Commitment, advertising and efficiency of two-sided investment in competitive search equilibrium

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MOTIVATION

- The basis on which search can be directed depends on commitment and information
- Some characteristics are more "committable to" than others
- I consider human capital, physical capital and wages
- I address
  1. how outcomes depend on the extent to which commitment and/or advertising are possible
  2. the extent to which the efficiency properties of benchmark models pass through to the more general environment?

LITERATURE

- Moen *JPE* (1997)
- Acemoglu and Shimer *IER* (1999)
- Menzio *JPE* (2007)
- Acemoglu (1996) Masters (1998)

ENVIRONMENT: Time and Demography

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- All jobs face destruction at the rate λ.

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- (Other than that induced by death and job destruction there is no discounting.)

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  - Vacancies meet workers at rate $m(\theta_j)/\theta_j$

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- Equating steady state inflow and outflow to unemployment:

$$\delta + \lambda(1-u) = (m(\theta) + \delta)u$$

EFFICIENCY (cont.)

$$W(k,h,\theta;b) = \frac{m(\theta)\left[f(k,h) - \delta c(h) - \lambda k\right] + (\delta+\lambda)\left[b - \delta c(h) - \lambda\theta k\right]}{m(\theta)+\delta+\lambda}$$

First order conditions, respectively for k, h and θ, for a maximum yield,

$$\begin{aligned} m(\theta^*)\left[f_1(k,h) - \lambda\right] - \lambda(\delta+\lambda)\theta^* &= 0 \\ m(\theta^*)\left[f_2(k^*,h^*) - \delta c'(h^*)\right] - \delta(\delta+\lambda)c'(h^*) &= 0 \\ m'(\theta^*)[f(k^*,h^*) - b] - \lambda[\delta+\lambda+m(\theta^*)+(1-\theta^*)m'(\theta^*)]k^* &= 0 \end{aligned}$$

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- A finite solution (k^*, h^*, θ^*) exists in positive orthant.
- If b not too large, k^*, h^* and θ^* are each strictly positive.

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    - Hidden choices are obvious in bilateral meetings

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- When neither side can commit to the wage there is generalized Nash bargaining

ALLOCATIONS

Definition

A symmetric steady state allocation is a tuple, $\{k, h, w, \theta\}$ such that all firms invest k, all workers invest h and receive payment w when hired and there is unique active market in which the ratio of vacancies to job seekers is θ.

ALLOCATIONS (cont.)

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- **For firms:**

$$\left.\begin{array}{c} \lambda V_v = \frac{m(\theta)}{\theta}\left[V_j - V_v\right] \\ \lambda V_j = f(k,h) - w - \delta\left[V_j - V_v\right] \\ V_c = -k + V_v \end{array}\right\} \Longrightarrow \left\{\begin{array}{c} V_v(k,h,w,\theta) \\ V_j(k,h,w,\theta) \\ V_c(k,h,w,\theta) \end{array}\right.$$

If workers do not accept offers to match then $V_v = 0$.

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$$\left.\begin{array}{c} \delta V_u = b + m(\theta)\left[V_e - V_u\right] \\ \delta V_e = w + \lambda\left[V_u - V_e\right] \\ V_b = \max\{b/\delta, V_u - c(h)\} \end{array}\right\} \Longrightarrow \left\{\begin{array}{c} V_u(w,\theta) \\ V_e(w,\theta) \\ V_b(h,w,\theta) \end{array}\right.$$

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- **The entrant firm solves**

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$$\text{subject to: worker indifference: } V_b(\hat{h}_f, \hat{w}_f, \theta_f) = V_b(h^*, w^*, \theta^*)$$

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WORKER'S PROBLEM

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EQUILIBRIUM

Definition

A (free entry) competitive search equilibrium is a symmetric steady state allocation, $\{k^*, h^*, w^*, \theta^*\}$, such that when everyone else conforms, it solves both the firm's and worker's problems, and $V_v(k^*, h^*, w^*, \theta^*) = 0$

TRANSPARENCY

Example: Firms advertise k_f and h_f workers advertise w_w.

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- So $\mu_f = 1/\mu_w$

TRANSPARENCY (Cont.)

- Necessary conditions for an equilibrium, $\{k^*, h^*, w^*, \theta^*\}$:

$$m(\theta^*)f_1(k^*, h^*) - \lambda[(\delta + \lambda)\theta^* + m(\theta^*)] = 0$$
$$m(\theta^*)f_2(k^*, h^*) - \delta[\delta + \lambda + m(\theta^*)]c'(h^*) = 0$$
$$m(\theta^*)m'(\theta^*)(w^* - b) - \lambda[m(\theta^*) - \theta^* m'(\theta^*)][\delta + \lambda + m(\theta^*)]k^* = 0$$
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- Firms and workers receive their marginal product
- Eliminating w^* yields Planner's optimality conditions.

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- No non-trivial equilibrium (Diamond Paradox)

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- **Hidden physical capital:** Given w^* and h^* workers do not care about k. Firms are residual claimants; their private and the social returns to investment coincide. Market equivalence applies. (*cf.* rental contracts)

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- Substituting into V_v and V_u taking continuation values of other side parametrically:

$$V_v^B = \frac{(1-\beta)m(\theta)[f(k,h) - \delta V_u]}{\lambda[(\delta+\lambda)\theta + (1-\beta)m(\theta)]}$$

$$V_u^B = \frac{\beta m(\theta)[f(k,h) - \lambda V_v] + [\delta+\lambda]b}{\delta[\delta+\lambda+\beta m(\theta)]}$$

ARRANGEMENTS WITH BARGAINING (cont.)

- In equilibrium $V_v^B = V_v$ and $V_u^B = V_u$. Solving yields

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- Competitive entry of vacancies: $V_v^B(k^*, h^*, \theta^*) - k^* = 0$ so,

$$(1-\beta)m(\theta^*)[f(k^*, h^*) - b] - \lambda\left[m(\theta^*)(1-\beta+\beta\theta^*) + (\delta+\lambda)\theta^*\right]k^* = 0$$

ARRANGEMENTS WITH BARGAINING (cont.)

Hosios condition: In Pissarides environment, equating worker's bargaining power to the elasticity of matching with respect to unemployment generates efficient vacancy creation.

- Here this means

$$\beta = \beta_H \equiv \frac{m(\theta) - \theta m'(\theta)}{m(\theta)}$$

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- Under Hosios, free entry condition same as planners F.O.C. for θ.

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- In general matching frictions mean inefficient levels of vacancy creation and investment
- Hosios condition restores efficiency on every margin

SIMULATIONS

Functional forms and parameters:

$$c(h) = \bar{c}h^{\sigma}, \quad f(k,h) = k^{\alpha}h^{1-\alpha} \quad \text{and} \quad m(\theta) = \bar{m}\theta^{\eta}.$$

Time unit: 1 year

b	$\bar{c}$	$\bar{m}$	α	δ	η	λ	σ
15	8×10^{-8}	4	0.35	0.05	0.5	0.2	8

Target	Parameter
65% labor share of output	α
5.5% unemployment	$\bar{m}$
Expected life of job $\approx$ 5.5 years	λ
Expected years in labor force $\approx$ 20	δ
human capital investment as share of output $\approx$ 7%	σ
average years of schooling $\approx$ 13.3	$\bar{c}$
value of leisure $\approx$ 40% of wage (Shimer [2005])	b
usual range for matching elasticity (Shimer [2005])	η

Results (% of efficient allocation value)

Model	β	k^*	h^*	f^*	w^*	u	Y	W
BTR	0.25	98.44	99.80	99.32	97.52	59.38	101.8	98.93
	0.5	100	100	100	100	100	100	100
	0.75	98.44	99.80	99.32	101.1	166.5	95.21	98.93
BHK	0.25	87.70	99.30	95.08	96.76	56.51	97.65	98.70
	0.5	90.44	99.56	96.26	99.27	95.87	96.51	99.83
	0.75	87.70	99.30	95.08	100.1	159.0	91.58	98.70
BHH	0.25	97.57	98.87	98.41	96.64	59.62	100.9	98.89
	0.5	99.22	99.19	99.20	99.20	100.3	99.18	99.97
	0.75	97.57	98.87	98.41	100.2	167.2	94.30	98.89
BCI	0.25	87.00	98.42	94.26	95.93	56.74	96.80	98.65
	0.5	98.79	98.78	95.53	98.51	96.20	95.76	99.78
	0.75	87.00	98.42	94.26	99.27	159.6	90.76	98.65

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