\partial \mu_t} = \dot{u}_t \quad \frac{\partial \mathcal{H}}{\partial b_t} + \frac{\partial \mathcal{H}}{\partial \theta_t}\left.\frac{d\theta_t}{db_t}\right|_{(4.14)} = 0.$$

From the first and second optimality conditions, after imposing steady state, we obtain

$$\mu = -\frac{\mathbb{E}_{[p\geq\hat{p}]}[p-z]+\theta a}{r+\lambda+m(\theta)}$$

and

$$u = \frac{(1-F(\hat{p}))\,\lambda}{\lambda+m(\theta)}.$$

Now we construct the third optimality condition element by element. First,

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial b} &= f(\hat{p})\left\{\frac{\left[1-F(\hat{p})-u\right]\mathbb{E}_{[p\geq\hat{p}]}\left(p-\hat{p}\right)}{1-F(\hat{p})} - \mathbb{E}_{[p\geq\hat{p}]}[p-z]-\mu\lambda\right\}\\
\frac{\partial \mathcal{H}}{\partial \theta} &= -u(a+\mu m'(\theta)).
\end{aligned}$$

Now, substitute for μ and u and recognize that

$$\mathbb{E}_{[p\geq\hat{p}]}[p-z] - \mathbb{E}_{[p\geq\hat{p}]}\left(p-\hat{p}\right) = \hat{p}-z.$$

Then, using Equation 4.14 and letting $\hat{f} \equiv f(\hat{p})$, $\hat{F} \equiv F(\hat{p})$, $m \equiv m(\theta)$, and $\hat{\mathbf{E}} \equiv \mathbb{E}_{[p \geq \hat{p}]}(p - \hat{p})$ we obtain

$$\frac{\partial \mathcal{H}}{\partial b} = \frac{\hat{f}\,[m(1-\beta) - r]\,\lambda \hat{\mathbf{E}}}{(\lambda + m)(r + \lambda + \beta m)} - \frac{(r+m)(\hat{p} - z)}{r + \lambda + m}$$

and

$$\frac{\partial \mathcal{H}}{\partial \theta} = \frac{(1-\hat{F})\lambda m \left\{[\beta - (1-\eta)](r+\lambda+m)\,\hat{\mathbf{E}} + \eta\,(r+\lambda+\beta m)(\hat{p} - z)\right\}}{\theta^* (\lambda + m)(r+\lambda+\beta m)(r+\lambda+m)}$$

After rewriting Equation 4.14 as

$$m(1-\beta)\hat{\mathbf{E}} - a\theta^*(r + \lambda + \beta m) = 0 \tag{B.11}$$

we have,

$$\left.\frac{d\theta^*}{db}\right|_{(4.14)} = \frac{-\frac{\partial \text{LHS}(B.11)}{\partial b}}{\frac{\partial \text{LHS}(B.11)}{\partial \theta^*}}$$

where

$$\frac{\partial \text{LHS}(B.11)}{\partial b} = \frac{m(1-\beta)\left[\hat{f}\hat{\mathbf{E}} - (1-\hat{F})\right]}{1-\hat{F}}$$

and

$$\begin{aligned} \frac{\partial \text{LHS}(B.11)}{\partial \theta^*} &= m'(1-\beta)\hat{\mathbf{E}} - a\left(r + \lambda + \beta m + \beta\theta^* m'\right) \\ &= -\left[(1-\eta)(r+\lambda) + \beta m\right] a. \end{aligned}$$

So,

$$\left.\frac{d\theta^*}{db}\right|_{(4.14)} = \frac{m(1-\beta)\left[\hat{f}\hat{\mathbf{E}} - (1-\hat{F})\right]}{a\left(1-\hat{F}\right)\left[(1-\eta)(r+\lambda) + \beta m\right]}. \tag{B.12}$$

To obtain the optimal value of b, we numerically substitute these into the formula

$$\frac{\partial \mathcal{H}}{\partial b} + \frac{\partial \mathcal{H}}{\partial \theta}\left.\frac{d\theta^*}{db}\right|_{(4.14)} = 0$$

and solve for b. Existence of optimal b follows from the logic of that section in the static model.

C Appendix C: Additional Material

C.1 Static model: Planner assignment with type indexed locations

Here the Planner can send workers to matching locations based on their type. The Planner can also control the number of vacancies created at those locations. As workers with $p \leq z$ contribute at least as much to aggregate welfare by not working, the Planner does not create vacancies at locations at which $p < z$. Moreover, as vacancy creation is costly, the match output has to be expected to cover that cost too. Notice that,

$$\lim_{\theta \to 0} \frac{m(\theta)}{\theta} = m'(0) = 1.$$

This means that for the marginal worker where the firm has to have a guaranteed hire this can happen only when $\theta = 0$. So the actual threshold ability above which the Planner creates vacancies is $a + z$. The ratio of vacancies to workers in each active location, $\theta(p) = v(p)/f(p)$. The Planner effectively chooses $\theta(p)$ to maximize aggregate welfare:

$$W = F(z)z + \int_{z+a}^{\bar{p}} \{m(\theta(p))p + [1 - m(\theta(p))]\, z - a\theta(p)\}\, dF(p)$$

This can be solved at each p to obtain $m'(\theta_p(p))(p - z) = a$ for all p in $(a + z, \bar{p}]$.

This allocation can be decentralized in directed search equilibrium as follows. Firms commit vacancies to a market which is indexed by worker ability, p, the wage, w, and market tightness, θ. A worker of type p then solves

$$\max_{\theta,w} m(\theta)w + (1 - m(\theta))z$$
$$\text{s.t.:} \qquad \frac{m(\theta)}{\theta}(p - w) - a = 0$$

The Lagrangian is

$$\mathcal{L} = m(\theta)w + (1 - m(\theta))z + \mu\, [m(\theta)(p - w) - a\theta]$$

and the first order conditions are

$$m'(\theta)\,(w - z) + \mu\,\big[m'(\theta)(p - w) - a\big]$$

$$m(\theta) - \mu m(\theta) = 0$$

from which we obtain

$$m'(\theta^*(p))(p - z) = a$$

To avoid negative expected profits firms do not create vacancies in markets with $p \leq z + a$.

It is important to notice the level of commitment required to support this allocation. Firms not only commit to a wage, they commit to hiring only the appropriate worker type for the market they are operating in. Otherwise, it would be in any worker's interest to enter the market in which their ability equals the wage being paid in that market. Ex post the firm would still hire the worker.

C.2 Static model: Winners and losers

Another way to compare these policies is in terms of who are the winners and who are the losers vis-a-vis laissez-faire. The minimum wage is quite straightforward on this. People with $p \in [0, z]$ are unaffected. Those with $p \in [z, \hat{p}]$ are clearly made worse off. Looking only at Figure 3.1 it looks like those with $p \in [\hat{p}, \tilde{p}]$ see an increase in income but of course those workers' propensity to get a job could be impacted by the change in θ^*. As it happens though, whenever a binding minium wage increases welfare, those workers must be better off – someone has to be and they are the first beneficiaries. Workers with $p > \tilde{p}$ are better off if θ^* increases with $\bar{w}$. From Equation 3.18 it is clear that this happens at least for values of $\bar{w}$ close to z.

Unfortunately, Figure 3.1 is somewhat deceptive when it comes to UI on this matter as it does not reflect the tax, τ, levied on workers. Balanced budget requires that

$$\tau = [1 - m(\theta_b^*)(1 - F(\hat{p}^*))]\, b$$

where θ_b^* is equilibrium market tightness with UI payments, b, in place. As $\tau < b$, people with $p \in [0, z]$ are better off by $m(\theta_b^*)(1 - F(\hat{p}^*))b$. For people with $p \in (z, \hat{p}]$ their expected income changes by

$$m(\theta_b^*)(1 - F(\hat{p}^*))b - m(\theta_0^*)\beta(p - z)$$

where θ_0^* is the equilibrium market tightness under laissez-faire. This obviously depends on how θ^* changes with b but, for low enough values of p in this range, it is positive. The net gain to these folks decreases with p but whether it actually goes negative for $p = \hat{p}^*$ depends on the actual parametric arrangement. For people with $p \in (\hat{p}, \bar{p}]$ their expected income changes by

$$[m(\theta_b^*) - m(\theta_0^*)]\, \beta(p - z) + b\, [1 - F(\hat{p}^*) - \beta]\,.$$

From Equation 3.25 we know that θ^* can decline with b even at $b = 0$ so this effect cannot be signed in general.

From Figure 3.1 we know that market tightness is higher with a minimum wage than with UI that implements the same value of $\hat{p}$. Ultimately, exactly who wins and looses by these policies, though, depends on the particular parametric arrangement. What does emerge from the preceding discussion is that the minimum wage benefits higher productivity workers more than it does the low productivity workers. Even when the minimum wage increases welfare, it is never Pareto improving. Meanwhile UI necessarily benefits the low ability workers and could benefit all workers at modest enough payouts.

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