Commitment, advertising and efficiency of two-sided investment in competitive search equilibrium

Adrian Masters*

Department of Economics, University at Albany, SUNY

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Abstract

Competitive search entails both commitment to and advertising of pay-off relevant aspects of market participants. This paper considers incrementally the implications of each in a labor market where both workers and firms invest prior to market entry. A wide range of institutional arrangements are addressed within the same general framework. When the characteristics of jobs or workers are advertised the efficient outcome pertains. Commitment without advertising typically leads to market unravelling: the Diamond paradox. But, whenever wages and human capital are advertised, firms become residual claimants; the private and social returns to investment coincide. Absent wage commitment, the Hosios condition implies efficiency when investments are advertised.

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*1400 Washington Avenue, Room BA109, Albany, New York, 12222, USA. Phone: (518)437-4418, Fax: (518)442-4736, e-mail: amasters@albany.edu

1 Introduction

This paper looks at how labor market matching frictions affect incentives to invest in the quality of jobs and in human capital. It has two main goals. The first, is to examine the extent to which the established efficiency results from competitive search extend to an environment with investment decisions on both sides of the market. The second follows from the recognition that competitive search requires that market participants can both commit to characteristics of the employment relationship, and make that commitment public (through advertising) – the paper examines incrementally the roles of commitment and advertising in shaping outcomes in this extended environment.

For context, consider a labor market with matching frictions and endogenous vacancy creation, but no investment decisions. Moen [1997] showed that competitive search equilibrium generates efficient vacancy creation. Now, if the workers' ability to observe individual firms' wages (which I will call advertising) is removed, Diamond [1971] implies that the equilibrium terms of trade assign the entire match surplus to firms. Workers facing any positive cost of entry will choose autarky. Beyond this, we can also remove the ability to commit to the terms of trade. If the wage is then determined *ex post* by generalized Nash bargaining, the environment becomes that of Pissarides [2000]. As long as both sides get some share of the match surplus, for sufficiently small cost of entry, equilibria other than autarky exist. And, we know that efficient vacancy creation transpires under the Hosios [1990] condition (that equates the elasticity of the matching function with respect to vacancies to the firm's share of the match surplus). The objective in the current paper, therefore, is to examine the extent to which these results pass through to a more general environment with two-sided investment.

The environment considered here, is one of a fixed number of workers (death and birth rates are equal) and a set of firms which can create as many vacancies as they like. Prior to market entry, workers acquire irreversible human capital and firms invest in job specific physical capital. Investments cannot be augmented after match formation so they are fully committed to. Output occurs in any match according to a neoclassical production function which requires both physical and human capital as inputs. Possible characteristics of a worker (resp. job) are human capital investment (resp. human capital requirement), physical capital requirement (resp. physical capital investment) and the wage.[1] Committed to values of characteristics are observed in bilateral matches. When they are advertised they also become public knowledge. Potential partners can then direct their search on the basis of these known characteristics. Extending Moen [1997], when anyone advertises a particular value of a characteristic they create a submarket at that value for that characteristic. In all submarkets, unemployed workers and vacancies find each other according to a constant returns to scale matching function.

The generalization of the Moen [1997] efficiency result is that whenever characteristics of jobs and workers to which they commit are advertised, competitive search provides both sides of the market with the "right" incentives. Thus, when wages, human capital or physical capital choices are made public (by either side of the market) the equilibrium conditions coincide with a Social Planner's optimality conditions for those characteristics.

When characteristics are committed to in private, there is a hidden action moral hazard problem. How this affects the outcome depends on the extent to which private marginal costs and benefits of any choice coincide with social costs and benefits.

Regardless of how investments are determined, if either side of the market can commit to a wage that is not advertised, search frictions ensure that the other side of the market will always accept a wage that is slightly worse for

[1] Symmetrically, the terms of trade could be characterized by a rental payment to the firm. How this plays out will be indicated at various points of the exposition.

them than the efficient one. The private marginal cost of changing the wage is zero while the private marginal benefit is positive. The economy unravels as in Diamond [1971] and the only equilibrium is autarky; workers simply enjoy leisure and firms do not create vacancies.

What happens when investments are not advertised depends on how wages are determined. The two remaining possibilities are that wages are advertised and that they are determined by *ex post* bargaining. If advertised, wages are determined by competitive search. Firms become residual claimants on match output and directly incur all the costs and benefits from their own physical capital investments. It does not matter whether physical capital is advertised or not, firms always receive their marginal product and, given the wage and prevailing level of human capital, they make efficient investments. Consequently, the advertising of wages and human capital choices alone is sufficient to support the efficient allocation in equilibrium. Human capital choices, however, do have to become public knowledge for workers to make the right decisions. Search frictions ensure that when human capital investments are hidden and workers take the wage as given, the marginal benefit from investment is zero while the marginal cost is strictly positive. Only autarky can be supported as an equilibrium.[2]

When wages are determined by Nash bargaining and investment choices are hidden, the model becomes essentially that of Acemoglu [1996] or Masters [1998] within the Pissarides [2000] framework. Consistent with those papers there is a hold-up problem that means both sides of the market underinvest. Taking investments as given, the Hosios [1990] condition generates efficient vacancy creation but there is no bargaining power that is also able to prevent hold-up.

Efficiency on every margin is achieved only when there is directed search

[2] If, instead of a wage, a rental payment to firms is advertised, workers become residual claimants. Human capital can be hidden without loss of efficiency but hidden physical capital leads to autarky.

with respect to both physical and human capital and the terms of trade are determined by the Hosios condition. Even though match output is divided by an *ex post* rule, the fact that either side can create a market at any level of investment means they can use their decisions to attract match partners. Away from the Hosios rule, however, the matching externalities distort returns to investment decisions and vacancy creation.

Papers that incorporate investment in similar environments include Acemoglu and Shimer [1999] and Shi [2001, 2005]. Acemoglu and Shimer [1999] incorporates physical but not human capital investment. It shows that commitment to and advertising of the wage alone is sufficient for both efficient investment and vacancy creation. They investigate the source of efficiency by considering variants of their model that are essentially special cases of those considered in this paper. Because efficiency transpires when workers can observe the capital investment of firms and the Hosios condition holds, they conclude that efficiency in their baseline environment follows from the ability of workers to effectively search across firms according to the level of capital invested. The current paper casts doubt on that interpretation because a similar result pertains regardless of which side does the searching. Instead, efficient physical capital investment occurs because with a wage contract, the firm becomes the residual claimant on output.[3] Shi [2001, 2005] builds on the work of Acemoglu and Shimer by introducing an exogenous distribution of human capital. He shows that efficiency transpires when firms advertise and commit to a wage and a skill level.[4]

This paper does not allow for decisions being advertised without commitment (i.e. cheap talk), nor does it allow for advertisements to be inaccurate. These possibilities are addressed in Menzio [2007]. In his paper, firms com-

[3]See Mailath *et al* [2010] which also recognizes the role of the residual claimant for efficient pricing.

[4]Various papers in the monetary search literature (e.g. Rocheteau and Wright [2005], Masters [2010]) have also shown that competitive search is able to bring about efficiency on multiple decision margins.

mit to a job type they cannot advertise and choose a wage that is advertised but cannot be committed to. He shows that non-autarkic equilibria can be supported by cheap talk because wage announcements signal job type. Other papers that have considered private information in competitive search equilibrium include Guerrieri [2008], Guerrieri *et al* [2010] and Eeckhout and Kircher [2010]. These consider *ex post* adverse selection in that individuals' types remain their private information even after meeting one another (types are typically revealed in equilibrium). By contrast, in the current paper, hidden choices become obvious at bilateral meetings.

The paper is organized as follows. Section 2 describes the physical environment. Section 3 identifies conditions for an efficient allocation. Section 4 describes the solution concept for the market economy models. Section 5 considers market economy outcomes when there is wage commitment. Section 6 looks at outcomes when wages are determined by bilateral bargaining. Section 7 provides an illustrative numerical example.

2 Environment

A continuum, mass 1, of workers exist in continuous time. Longevity is distributed exponentially with parameter δ. To keep the population constant every one who dies is replaced by a new entrant. Workers are risk neutral. Other than that induced by the possibility of death, there is no discounting.[5] New entrant workers can instantaneously acquire human capital, h, at a cost of $c(h)$. The cost function $c(.)$ is strictly increasing and strictly convex with $c(0) = c'(0) = 0$ and $\lim_{h\to\infty} c'(h) = \infty$. Once educated they look for work. Workers value leisure at the rate $b \geq 0$.

There are a large number of firms which can each create any number

[5] Using subjective discount rates that are different from zero complicates the welfare analysis without any qualitative effect on the results. For realistic parameter values even the quantitative effects are small. See Hosios [1990] and Pissarides [2000] for a discussion.

of vacancies. A vacancy is characterized by its set-up cost k which can also be interpreted as the quantity of specific capital invested in the job. From the moment of creation, jobs are subject to destruction shocks at a Poisson arrival rate λ. Other than that induced by job destruction, firms do not discount the future. A match between a worker of type h and job of type k produces $f(k,h)$ units of the consumption good. Here, $f(k,h)$ is homogeneous of degree 1, and increasing in both arguments. It is strictly concave and satisfies the Inada conditions.[6] Employment lasts until either the worker dies or the job is destroyed; there is no on-the-job search.

Matching can occur in any of a large number of potential submarkets. The number of meetings M^j per unit time in submarket j is given by $M^j = M(u_j, v_j)$ where u_j is the mass of workers in the market and v_j is the mass of vacancies. The meeting function, M, is strictly increasing in both arguments, twice differentiable, strictly concave and homogeneous of degree 1. This means that workers in submarket j meet firms at the Poisson arrival rate $m(\theta_j) = M^j/u_j$ where $\theta_j \equiv v_j/u_j$ and $m(.) \equiv M(1,.)$. Vacancies created for that submarket can expect to encounter unemployed workers at the rate $m(\theta_j)/\theta_j$. To ensure existence of non-autarkic solutions, I impose Inada type conditions on $M(.,.)$ so that $\lim_{\theta\to\infty} m'(\theta) = 0$ and $\lim_{\theta\to 0} m'(\theta) = \infty$. I also require that the elasticities of the matching function, $M(.,.)$, be bounded away from zero. The specific requirements on $m(.)$ are that

$$\lim_{\theta\to\infty} \frac{\theta m'(\theta)}{m(\theta)} > 0 \quad \text{and} \quad \lim_{\theta\to 0} \frac{m(\theta) - \theta m'(\theta)}{m(\theta)} > 0.$$

Moving between active submarkets is costless for both vacancies and workers seeking employment. Whenever multiple active submarkets are identical with each other in terms of the market tightness, θ, and the distributions of characteristics of workers and vacancies that can be advertised I will call

[6] Specifically: $x_i = 0$ and $x_j \geq 0$ implies $f(x_1, x_2) = 0$ for $i = 1, 2, j \neq i$; $\lim_{x_i \to 0} \frac{\partial f(x_1,x_2)}{\partial x_i} = \infty$, for $i = 1, 2$ and $x_j > 0$, $j \neq i$; $\lim_{x_i\to\infty} \frac{\partial f(x_1,x_2)}{\partial x_i} = 0$, for $i = 1, 2$ and $x_j > 0$, $j \neq i$.

that one submarket. (This simply avoids the proliferation of allocations that are pay-off equivalent.) Participants are thus able to direct their search according to the publicly observed characteristics of the individuals with whom they would like to match. Once meetings occur, all pay-off relevant characteristics of the other party are revealed so that there is no private information at the level of the individual meeting.

How much workers educate, how much capital is invested in any job, which realized matches are formed, and how the output of any match is distributed between employer and worker depends on the specific institutional arrangements and informational assumptions made. Various arrangements are discussed below but first we need to consider the efficient allocation.

3 Efficiency

The Social Planner weights all firms and every generation of individuals' utility equally in her measure of welfare. Consequently, she does not discount the future and simply chooses the best steady-state for the economy. Because everyone is risk neutral, flow welfare, W, is equal to aggregate flow benefits less flow costs. So,

$$W = (1-u)f(k,h) + ub - \delta c(h) - sk \tag{1}$$

where u is the aggregate unemployment rate and s is the number of vacancies created per unit time. In steady-state the inflow to unemployment is $\delta+\lambda(1-u)$. The outflow is $(m(\theta)+\delta)u$. Thus,

$$u = \frac{\delta+\lambda}{m(\theta)+\delta+\lambda}.$$

A steady-state also implies that the rate of vacancy creation has to equal the rate at which jobs are destroyed. The total number of jobs in the economy is $v+(1-u)$ where $v=\theta u$ is the mass of vacancies. So $s=\lambda(\theta u+1-u)$.

Substitution into equation (1) for s and u and yields

$$W = W(k,h,\theta;b) \equiv \frac{m(\theta)\left[f(k,h)-\delta c(h)-\lambda k\right]+(\delta+\lambda)\left[b-\delta c(h)-\lambda\theta k\right]}{m(\theta)+\delta+\lambda}. \tag{2}$$

The Planner's problem is to maximize W with respect to k, h and θ over the positive orthant. Depending on the size of b, $\{k^*, h^*, \theta^*\} = \{0,0,0\}$ is a potential solution. In that case $W = b$. The objective here is to show that for sufficiently small but positive values of b, a non-autarkic finite solution $\{k^*, h^*, \theta^*\}$ exists and that it is characterized by the first order conditions:

$$\begin{aligned} m(\theta^*)\left[f_1(k,h)-\lambda\right]-\lambda(\delta+\lambda)\theta^* &= 0 \quad (3)\\ m(\theta^*)\left[f_2(k^*,h^*)-\delta c'(h^*)\right]-\delta(\delta+\lambda)c'(h^*) &= 0 \quad (4)\\ m'(\theta^*)[f(k^*,h^*)-b]-\lambda[\delta+\lambda+m(\theta^*)+(1-\theta^*)m'(\theta^*)]k^* &= 0. \quad (5) \end{aligned}$$

A complication is that without further restrictions on the functional forms of f, m and c, $W(k,h,\theta;b)$ is not necessarily concave.

Lemma 1 *Consider a system of the form*

$$\begin{aligned} f_1(k,h)-\lambda-C(\theta) &= 0 \quad (6)\\ f_2(k,h)-\delta c'(h)-\delta c'(h)D(\theta) &= 0 \quad (7) \end{aligned}$$

where $C(.)$ and $D(.)$ are positive continuous functions such that

$$\begin{aligned} \lim_{\theta\to 0}\{C(\theta),D(\theta)\} &= \{0,\infty\}\\ \lim_{\theta\to \infty}\{C(\theta),D(\theta)\} &= \{\infty,0\}. \end{aligned}$$

Then there is a unique solution, $\{\hat{k}(\theta),\hat{h}(\theta)\} \in \mathbb{R}^2_{++}$, to the system {(6),(7)} which has the following properties:

$$\lim_{\theta\to\infty}\{\hat{k}(\theta),\hat{h}(\theta)\} = \lim_{\theta\to 0}\{\hat{k}(\theta),\hat{h}(\theta)\} = \{0,0\}$$

$$\lim_{\theta\to 0}\frac{f(\hat{k}(\theta),\hat{h}(\theta))}{\hat{k}(\theta)} > \lambda.$$

Proof. See Appendix. ■

Consider equations (3) and (4). After dividing both by $m(\theta^*)$, Lemma 1 clearly applies. Let $\{k(^*(\theta), h^*(\theta)\}$ be the implied solution. It is simple to verify that W is strictly concave with respect to k and h so $\{k^*(\theta), h^*(\theta)\}$ also represents an optimum for a given value of $\theta > 0$. Any global maximizer, $\{k^*, h^*, \theta^*\}$ of W must also have the property that $\{k^*, h^*\} = \{k^*(\theta^*), h^*(\theta^*)\}$. Otherwise, holding $\theta = \theta^*$ one could increase W.

Claim 1 $W(k, h, \theta; 0)$ *attains a global maximum on the interior of the positive orthant of* $\mathbb{R}^3$.

Proof. See Appendix. ■

Now, if $\{k_0^*, h_0^*, \theta_0^*\} \in \arg\max W(k, h, \theta; 0)$ and $W_0 \equiv W(k_0^*, h_0^*, \theta_0^*; 0)$ then a non-autarkic finite solution to the Planner's problem exists for any $b < W_0$. This is because the Planner could always pick $\{k, h, \theta\} = \{k_0^*, h_0^*, \theta_0^*\}$ and, from equation (2),

$$W(k_0^*, h_0^*, \theta_0^*; b) > W_0 > b = W(0, 0, 0; b).$$

But, from Lemma 1 and equation(2),

$$\lim_{\theta \to \infty} W(k^*(\theta), h^*(\theta), \theta; b) < b.$$

The foregoing establishes that for b sufficiently small, a solution $\{k^*, h^*, \theta^*\}$, to the Planner's problem exists in the interior of the positive orthant. When b is large the solution is $\{k^*, h^*, \theta^*\} = \{0, 0, 0\}$ and $W = b$. As the first order conditions, (3), (4) and (5), are necessary for an interior optimum, existence of an interior optimum implies the existence of a solution to those conditions. Of course, there may be other solutions to the first order conditions but whenever the solution is unique (and $b < W_0$) it corresponds to the solution of the planner's problem. It should be noted that b has no direct impact on the results. It is included to show robustness to this common component of matching models.

4 Competitive Search Equilibrium

This section of the paper develops the solution concept for the set of decentralized economies that will be considered. The central concept will be an adaptation of symmetric competitive search as in Moen (1997). Implementation, however, requires that we specify which (if any) side of the market is able to commit to choices they make with respect to which match characteristics. Whether or not those commitments are advertised also has to be specified. Such a specification will be referred to as an institutional arrangement.

4.1 Institutional Arrangements

Firms decide on how many vacancies to create, the associated capital stock, k, the wage, w, to pay to anyone hired, and the required human capital h for the job. Workers decide on their own level of education, h, the required wage, w, and the required level of physical capital associated with an acceptable job, k.

Definition 1 *An institutional arrangement specifies:*

- *the decisions to which each individual is committed*
- *whether or not decisions made with commitment are advertised (i.e. become public knowledge)*

Choices to which individuals cannot commit are vacuous; the realized values are determined at the point of match formation. Firms' and workers' capital investments are fully committed to and cannot be further augmented once matching has occurred.

This general framework allows for a large number of possible institutional arrangements. To avoid equilibrium indeterminacy arising from conflicting commitments, I rule out institutional arrangements that permit both sides of the market to advertise decisions made with respect to the same variable and arrangements in which both sides commit to a variable neither can advertise. I also allow at most one side of the market to make wage commitments.

These restrictions mean that with respect to the wage: either side can advertise, either side can commit to a value without advertising, or no side can commit to a value. For capital: either side can advertise a level (or requirement) for either type of investment but only firms can commit to physical capital without advertising and only workers can commit to human capital without advertising.

4.2 Allocations

Individuals' optimal strategies maximize their expected utility while taking institutions and the strategies of everyone else as given.[7] So, for instance, if a firm deviates from equilibrium behavior with respect to a characteristic that is advertised by firms, the deviation sets up a market to which workers will be immediately attracted. Workers will enter the deviant's market until they are indifferent between entering that market and remaining in the market specified by the equilibrium. If a firm deviates from equilibrium behavior with respect to a decision that is not advertised, no new market is made - the firm's ability to attract workers is unaffected by the deviation.

Definition 2 *A symmetric steady-state allocation is a tuple,* $\{k, h, w, \theta\}$ *such*

[7] Formally, an individual's strategy is

1. a mapping from all possible distributions of match attributes, market tightnesses and contingent acceptance decisions observed at birth, into a zero/one market entry decision and a choice of characteristics.
2. a mapping from all possible distributions of attributes, market tightnesses and contingent acceptance decisions observed once a potential trading partner has been met, into a zero/one match acceptance decision.

For workers market entry amounts to whether or not to seek work. For firms it is vacancy creation.

that all firms invest k, all workers invest h and receive payment w when hired and there is unique active market in which the ratio of vacancies to job seekers is θ.

In any symmetric steady-state allocation, $\{k, h, w, \theta\}$, in which all offers to match are accepted we have

$$\lambda V_v = \frac{m(\theta)}{\theta}[V_j - V_v] \tag{8}$$

$$\lambda V_j = f(k,h) - w - \delta[V_j - V_v] \tag{9}$$

where V_v is the value to holding the vacancy open and V_j is the value to the filled job. If V_c is the value to creating a vacancy in that market then $V_c = -k + V_v$. If workers do not accept offers to match then $V_v = 0$.

Similarly, for workers,

$$\delta V_u = b + m(\theta)[V_e - V_u] \tag{10}$$

$$\delta V_e = w + \lambda[V_u - V_e] \tag{11}$$

where V_u is the value to unemployment and V_e is the value to employment. Let V_b represent the equilibrium value to being born into this economy. Then $V_b = \max\{b/\delta, V_u - c(h)\}$. If firms do not accept offers to match then $V_u = b/\delta$.

Under the presumption workers enter the market and that all meetings lead to match formation we obtain

$$\begin{aligned}
V_u &= V_u(w,\theta) \equiv \frac{m(\theta)w + (\delta+\lambda)b}{\delta(\delta+\lambda+m(\theta))} \\
V_e &= V_e(w,\theta) \equiv \frac{[\delta+m(\theta)]w + \lambda b}{\delta(\delta+\lambda+m(\theta))} \\
V_b &= V_b(h,w,\theta) \equiv V_u(w,\theta) - c(h) \\
V_v &= V_v(k,h,w,\theta) \equiv \frac{m(\theta)[f(k,h)-w]}{\lambda[(\delta+\lambda)\theta + m(\theta)]} \\
V_j &= V_j(k,h,w,\theta) \equiv \frac{[\lambda\theta + m(\theta)][f(k,h)-w]}{\lambda[(\delta+\lambda)\theta + m(\theta)]} \\
V_c &= V_c(k,h,w,\theta) \equiv V_v(k,h,w,\theta) - k.
\end{aligned} \tag{12}$$

To obtain a formal definition of equilibrium we need to specify the problems faced by individuals in the economy.

4.2.1 The Firm's problem

Let $\{k^*, h^*, w^*, \theta^*\}$ be a symmetric steady-state allocation to which everyone else in the economy adheres and under which every firm/worker meeting leads to match formation. The individual firm will solve

$$\max_{k_f, h_f, w_f, \theta_f} V_c(\tilde{k}_f, \tilde{h}_f, \tilde{w}_f, \theta_f) \tag{P1}$$

subject to:

$$\text{worker indifference: } V_b(\hat{h}_f, \hat{w}_f, \theta_f) = V_b(h^*, w^*, \theta^*)$$

$$\text{worker acceptance: } V_e(\tilde{w}_f, \theta_f) \geq V_u(w^*, \theta^*).$$

What the variables which carry the tildes ($\tilde{}$) and carats ($\hat{}$) will be equal to depends on specific institutional arrangement:

- If physical capital is advertised by firms or if it is hidden then $\tilde{k}_f = k_f$. If it is advertised by workers, $\tilde{k}_f = k^*$.
- If firms' human capital requirements are advertised, $\tilde{h}_f = \hat{h}_f = h_f$. If workers advertise their human capital investment, or if it is hidden, $\tilde{h}_f = \hat{h}_f = h^*$.
- If the wage is advertised by firms, $\tilde{w}_f = \hat{w}_f = w_f$. If it is a hidden choice of firms workers will have insufficient information on which to direct their search so $\hat{w}_f = w^*$ but $\tilde{w}_f = w_f$. If firms cannot commit to the wage then $\tilde{w}_f = \hat{w}_f = w^*$.

4.2.2 Worker's problem

Given everyone else follows the symmetric steady-state allocation $\{k^*, h^*, w^*, \theta^*\}$, individual workers solve

$$\max_{k_w,h_w,w_w,\theta_w} V_b(\tilde{h}_w, \tilde{w}_w, \theta_w) \tag{P2}$$

$$\begin{aligned}
\text{subject to} &: \\
\text{firm indifference} &: \quad V_c(\hat{k}_w, \hat{h}_w, \hat{w}_w, \theta_w) = V_c(k^*, h^*, w^*, \theta^*) \\
\text{firm acceptance} &: \quad V_j(\tilde{k}_w, \tilde{h}_w, \tilde{w}_w, \theta_w) \geq V_v(k^*, h^*, w^*, \theta^*).
\end{aligned}$$

- If workers' physical capital requirement is advertised, $\tilde{k}_w = \hat{k}_w = k_w$. If it is advertised by firms or chosen with commitment by firms then $\tilde{k}_w = \hat{k}_w = k^*$.

- If human capital is advertised by workers, $\tilde{h}_w = \hat{h}_w = h_w$. If it is advertised by firms, $\tilde{h}_w = \hat{h}_w = h^*$. If its choice (by workers) is hidden, firms cannot direct their search on that basis so $\hat{h}_w = h^*$ but $\tilde{h}_w = h_w$.

- If the wage is advertised by workers, $\tilde{w}_w = \hat{w}_w = w_w$. If it is chosen with commitment alone by workers, firms will not be able to direct their search based on wages so $\hat{w}_w = w^*$ but $\tilde{w}_w = w_w$. If workers cannot commit to the wage, $\tilde{w}_w = \hat{w}_w = w^*$.

4.3 Equilibrium

Definition 3 *A (free entry) competitive search equilibrium is a symmetric steady-state allocation, $\{k^*, h^*, w^*, \theta^*\}$, such that when everyone else conforms to it, it solves both the firms' problem (P1) and the workers' problem (P2) and, $V_c(k^*, h^*, w^*, \theta^*) = 0$.*

In general, equilibrium requires that there are no profitable deviations on either side of the market. For a characterization of equilibrium we need to find an allocation that simultaneously solves the firm's and the worker's problems. Notice though, that whenever one side of the market chooses a hidden attribute with commitment the other side of the market has insufficient information on which to direct their search and the indifference constraint becomes moot.

Assuming that when workers are indifferent between market entry and simply enjoying leisure they take the latter option, equations (12) imply that in any non-autarkic equilibrium, $w^* > b$ and neither the firm's nor the worker's acceptance condition binds.

4.4 Market equivalence

In any arrangement in which both sides of the market have sufficient information to direct their search, both indifference constraints apply. Let the Lagrange multiplier on the worker indifference constraint in the firm's problem, (P1), be μ_f and Lagrange multiplier on the firm indifference constraint in the worker's problem, (P2), be μ_w.

The first order condition with respect to θ_f problem (P1) is

$$\frac{\partial V_c}{\partial \theta_f} - \mu_f \frac{\partial V_b}{\partial \theta_f} = 0$$

while the first order condition with respect to θ_w problem (P2) is

$$\frac{\partial V_b}{\partial \theta_w} - \mu_w \frac{\partial V_c}{\partial \theta_w} = 0.$$

In any arrangement in which both indifference constraints bind, a non-autarkic equilibrium will therefore imply $\mu_f = 1/\mu_w$. Problem (P1) and problem (P2) are equivalent. In these cases, it will not matter which side of the market advertises which match attribute.[8] In the sequel, I will refer to this result as "Market Equivalence".

[8] Rogerson *et al* [2005] point out that this is true with respect to the wage level in the standard directed and competitive search environments.

5 Institutional arrangements with wage commitment

We are now in a position to ascertain the nature of equilibrium for each of the permissible institutional arrangements. To do so, it is helpful to divide them into two classes. Those that involve wage commitment are considered here. Those that rely on *ex post* Nash bargaining for the determination of the terms of trade are considered in Section 6. Although there are a large number of institutional arrangements within each class, we will be able to invoke Market Equivalence to reduce the extent of the analysis.

5.1 Transparency

Under full transparency all the features of jobs and workers that are salient to match formation are public knowledge. The specific arrangement considered here is that firms advertise their physical capital input, the wage, and a human capital requirement for the job. Workers simply invest in human capital and direct their search accordingly. Market equivalence implies that the same allocation would arise if any or all of the following were true:

1. instead of firms advertising an educational requirement, workers advertise their level of educational attainment,

2. instead of firms, workers commit to a wage and advertise it,

3. instead of firms advertising physical capital, workers advertise a physical capital requirement.

For this particular arrangement we need only solve Problem (P1) which becomes,

$$\{k^*, h^*, w^*, \theta^*\} = \arg \max_{k_f, h_f, w_f, \theta_f} V_c(k_f, h_f, w_f, \theta_f)$$
$$\text{subject to:} \qquad V_b(h_f, w_f, \theta_f) = V_b(h^*, w^*, \theta^*) \tag{13}$$
$$V_e(w_f, \theta_f) \geq V_u(w^*, \theta^*).$$

Ignore, for now, the worker acceptance condition so the appropriate Lagrangian (after dropping the subscripts) is

$$\begin{aligned} \mathcal{L} &= \frac{m(\theta)\,[f(k,h) - w]}{\lambda[(\delta+\lambda)\theta + m(\theta)]} - k \\ &\quad -\mu_f \left[(V_b^* + c(h))\, \delta(\delta + \lambda + m(\theta)) - (\delta+\lambda)b - m(\theta)w \right] \end{aligned}$$

where $V_b^* \equiv V_b(h^*, w^*, \theta^*)$. The implied first order conditions are:

$$\begin{aligned} k &: \frac{m(\theta) f_1(k,h)}{\lambda[(\delta+\lambda)\theta + m(\theta)]} - 1 = 0 \\ h &: \frac{m(\theta) f_2(k,h)}{\lambda[(\delta+\lambda)\theta + m(\theta)]} - \mu_f \delta\, [\delta + \lambda + m(\theta)]\, c'(h) = 0 \\ w &: \frac{-m(\theta)}{\lambda[(\delta+\lambda)\theta + m(\theta)]} + \mu_f m(\theta) = 0 \\ \theta &: \frac{[f(k,h) - w]\,(\delta+\lambda)(\theta m'(\theta) - m(\theta))}{\lambda[(\delta+\lambda)\theta + m(\theta)]^2} - \mu_f \left([V_b^* + c(h)]\, \delta - w \right) m'(\theta) = 0. \end{aligned}$$

To proceed, use the wage equation to solve for μ_f and substitute it into the remaining equations. Then substitute for V_b^* from equation (12). The implied necessary conditions for an equilibrium, $\{k^*, h^*, w^*, \theta^*\}$ are

$$m(\theta^*) f_1(k^*, h^*) - \lambda[(\delta+\lambda)\theta^* + m(\theta^*)] = 0 \tag{14}$$
$$m(\theta^*) f_2(k^*, h^*) - \delta[\delta + \lambda + m(\theta^*)] c'(h^*) = 0 \tag{15}$$
$$m(\theta^*) m'(\theta^*)(w^* - b) - \lambda[m(\theta^*) - \theta^* m'(\theta^*)][\delta + \lambda + m(\theta^*)] k^* = 0 \tag{16}$$
$$m(\theta^*)[f(k^*, h^*) - w^*] - \lambda[(\delta+\lambda)\theta^* + m(\theta^*)] k^* = 0 \tag{17}$$

where equation (17) comes from the competitive entry condition, $V_c = 0$.

It is immediate from (14) and (17) that firms receive the marginal product on their investments. Linear homogeneity of f, implies the same is true for workers. Eliminating w^* from (16) and (17) yields

$$m'(\theta^*)[f(k^*, h^*) - b] - \lambda[\delta + \lambda + m(\theta^*) + (1 - \theta^*)m'(\theta^*)]k^* = 0. \qquad (18)$$

The market equilibrium under transparency and the Planner's model share the same characterization (equations (14), (15), (18) are identical to (3), (4), (5)). For small enough b, a non-autarkic equilibrium exists and the worker acceptance condition does not bind.[9] Thus, if the right commitments can be made and those commitments can be made public, search can overcome the hold up problems associated with two-sided investments as well as overcome the matching externalities.

Notice that all the firm really needs to specify is a minimum human capital requirement. Workers would never educate themselves beyond the minimum requirement of the job. This a consequence of the bilateral nature of matching within the competitive search framework. Moen [1999] provides a model in which firms potentially meet batches of workers at a time. In that case workers tend to become over-educated in an attempt to stand out from the crowd.

5.2 Hidden wages

Claim 2 *When either workers or firms commit to wages which are not advertised, the only equilibrium is autarky.*

[9] Strictly speaking this statement does not follow from the foregoing. We do know that a solution to the Planner's problem exists and has to solve the system of equations, (14), (15) and (18). We have not shown that this system represents a solution to problem (13); it might be a saddle point for instance. Market equivalence tells us, however, that problem (13) is identical to maximizing V_b subject to $V_c = 0$. Using equations (12) the wage can be eliminated from this problem which transforms it precisely into the Planner's problem.

Proof. From the definition of equilibrium we know that if wages are hidden, the relevant indifference conditions do not bind because the side which does not set the wage has no basis for directing their search. Fix a candidate non-autarkic equilibrium, $\{k^*, h^*, w^*, \theta^*\}$ and consider what happens when firms choose wages. By definition, $w^* > b$. Offering a wage slightly below w^* would increase a firm's profits while leaving its matching rate unaffected. The firm would choose the lowest wage acceptable to workers, $w = \delta V_u(w^*, \theta^*) < w^*$. Similarly if workers set wages, as $V_j(k^*, h^*, w^*, \theta^*) > V_v(k^*, h^*, w^*, \theta^*)$ any worker who could secretly commit to a wage would set that wage higher than w^*. ■

That is, when either side of the market gets to commit to a wage that does not become public knowledge, the Diamond [1971] result applies.

5.3 Hidden Investments

First, it should be noted that whenever there is commitment with full transparency with respect to the wage, whether the physical capital investment of the firms is advertised or not is moot. Because V_b does not depend on k at all, the firm becomes the residual claimant and can fully internalize the value of its own investment. Market Equivalence applies and decentralizing the efficient allocation requires only that w and h be advertised.

Remark 1 *This result highlights an asymmetry in the model as currently formulated. If, instead, the terms of trade were determined by a rental rate, r, paid to firms then the worker would be the residual claimant - efficiency would pertain whenever r and k were advertised.*

With respect to hidden human capital investment, however, the Diamond Paradox result applies:

Claim 3 *When human capital investment is hidden the only equilibrium is autarky.*

Proof. Fix a non-autarkic equilibrium, $\{k^*, h^*, w^*, \theta^*\}$. The firm indifference condition in the workers problem, (P2), does not apply because they have insufficient information to accurately direct their search. Workers choose h subject only to firm acceptance, $V_j = V_v$. As $V_v = k$, firms will be willing to hire any worker with $h \geq h_r$ where h_r solves $V_j = k$. From equations (12), this means

$$\frac{[\lambda\theta^* + m(\theta^*)]\,[f(k^*, h_r) - w^*]}{\lambda[(\delta+\lambda)\theta^* + m(\theta^*)]} = k^*$$

But for $\{k^*, h^*, w^*, \theta^*\}$ to be an equilibrium, $V_v = k$ too. So that

$$\frac{m(\theta^*)\,[f(k^*, h^*) - w^*]}{\lambda[(\delta+\lambda)\theta^* + m(\theta^*)]} = k^*.$$

As $\lambda\theta^* > 0$ and $f(k^*, h)$ is increasing in h, this means $h_r < h^*$. As education is costly, workers will all choose $h = h_r$ so that h^* cannot be supported as an equilibrium. Moreover, firms, faced with the prospect of zero post-match profit, will not create vacancies at all. ■

Commitment without advertising leads to a hidden action moral hazard problem. The severity of the implied distortion is reflected in the gap between the private marginal benefit and the private marginal cost of changing that action at the efficient allocation. When the deciding party is a residual claimant, the gap is zero and private choices coincide with socially optimal choices. When the party with the power to decide is not a residual claimant, wage commitment means that the wage or investment level can be changed at zero cost while the benefit to the change is strictly positive. The next section looks at *ex post* bargaining over the wage which leads to a division of the match surplus. In that case we will see that because an individual's investment increases match output, the marginal cost of adjustment is positive. This prevents the complete unravelling of equilibrium.

6 Institutional arrangements without wage commitment

6.1 Bargaining

When neither side can commit to a wage, the terms of trade are determined by generalized Nash bargaining in which $\beta \in (0,1)$ represents the bargaining power of the worker. The formulae for the asset values of each state given in equations (12) are still valid. However, individuals will now take into account the dependence of the wage on any of their own choices with respect to capital investments or requirements and, on the choices made by everyone else. As such, the formulation of problems (P1) and (P2) should now reflect this dependence. For this section, though, it is simpler to address each set of distinct institutional arrangements in turn.

For the purpose of negotiations the continuation values of counterparties, V_u and V_v are taken as given. Nash bargaining (see Pissarides [2000]) implies,

$$V_e - V_u = \beta \left(V_j - V_v + V_e - V_u \right). \tag{19}$$

As long as the match surplus (the right hand side of equation (19)) is positive, there will be match formation and the values of V_j and V_e can be obtained from equations (9) and (11) respectively. Thus,

$$w = \delta V_u + \beta \left[f(k,h) - \lambda V_v - \delta V_u \right]. \tag{20}$$

Eliminating V_j from equations (8) and (9), substituting for w and solving for V_v yields

$$V_v^B = \frac{(1-\beta)m(\theta)[f(k,h) - \delta V_u]}{\lambda[(\delta+\lambda)\theta + (1-\beta)m(\theta)]} \tag{21}$$

where the superscript B indicates that wage formation is by bargaining. Similarly eliminating V_e from equations (10) and (11), substituting for w and solving for V_u yields

$$V_u^B = \frac{\beta m(\theta)[f(k,h) - \lambda V_v] + [\delta+\lambda]b}{\delta[\delta+\lambda+\beta m(\theta)]}. \tag{22}$$

Variables V_v^B and V_u^B represent respectively the value to holding a vacancy, and the value to unemployment when wages are determined by bargaining and the continuation values, V_u and V_v, of the counterparties are taken as given.

Of course, in equilibrium, $V_v^B = V_v$ and $V_u^B = V_u$. Solving equations (21) and (22) implies

$$V_v^B(k^*, h^*, \theta^*) \equiv \frac{(1-\beta)m(\theta^*)[f(k^*, h^*) - b]}{\lambda[(\delta+\lambda)\theta^* + (1-\beta+\theta^*\beta)m(\theta^*)]} \tag{23}$$

$$V_u^B(k^*, h^*, \theta^*) \equiv \frac{\theta^*\beta m(\theta^*)[f(k^*, h^*) - b]}{\delta[(\delta+\lambda)\theta^* + (1-\beta+\theta^*\beta)m(\theta^*)]} + \frac{b}{\delta}. \tag{24}$$

Competitive entry of vacancies means $V_v^B(k^*, h^*, \theta^*) - k^* = 0$ which implies,

$$(1-\beta)m(\theta^*)[f(k^*, h^*) - b] - \lambda\left[m(\theta^*)(1-\beta+\beta\theta^*) + (\delta+\lambda)\theta^*\right]k^* = 0. \tag{25}$$

It is well known that in the basic labor market environment discussed in the second paragraph of the introduction, hidden wage commitment is indistinguishable from *ex post* bargaining in which one side of the market is given all the bargaining power. There is, however, a timing distinction between to the two approaches to modelling wage formation which potentially has relevance here. Hidden wage commitments do not respond to the specific types of individual encountered whereas take-it-or-leave-it offers do. As it happens, equilibrium outcomes are identical but this may not have been obvious *a priori.* We are now in a position to analyze specific institutional arrangements with bargaining.

6.2 Transparency

Here, firms commit to and advertise their physical capital stock and a human capital requirement. As the wage is determined *ex post*, both sides of the market have sufficient information to direct their search and Market Equivalence applies. This means that either or both of the following would yield the same equilibrium conditions,

1. rather than firms advertising an educational requirement, workers advertise their level of human capital investment,

2. rather than firms advertising their physical capital investment, workers advertise a physical capital requirement.

Because firms' choices with respect to their capital stock and educational requirement are public knowledge, workers will respond to any deviation a firm makes from equilibrium behavior. Market tightness will adjust and the continuation values for workers and firms in any deviant's submarket will be determined within that submarket. The appropriate equations for V_v^B and V_u^B are therefore (23) and (24). Of course, in equilibrium, the value of θ in any market to which a firm deviates will be such that workers are indifferent between entering that market and staying in the candidate equilibrium market. With the wage determined by bargaining, problem (P1) becomes

$$\begin{aligned} \{k^*, h^*, \theta^*\} &= \arg\max_{k,h,\theta} V_v^B(k, h, \theta) - k \\ \text{subject to } V_u^B(k, h, \theta) &= V_u^B(k^*, h^*, \theta^*). \end{aligned}$$

The implied equilibrium conditions for k^* and h^* are (see Appendix for derivation)

$$(1-\beta)m^2(\theta^*)\left[f_1(k^*, h^*) - \lambda\right] - \lambda(\delta+\lambda)\theta^{*2}m'(\theta^*) = 0 \tag{26}$$

$$\beta m^2(\theta^*)\left[f_2(k^*, h^*) - \delta c'(h^*)\right] - \delta(\delta+\lambda)\left[m(\theta^*) - \theta^* m'(\theta^*)\right]c'(h^*) = 0. \tag{27}$$

Equations (25), (26) and (27) are the equilibrium conditions for this model.

Claim 4 *For b small enough there exists a non-autarkic equilibrium under transparency (without wage commitment)*

Proof. We need to show that the conditions of Lemma 1 apply. Here, the relevant equations are (26), (27) meaning that

$$C(\theta) \equiv \frac{\lambda(\delta+\lambda)\theta^2 m'(\theta)}{(1-\beta)m^2(\theta)} \quad \text{and} \quad D(\theta) \equiv \frac{(\delta+\lambda)\left[m(\theta)-\theta m'(\theta)\right]}{\beta m^2(\theta)}.$$

Consider what happens as θ approaches 0. As $\theta m'(\theta)/m(\theta)$ remains finite and $\theta/m(\theta) \to 0$, $\lim_{\theta\to 0} C(\theta) = 0$. As, by assumption, $\lim_{\theta\to 0}\left[m(\theta)-\theta m'(\theta)\right]/m(\theta) > 0$ and $\lim_{\theta\to 0} m(\theta) = 0$, $\lim_{\theta\to 0} D(\theta) = \infty$.

Now consider what happens as θ approaches ∞. As, by assumption, $\theta m'(\theta)/m(\theta)$ remains strictly positive but $\theta/m(\theta) \to \infty$, $\lim_{\theta\to\infty} C(\theta) = \infty$. As $\left[m(\theta)-\theta m'(\theta)\right]/m(\theta) < 1$ and $m(\theta) \to \infty$, $\lim_{\theta\to\infty} D(\theta) = 0$. So Lemma 1 applies.

This means that for small enough b there must be a value of θ such that LHS (25) is positive. And, for large enough values of θ, $f(k^*(\theta), h^*(\theta)) < b$ for any b so that LHS (25) becomes negative. ■

In general, the equilibrium conditions do not coincide with those of the first order conditions of Planner's problem, (3), (4) and (5). We can, however, ask if the Hosios [1990] result applies here. His result was that in the Pissarides [2000] framework, if the bargaining power of the worker just so happened to equal the elasticity of the matching function with respect to unemployment, that the equilibrium conditions for the model of the decentralized economy were the same as the Planner's optimality conditions. In the current context that would mean setting

$$\beta = \beta_H \equiv \frac{m(\theta)-\theta m'(\theta)}{m(\theta)}. \tag{28}$$

It is simple to show that when we set $\beta = \beta_H$, equations (25), (26) and (27) become identical to equations (3), (4), and (5). That is, if the bargaining power of the workers is just right, equilibrium will be consistent with efficiency on every margin.

To see what happens away from the Hosios condition, substitute from equation (24) into (20) and set $V_v = k^*$. Then eliminate b from the competitive entry condition (25) to obtain,

$$m(\theta^*)[f(k^*,h^*)-w^*]-\lambda[(\delta+\lambda)\theta^*+m(\theta^*)]k^*=0. \tag{29}$$

This is identical to equation (17).

Now suppose that compared to the Hosios condition, bargaining favors the firms (i.e. $\beta < \beta_H$). Then

$$(1-\beta_H)m^2(\theta^*)\left[f_1(k^*,h^*)-\lambda\right]-\lambda(\delta+\lambda)\theta^{*2}m'(\theta^*)<0$$

so

$$m(\theta^*)(f_1(k^*,h^*)-\lambda)-\lambda(\delta+\lambda)\theta^*<0$$

multiplying through by k and comparing to equation (29) indicates that

$$f_1(k^*,h^*)k^*<f(k^*,h^*)-w^*.$$

This means that whenever $\beta < \beta_H$, firms receive more than the marginal contribution of their investments to output. By the same token, workers receive less than the marginal contribution of their investments to output. (The results are reversed if $\beta > \beta_H$.) The question that follows from this is whether this leads to over investment relative to the Planner's optimum. This question will be addressed in Section 7 by simulation.

6.3 Hidden Investments

Consider now the other extreme case in which no aspect of the worker or job is public knowledge (i.e. there is complete ignorance). Firms seek to maximize $V_c^B \equiv V_v^B - k$ by picking k. As they cannot influence the arrival rate of workers by their choice, they take V_u as given and the relevant equation for V_v^B is (21). The implied first order condition yields

$$(1-\beta)m(\theta^*)\left[f_1(k^*,h^*)-\lambda\right]-\lambda(\delta+\lambda)\theta^*=0. \tag{30}$$

Similarly, workers choose h to maximize $V_b^B \equiv V_u^B - c(h)$. As their choice cannot influence the behavior of firms, V_u^B is obtained from (22) and the implied first order condition yields

$$\beta m(\theta^*)\left[f_2(k^*, h^*) - \delta c'(h^*)\right] - \delta(\delta + \lambda)c'(h^*) = 0. \tag{31}$$

Equations (25), (30) and (31) represent the equilibrium conditions for this model.

Claim 5 *For b small enough there exists a non-autarkic equilibrium under complete ignorance*

Proof. After dividing (30) and (31) through by $(1-\beta)m(\theta^*)$ and $\beta m(\theta^*)$ respectively it should be clear that the conditions for Lemma 1 apply. Hence, for small enough b there must be a value of θ such that LHS (25) is positive and for large enough values of θ, $f(k^*(\theta), h^*(\theta)) < b$ for any b so that LHS (25) becomes negative. ■

We would like to know if the Hosios condition (28) plays any role here. After making the substitution it is immediate that equations (25) and (5) are identical. It should also be clear that there is no substitution that will yield equations (3) and (4) from equations (30) and (31). Indeed, simple inspection indicates that there is always under-investment in this model caused by the hold-up problem.[10] The workers' inability to pre-contract with firms along with bargaining means both sides only get some fraction of their marginal contribution to match output.

Consistent with Hosios, then, given k and h, the "right" bargaining power delivers efficient vacancy creation but it does not lead to efficient investment. This should be viewed in comparison to Pissarides [2000] who finds that the Hosios condition is sufficient to bring about efficient search and advertising

[10] This model is essentially that of Acemoglu [1996] or Masters [1998] brought into the Pissarides [2000] framework.

intensity decisions.[11] The difference is that search intensity only affects one's own immediate pay-off where as capital investments affect individual continuation values which can be appropriated by the other side of the market through the bargaining process.

When either side advertises human capital, and physical capital is still a hidden choice of firms, it is straightforward to show that the relevant equilibrium conditions are (25), (30) and (27). Lemma 1 implies that there will be a non-autarkic equilibrium as long as b is not too large. When either side advertises physical capital, and human capital is still a hidden choice of workers, the relevant equilibrium conditions are (25), (26) and (31). Again, Lemma 1 implies there will be a non-autarkic equilibrium as long as b is not too large. In these models, under the Hosios condition all but one of the equilibrium conditions coincides with the Planner's optimality conditions. Notice also that the asymmetry associated with the characterization of the terms of trade referred to in Remark 1 does not apply when they are determined by bargaining. As long as some proportion of the match surplus goes to both participants neither is the residual claimant.

7 Simulations

Here I provide a numerical example to demonstrate how the various institutional arrangements work and I address some issues that have proven too difficult to consider analytically. An appropriate choice of parameter values also permits a quantitative assessment of how far the various models' equilibrium allocations are from efficiency.

[11] Moen [1997] shows that competitive search induces efficient search intensity decisions.

7.1 Functional forms and parameter values

I used $c(h) = \bar{c}h^\sigma$, $f(k,h) = k^\alpha h^{1-\alpha}$ and $m(\theta) = \bar{m}\theta^\eta$. The parameter values, based on a time unit of 1 year, are in Table 1.

b	$\bar{c}$	$\bar{m}$	α	δ	η	λ	σ
15	8×10^{-8}	4	0.35	0.05	0.5	0.2	8

Table 1: Parameter values

The parameter values were chosen to achieve certain targets for the efficient allocation as laid out in Table 2.[12] The parameter associated with each target is that most relevant for achieving it.

Target	**Parameter**
65% labor share of output	α
5.5% unemployment	$\bar{m}$
Expected life of job $\approx$ 5.5 years	λ
Expected years in labor force $\approx$ 20	δ
human capital investment as share of output $\approx$ 7%	σ
average years of schooling $\approx$ 13.3	$\bar{c}$
value of leisure$\approx$ 40% of marginal product (Shimer [2005])	b
usual range for matching elasticity (Shimer [2005])	η

Table 2: Empirical targets

7.2 Results

Table 3 contains the simulation results. The reported values are percentages of those for the efficient allocation.

[12] Of course it would be preferable to calibrate the model that is believed to most comport with reality. The results show, that the difference would be very small.

Model	β	k^*	h^*	f^*	w^*	u	Y	W
BTR	0.25	98.44	99.80	99.32	97.52	59.38	101.8	98.93
	0.5	100	100	100	100	100	100	100
	0.75	98.44	99.80	99.32	101.1	166.5	95.21	98.93
BHK	0.25	87.70	99.30	95.08	96.76	56.51	97.65	98.70
	0.5	90.44	99.56	96.26	99.27	95.87	96.51	99.83
	0.75	87.70	99.30	95.08	100.1	159.0	91.58	98.70
BHH	0.25	97.57	98.87	98.41	96.64	59.62	100.9	98.89
	0.5	99.22	99.19	99.20	99.20	100.3	99.18	99.97
	0.75	97.57	98.87	98.41	100.2	167.2	94.30	98.89
BCI	0.25	87.00	98.42	94.26	95.93	56.74	96.80	98.65
	0.5	98.79	98.78	95.53	98.51	96.20	95.76	99.78
	0.75	87.00	98.42	94.26	99.27	159.6	90.76	98.65

Table 3: Simulation Results

The four models are: BTR, bargaining with transparent investment on both sides; BHK, bargaining with hidden physical capital; BHH, bargaining with hidden human capital and BCI, complete ignorance (both types of capital are hidden choices). The column headings of Table 3 generally have the same meanings as in the text except that $f^* \equiv f(k^*, h^*)$ and Y is aggregate output, $f^*(1-u)$. As the elasticity of matching with respect to vacancies is 0.5, the Hosios condition applies whenever $\beta = 0.5$.

The results show a striking amount of symmetry. For each model, the rows associated with the extreme values of β contain many identical numbers. This comes from the symmetry of the model but points to possible policy implications. If lowering unemployment is an objective, policies that reduce the bargaining power of individual workers have a small effect on wages but can significantly lower the unemployment rate. Furthermore, even in the model with complete ignorance, investment levels and output are surprisingly close to the efficient levels.

A question of interest here is whether there can be over-investment. We saw in the case of bargaining with transparent investments, that the side of the market which has excess bargaining power relative to the Hosios condition, receives more than its marginal product on investments. Taking both market tightness and the investment on the other side of the market as given this would certainly lead to over-investment. However, the results here (and for a broad set of alternative parameter values) indicate that when they are endogenous, no such over-investment occurs.

A more general issue raised by this analysis is the role of transparency. That is, we know that complete transparency with commitment leads to fully efficient outcomes but is more transparency necessarily better than less? The theory of second best would suggest a negative answer. Nevertheless, in this example (for a given value of β) welfare is increasing as we move up Table 3. Again no parameter values have been found for which this does not hold true.

8 Conclusion

This paper provides an assessment of the efficiency properties of a general class of search models with two-sided investment. When characteristics of the job and worker are knowable (i.e. either observed directly or, given the environment, can be ascertained from what is directly observable) we find that the efficiency result of Moen [1997] passes through to the more general environment. When neither side can commit to a wage so that the terms of trade are determined by Nash bargaining the Hosios condition ensures efficient vacancy formation but its implications for investment depend on the extent to which firms and workers advertise their investments. When investments are private information to the investor (only revealed in one-on-one meetings) the classical hold-up problem leads to under investment. However, if investments become public knowledge, then competitive search disciplines investors into making the right choices.

A general theme that emerges from this analysis is that while competitive search leads to desirable outcomes it requires strong informational assumptions on the environment. Specifically, for the equilibrium condition associated with some job or worker characteristic to coincide with that of the optimality condition of Social Planner requires both commitment and advertising. When it is reasonable to make these assumptions depends on the specific context.

Furthermore, we have seen that if advertising does not convey sufficient information, very bad outcomes are possible. For instance, when investments are hidden but wage commitments are advertised, autarky is the only possible outcome. In this case, *ex post* bargaining mitigates the severity of the moral hazard problem by providing a fixed share of the match surplus to investors. Thus, a job posting agency that requires payoff relevant aspects of vacancies be made public and accurate would achieve efficient matching and investment. But in the absence of any enforcement of truth-in-advertising we might be better off with random matching.

9 Appendix

9.1 Proof of Lemma 1

Part 1: Existence and uniqueness of $\{\hat{k}(\theta), \hat{h}(\theta)\}$.

As f is constant returns to scale, for given θ, (6) implies $\hat{k}/\hat{h}$ is a constant. Thus $f_2(\hat{k}, \hat{h})$ is also a constant and since $c'(.)$ is strictly increasing with range $\mathbb{R}_+$, (7) implies a unique $\hat{h} > 0$ must exist. Then $\hat{k}/\hat{h}$ being a strictly positive constant also implies that $\hat{h} > 0$.

Part 2: $\lim_{\theta \to 0} \hat{h}(\theta) = \lim_{\theta \to 0} \hat{k}(\theta) = 0$, $\lim_{\theta \to 0} \frac{f(\hat{k}(\theta), \hat{h}(\theta))}{\hat{k}(\theta)} > \lambda$

Notice that L'Hospital's rule indicates that as $\theta \to 0$, $C(\theta) \to 0$ implies $f_1(\hat{k}(\theta), \hat{h}(\theta)) \to \lambda$ and therefore $\hat{k}(\theta)/\hat{h}(\theta) \to \nu$ where ν is a strictly positive constant. Furthermore, $f_2(\hat{k}(\theta), \hat{h}(\theta)) \to \gamma$ where γ is a strictly positive constant. As $D(\theta) \to \infty$, (7) implies $c'(\hat{h}(\theta)) \to 0$ so $\lim_{\theta\to 0} \hat{h}(\theta) = \lim_{\theta\to 0} \hat{k}(\theta) = 0$. As both f and k approach 0 we need to use L'Hospital's rule to evaluate their ratio. Thus,

$$\lim_{\theta\to 0} \frac{f(\hat{k}(\theta), \hat{h}(\theta))}{\hat{k}(\theta)} = \lim_{\theta\to 0} \frac{f_1 \frac{d\hat{k}}{d\theta} + f_2 \frac{d\hat{h}}{d\theta}}{\frac{d\hat{k}}{d\theta}} = \lambda + \gamma \lim_{\theta\to 0} \frac{d\hat{h}}{d\hat{k}} = \lambda + \gamma\nu > \lambda.$$

Part 3: $\lim_{\theta\to\infty}\{\hat{k}(\theta), \hat{h}(\theta)\} = \{0, 0\}$

From (6)

$$\lim_{\theta\to\infty} C(\theta) = \infty \Longrightarrow \lim_{\theta\to\infty} f_1(\hat{k}(\theta), \hat{h}(\theta)) = \infty$$
$$\Longrightarrow \lim_{\theta\to\infty} \frac{\hat{k}(\theta)}{\hat{h}(\theta)} = 0 \Longrightarrow \lim_{\theta\to\infty} f_2(\hat{k}(\theta), \hat{h}(\theta)) = 0.$$

In (7),

$$\lim_{\theta\to\infty} f_2(\hat{k}(\theta), \hat{h}(\theta)) = 0 \Longrightarrow \lim_{\theta\to\infty} c'(\hat{h}(\theta)) = 0 \Longrightarrow \lim_{\theta\to\infty} \hat{h}(\theta) = 0.$$

9.2 Proof of Claim 1

Let $\tilde{W}(\theta) \equiv W(k^*(\theta), h^*(\theta), \theta; 0)$. From Lemma 1, $\lim_{\theta\to 0} \tilde{W}(\theta) = \lim_{\theta\to\infty} \tilde{W}(\theta) = 0$.

The envelope theorem implies

$$\frac{d\tilde{W}}{d\theta} = \frac{(\delta+\lambda)}{(m(\theta)+\delta+\lambda)^2} \left\{ \begin{array}{c} m'(\theta)[f(k^*(\theta), h^*(\theta))] \\ -\lambda[\delta+\lambda+m(\theta)+(1-\theta)m'(\theta)]k^*(\theta) \end{array} \right\}.$$

In the neighborhood of $\theta = 0$, $m'(\theta) \gg m(\theta) > \theta m'(\theta)$ so in that neighborhood

$$\frac{d\tilde{W}}{d\theta} \approx \frac{m'(\theta)}{\delta+\lambda}[f(k^*(\theta), h^*(\theta)) - \lambda k^*(\theta)] = \frac{m'(\theta)k^*(\theta)}{\delta+\lambda}\left[\frac{f(k^*(\theta), h^*(\theta))}{k^*(\theta)} - \lambda\right]. \tag{32}$$

Now,

$$\begin{aligned}
\lim_{\theta\to 0}\frac{d\tilde{W}}{d\theta} &= \lim_{\theta\to 0}\frac{m'(\theta)k^*(\theta)}{\delta+\lambda}\left[\frac{f(k^*(\theta),h^*(\theta))}{k^*(\theta)}-\lambda\right] \\
&= \lim_{\theta\to 0}\frac{m(\theta)}{(\delta+\lambda)\,\theta}\left[f_1(k^*(\theta),h^*(\theta))-\lambda\right]k^*(\theta) \\
&= \lim_{\theta\to 0}\lambda k^*(\theta) \\
&= 0
\end{aligned}$$

where the next to last line follows from (3). However, (32) implies that away from, but close to $\theta = 0$, $\frac{d\tilde{W}}{d\theta} > 0$. Consequently there exists some $\tilde{\theta}$ close to 0 such that $\tilde{W}(\tilde{\theta}) > 0$ and $\tilde{W}(\theta) < \tilde{W}(\tilde{\theta})$ for all $\theta \in [0, \tilde{\theta})$.

As $\lim_{\theta\to\infty}\tilde{W}(\theta) = 0$, either $\tilde{\theta} = \arg\max_\theta \tilde{W}(\theta)$ or there exists at least one other value of θ such that $\tilde{W}(\theta) = \tilde{W}(\tilde{\theta})$. Let θ_H represent the largest such value of θ (which has to exist because $\tilde{W}(\theta)$ eventually approaches 0. As $\{k^*(\theta^*), h^*(\theta^*)\}$ are continuous on $[\tilde{\theta}, \theta_H]$ (by the theorem of the maximum) so is $\tilde{W}(\theta)$. Hence, a maximizing value θ^* of $\tilde{W}(\theta)$ exists such that $0 < \theta^* < \infty$ and $\{k^*, h^*\} = \{k^*(\theta^*), h^*(\theta^*)\}$.

9.3 Derivation of Equations (26), (27)

Using μ for the multiplier on the (first) constraint leads to the Lagrangian

$$\begin{aligned}
\mathcal{L} &= \frac{(1-\beta)m(\theta)[f(k,h)-b]}{\lambda[(\delta+\lambda)\theta+(1-\beta+\theta\beta)m(\theta)]} \\
&\quad -\mu\left\{\begin{array}{c}\left[V_u^{B*}+c(h)\right]\delta\left[(\delta+\lambda)\theta+(1-\beta+\theta\beta)m(\theta)\right] \\ -\left(\theta\beta m(\theta)[f(k,h)-b]+[(\delta+\lambda)\theta+(1-\beta)m(\theta)]b\right)\end{array}\right\}
\end{aligned}$$

where $V_u^{B*} = V_u^B(k^*, h^*, \theta^*)$. The first order conditions are

k :

$$\frac{(1-\beta)m(\theta)f_1(k,h)}{\lambda[(\delta+\lambda)\theta+(1-\beta+\theta\beta)m(\theta)]} - 1 - \mu\theta\beta m(\theta)f_1(k,h) = 0$$

h :

$$\frac{(1-\beta)m(\theta)f_2(k,h)}{\lambda[(\delta+\lambda)\theta+(1-\beta+\theta\beta)m(\theta)]} \\ -\mu\left\{c'(h)\delta\left[(\delta+\lambda)\theta+(1-\beta+\theta\beta)m(\theta)\right]-\mu\theta\beta m(\theta)f_2(k,h)\right\}=0$$

θ :

$$\frac{(1-\beta)[f(k,h)-b]\left[m'(\theta)(\delta+\lambda)\theta+m(\theta)(\delta+\lambda+\beta m(\theta))\right]}{\lambda\left[(\delta+\lambda)\theta+(1-\beta+\theta\beta)m(\theta)\right]^2} \\ -\mu\left\{\begin{array}{c}\left[V_u^{B*}+c(h)\right]\left[\delta+\lambda+m'(\theta)(1-\beta+\theta\beta)+\beta m(\theta)\right] \\ -\left[\delta+\lambda+m'(\theta)(1-\beta)\right]b-\beta(m(\theta)+\theta m'(\theta))f(k,h)\end{array}\right\}=0.$$

Now, substitute for $V_u^B(k^*,h^*,\theta^*)$ from equation (24) into the θ equation, solve for μ and substitute into the k and h equations.

10 References

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