Efficiency in a search and matching model with participation policy

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Abstract

This note presents a random search and matching model with ex ante heterogeneity in worker productivity. A Social Planner with the power to prevent labor market participation by the least productive is shown to increase economic efficiency. This is true even at the Hosios rule which, under worker homogeneity, implements the constrained efficient allocation.

1 Introduction

Albrecht et al (2010) (henceforth ANV) provide a variant of the Pissarides (2000) search and matching model in which workers are ex ante heterogenous with respect to their productivity. They show that if workers have a participation choice in which labor market participants forego their value of leisure, z, the Hosios (1990) rule (that sets the elasticity of the matching function with respect to vacancies equal to the Nash bargaining power of the firms) no longer implements the constrained efficient allocation. In particular, at the Hosios rule there is excess labor force participation. In their framework, when $z = 0$ there is no opportunity cost of participation and Hosios rule result is restored.

This note points out that compared to when a social planner gets to choose which workers participate, there is too much participation in a market equilibrium at the Hosios rule even when $z = 0$. Low productivity workers do not account of their dampening effect on the market when they decide to participate. Excluding them from the market raises the average productivity of participants and raises the number of vacancies per participant. The resulting increase in employment of the more productive workers more than offsets the loss of output caused by the forced nonparticipation of the less productive workers.

I will obtain the Planner's optimal allocation and show that at the Hosios rule, a government with the ability to control participation can achieve that optimal allocation. At the end I will discuss the extent to which standard labor market policy instruments might be used to implement the Planner's allocation.

2 Model

The framework used here will follow that of ANV with $z = 0$. I will then go on to discuss how the result can be generalized. The model has a single time period. The mass 1 of workers are ex ante heterogeneous in their productivity, y, which is continuously distributed according to the function $F(.)$ on $[0, 1]$. Employed workers of type y receive βy in compensation. The remainder, $(1 - \beta)y$, goes to the firm as profit. There is a large mass of firms that can each create a single vacancy at the cost c. Firms who fail to match, workers who do not participate, and workers who fail to match get a pay-off of zero.

(By comparison, ANV assume that nonparticipants get $z \geq 0$.)

Workers find a job with probability $m(\theta)$ where $m(.)$ is twice differentiable, increasing and concave and θ is the labor market tightness, v/u. Here v is the mass of vacancies and u is the mass of workers who seek employment. Consequently, firms who hold a vacancy meet a worker with probability $m(\theta)/\theta$.

Now let $\tilde{y}$ represent a threshold level of worker ability above which the worker participates in the labor market and below which they do not. A direct implication of ANV is that in a market context in which $\tilde{y}$ is taken as given, the equilibrium market tightness solves

$$\frac{m(\theta)}{\theta}(1-\beta)\mathbb{E}[y|y \geq \tilde{y}] = c \tag{1}$$

where $\mathbb{E}[y|y \geq \tilde{y}]$ represents the conditional expected value of y given $y \geq \tilde{y}$. And, under voluntary participation (with $z = 0$) the equilibrium value of $\tilde{y}$, represented by y^*, is 0. Without any opportunity cost of participation, everyone participates. ANV also show that with voluntary participation, constrained efficiency pertains at the Hosios rule:

$$1 - \beta = \frac{\theta m'(\theta)}{m(\theta)}.$$

The issue explored here is the extent to which a Social Planner would exclude workers from this market given the choice. The measure of welfare is

$$\Omega = m(\theta)[1 - F(\tilde{y})]\mathbb{E}[y|y \geq \tilde{y}] - cv = m(\theta)\gamma(\tilde{y}) - cv$$

where

$$\gamma(\tilde{y}) \equiv \int_{\tilde{y}}^{1} yf(y)dy$$

and $f(.)$ is the density of $F(.)$. Recognizing that $u = 1 - F(\tilde{y})$ we have

$$\theta = \frac{v}{1 - F(\tilde{y})}.$$

So that

$$\Omega = m(\theta)\gamma(\tilde{y}) - c\theta[1 - F(\tilde{y})].$$

The Planner chooses a labor market tightness, θ_p, and productivity threshold, y_p, to to maximize Ω. Taking first-order conditions with respect to θ and $\tilde{y}$ leads to

$$m'(\theta_p)\mathbb{E}[y|y \geq y_p] - c = 0 \tag{2}$$

and

$$m(\theta_p)y_p = c\theta_p. \tag{3}$$

Any interior solution must satisfy equations (2) and (3).

Unfortunately, Ω is not generally concave in θ and y_p. That $\tilde{y} = 0$ is not optimal, though, comes from taking repeated derivatives of Ω with respect to $\tilde{y}$ and evaluating at 0. Thus

$$\frac{\partial^n \Omega}{\partial \tilde{y}^n} = -m(\theta)\left[(n-1)f^{(n-2)}(\tilde{y}) + \tilde{y}f^{(n-1)}(\tilde{y})\right] + c\theta f^{(n-1)}(\tilde{y})$$

where $f^{(n)}(.)$ denotes the nth derivative of the density, $f(.)$. Starting from $n = 1$, if $f(0) > 0$, the result follows. If $f(0) = 0$ then we need to look at $f'(0)$ as the operative element of the next term in the Taylor series expansion of $\partial\Omega/\partial\tilde{y}$ around $\tilde{y} = 0$. Of course, $f'(0)$ may not exist but then, as 0 is the infimum of the support of $F(.)$, it must be the case that $f(0) > 0$. We need only consider the subsequent term of the Taylor series expansion if the previous one is zero. If $f^{(n-2)}(0) = 0$ then$f^{n-1}(0)$ must exist and

$$\left.\frac{\partial^n \Omega}{\partial \tilde{y}^n}\right|_{\tilde{y}=0} = c\theta f^{(n-1)}(0) \geq 0.$$

If $f^{n-1}(0) = 0$ then we consider the next term. The result follows by induction. As welfare cannot be strictly positive with $\tilde{y} = 1$, there exists an internal maximum which, given the smoothness of the functional forms, has to satisfy equations (2) and (3).

To show further that an appropriately empowered government would implement the Planner's solution, notice that the government has the same objective function as the Social Planner but is constrained by the market equilibrium condition (1) in which $\tilde{y}$ is replaced by y_g (the government's choice of the productivity threshold). Effectively substituting θ out of Ω, using (1) and taking the first order condition with respect to y_g yields

$$\frac{\partial\Omega}{\partial\theta}\left.\frac{d\theta}{dy_g}\right|_{(1)}+\frac{\partial\Omega}{\partial y_g}=0.$$

This equality is satisfied whenever the Planner's first order conditions hold.

Now, rearranging equations (2) and (3) we obtain

$$\theta m'(\theta)\mathbb{E}(y|y\geq y_p)=m(\theta)y_p=c\theta.$$

So if the Hosios condition holds (i.e. $\theta m'(\theta)=m(\theta)(1-\beta)$), the government can implement the Planner's solution by picking y_g to solve

$$(1-\beta)\mathbb{E}[y|y\geq y_g]=y_g.$$

Example 1 *Suppose F is uniform on $[0,1]$, $m(\theta)=\theta^{\frac{1}{2}}$, $\beta=\frac{1}{2}$ so that the Hosios condition holds and $c=\frac{1}{3}$. Then, the market equilibrium with voluntary participation, is $y^*=0$, $\theta^*=9/16$ and equilibrium welfare, $\Omega^*=3/16$. The Planner/government allocation is $y_p=1/3$, $\theta_p=1$ and welfare is $2/9$. Excluding the least productive third of the population, therefore, increases welfare by $5/144$ (or about 18.5%).*

3 Extension

Here we consider a generalization of the environment to reintroduce some value from leisure or non-market activity, z. In a departure from the ANV framework, however, all idle workers get z so there is still no opportunity cost of search. However, consistent with ANV I will consider participation in the laissez-faire economy as voluntary under the proviso that workers who are indifferent between participation and sitting out the labor market choose the latter. This behavior could be supported by an infinitesimal search cost. I will consider two specifications for z: $z(y) = \bar{z}$ for all y and $z(y) = \hat{z}y$. Both $\bar{z}$ and $\hat{z}$ will be restricted to being between 0 and 1. I also assume that, in the spirit of Nash bargaining, the workers should receive a share β of the match surplus. So, workers get, $w = \beta(y - z(y)) + z(y)$ and firms get $(1 - \beta)(y - z(y))$.

Economic welfare for a given threshold productivity, $\tilde{y}$, will now be

$$\Omega = \int_0^{\tilde{y}} z(y)f(y)dy + m(\theta)\int_{\tilde{y}}^1 yf(y)dy + (1 - m(\theta))\int_{\tilde{y}}^1 z(y)f(y)dy - cv.$$

So

$$\Omega = \mathbb{E}[z(y)] + m(\theta)\int_{\tilde{y}}^1 (y - z(y))f(y)dy - c\theta[1 - F(\tilde{y})] \tag{4}$$

where $\mathbb{E}[z(y)]$ is the unconditional expectation of $z(y)$. As $\mathbb{E}[z(y)]$ is the level of welfare associated with $\tilde{y} = 1$, we can define $\Gamma \equiv \Omega - \mathbb{E}[z(y)]$ to represent any additional welfare that can be attained through labor market activity. Meanwhile, in a market equilibrium in which workers of type $y < \tilde{y}$ are excluded, market tightness is determined by

$$\frac{m(\theta)}{\theta}(1 - \beta)\mathbb{E}[y - z(y)|y \geq \tilde{y}] = c. \tag{5}$$

First, with $z(y) = \bar{z}$, voluntary participation (under infinitesimal search cost) implies $y^* = \bar{z}$. Now let $s = y - \bar{z}$, equations (4) and (5) become

$$\Gamma = m(\theta)\int_{\tilde{s}}^{1-\bar{z}} sg(s)ds - cv$$

where

$$g(s) = \frac{f(s + \bar{z})}{1 - F(\bar{z})}$$

and

$$\frac{m(\theta)}{\theta}(1 - \beta)E(s|s \geq \tilde{s}) = c. \tag{6}$$

The support of $g(s)$ is $[0, 1-\bar{z}]$. As both $f(.)$ and the upper bound of the support were chosen arbitrarily, maximizing Γ subject to (6) is equivalent to maximizing Ω subject to (1).

Now if $z(y) = \hat{z}y$, voluntary participation implies $y^* = 0$ and equations (4) and (5) become

$$\Gamma = m(\theta)(1-\hat{z}) \int_{\tilde{y}}^{1} yf(y)dy - cv$$

and

$$\frac{m(\theta)}{\theta}(1-\beta)(1-\hat{z})\mathbb{E}[y|y \geq \tilde{y}] = c.$$

Replacing c with $c/(1-\hat{z})$ restores the problem to its original format. These extensions can be combined by letting $z(y) = \bar{z} + \hat{z}(y - \bar{z})$ with $\bar{z} + \hat{z} < 1$.

Example 2 *Continuing with the same parameters as in Example 1, let* $\bar{z} = \hat{z} = \frac{1}{4}$ *so that* $z(y) = (1+y)/4$. *Expected welfare in the absence of any labor market activity,* $\mathbb{E}[z(y)] = 3/8$. *With voluntary participation,* $y^* = \bar{z} = \frac{1}{4}$ *and* $\theta^* = 441/4096$ *which yields* $\Gamma^* = 441/16384$. *Meanwhile the Planner/Government allocation is* $y_p = 5/9$ *and* $\theta_p = \frac{1}{4}$ *so that* $\Gamma_p = 1/27$. *This represents a 37% increase in* Γ *and a 2.5% increase in total welfare,* Ω.

4 Conclusions and future work

This note has shown that the inability of firms to commit to not hiring low ability workers means that too many workers get hired in a market equilibrium even at the Hosios rule with zero value of leisure. When the workers who are indifferent between participation and sitting out the labor market are assumed to choose the latter, the result extends to an environment with strictly positive values of leisure.

To implement the Planner's allocation when the model parameters do not satisfy the Hosios rule, a government has to bring about both the optimal level of vacancy creation and exclude from the market those workers whose productivity is less than the optimal cut-off. The optimal vacancy creation can be implemented by lump-sum taxation of productive matches which is redistributed to the whole population of workers (or vice-versa). More relevant to this note is how a government might bring about the optimal level of market exclusion. A crude approach would be to use a minimum wage. Those with low enough productivity will never get hired and under any search cost will drop out of the market.

References

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