Matching with Interviews.

Adrian Masters*
University at Albany, SUNY
amasters@albany.edu

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Abstract

This paper explores how interviews affect the matching process when worker productivity is private information. Wages are determined by a single round of strategic bargaining after the worker is interviewed. The implications of this hiring process for the efficiency of matching and the incidence and severity of statistical discrimination are considered. The better are firms at identifying productive workers the worse the average quality of the unemployment pool so interviewing tends to slow down matching for every one. Multiple Pareto rankable equilibria are possible such that any social group in a "bad" equilibrium faces stricter hiring standards, longer spells of unemployment and lower welfare.

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1 Introduction

In the process of hiring a worker, firms use various means of assessing his or her appropriateness for the job. Pre-interview screening, cognitive tests and face-to-face interviews are among the most common.[1] This paper looks into how the technology used by firms to determine a worker's productivity helps to shape labor market outcomes.

The matching literature (e.g. Pissarides [2000], Rogerson *et al* [2005]) has historically avoided introducing private information over worker productivity because of the difficulty it poses for modelling wage determination. A consequence of this omission has been that while a great deal of work has been done on how the nature of the matching function affects outcomes (Petrongolo and Pissarides [2001]), the role of the interview technology has largely been ignored. This paper embeds a model of private information over worker productivity into a matching environment. Individual firms benefit from unilateral adoption of a more accurate means of worker assessment. Universal adoption, however, leads to a deterioration of the composition of the unemployment pool. This feeds back into hiring decisions. Two major implications emerge. The first, is that in the long run, interview technologies that firms willingly adopt can make everyone worse off than under random assignment. The second is that the interview technology can generate a purely statistical basis for discrimination.

In the model there are two types of worker: productive and unproductive. A worker's productivity is his private information. The proportion of productive workers flowing into unemployment is fixed but with sufficiently informative interviews they get hired more quickly. This makes the average quality of the unemployment pool endogenous. Firms use this average quality as their prior on a worker's productivity. When a firm and worker meet, the worker is interviewed. The outcome is known to both parties and is used via Bayes' rule to up-date the worker's expected productivity. This posterior belief forms the basis for any wage negotiations.

[1] Edder and Ferris [1989] provide a collection of articles on the hiring process.

The existing literature which introduces private information along with search frictions has focussed on environments in which the terms of trade are exogenous. Williamson and Wright [1994] provide a model in which the quality of a good is the producer's private information. Multiple equilibria occur due, as in Coate in Loury [1993], to a costly hidden action to which the returns are determined by market outcomes. Chade [2006] provides a model of marriage with private information in which acceptance decisions are indicative of individuals' true types.

The complications associated with private information when terms of trade are determined by bargaining, however, should not preclude analysis of such environments. The issue is that offers made by the party holding private information potentially convey information about his type. A huge number of equilibria are possible depending on how the person receiving the offer interprets it. All but an infinitessimal fraction of these equilibria provide some of the expected gains from trade to the worker. As long as this is true, high quality workers are more likely to be offered any job, and the mechanisms highlighted in this paper will be in operation.

Statistical discrimination occurs whenever the characteristics of a group are used to influence decisions made about individual members of the group. The clearest context for the relevance of the current model to statistical discrimination is when there are multiple steady-state equilibria. When they exist, multiple equilibria are Pareto rankable. Worse equilibria are characterized by a higher threshold interview performance, a lower proportion of productive workers among the unemployed and a greater level of joblessness. Now suppose, as in Coate and Loury [1993], that there are two groups of workers who differ *ex ante* only by their appearance. We can envision that for historical reasons one group is in a worse equilibrium than the other. Unlike Coate and Loury [1993] however, such discriminatory outcomes emerge without recourse to hidden actions. Rather, the members of the disadvantaged group are held to a higher standard, so the average quality of their unemployment pool is low which rationalizes the higher standards.

The point to take from this, though, is not that understanding the plight of minority groups is contingent on the existence of multiple equilibria. Even when equilibrium is unique, the mechanism that otherwise leads to multiplicity generates a multiplier effect which makes outcomes very sensitive to exogenous differences across groups. For instance, suppose for unmodeled reasons there are differences in the distributions of unobserved human capital across groups facing the same interview technology. The group with the lowest propensity to be qualified for the job will face more exacting hiring standards. These higher standards lead to a dilution of the quality of the unemployed in that group causing a further disparity in hiring standards; and so on.

It will be shown that unilateral adoption of a more accurate interview technology improves a firm's profitability. It also reduces the extent of possible discrimination by that firm because it attaches more importance to an objective productivity measure (the interview). This last point has been documented in the literature on aptitude testing (see Campion and Arvey [1989] and also Autor and Scarborough [2004]). What that literature generally misses though, is that the composition effects that ensue from *universal* adoption of the more accurate productivity measure will worsen a firm's prior on the likely productivity of any worker.[2] In the long run the inflow to unemployment has to be identical to the outflow; greater accuracy cannot affect the long run distribution of productivity among workers hired. What interviewing can do, however, is slow down the matching rate. In examples based on interviews producing normally distributed signals of worker productivity, greater accuracy can increasingly make everyone (qualified and unqualified alike) worse off.

[2] One notable exception is Hartigan and Wigdor [1989] p. 242 who suggest "If a firm uses tests to identify the able and if the firm can be selective, then it can improve the quality of its work force. The economy as a whole cannot; the economy as a whole must employ the labor force as a whole."

2 Model

2.1 Environment

Time passes continuously with an infinite horizon. A mass 1 of workers per unit time flow into an unemployment pool. The (endogenous) stock of unemployed workers in the pool has mass u. There is a large number of firms who can each freely create identical, atomistic vacancies but pay a flow advertising cost a to keep them open. The mass of vacancies is v. Participants on both sides of the market have infinite lives, are risk neutral and discount the future at rate r. Unemployed workers have zero flow value of leisure.

A fraction η of new entrant workers are "productive" the remainder are "unproductive". When hired, the flow output from a productive (unproductive) worker is 1 (0). Productivity is private information to the worker. Jobs are for life. Firms each employ a large number of workers and know their average productivity; the productivity of individual workers is never revealed to them.[3] The proportion, μ, of the unemployment pool that is productive is endogenous. Although the distribution of unemployment durations is common knowledge, an individual worker's duration of unemployment is also his private information.[4]

Unemployed workers encounter vacancies at a Poisson arrival rate α. The arrival rate of workers to vacancies is then $\alpha u/v$. The implied meeting function, αu, is a special case of a standard constant returns to scale technology (Petrongolo and Pissarides [2001]). This form was chosen for tractability and to highlight the role of interviews in the matching process. Some dis-

[3] Alternatively, we can assume that contracts are fully enforceable. Such assumptions are common in the literature on individual employment contracts (see Shapiro and Stiglitz [1984], Malcomson [1999]) but not in search and matching. A recent example is Shimer and Wright [2004].

[4] Assuming that workers' duration of unemployment is private information is non-standard. It is, however, common to assume that a worker's search intensity is not observed so the effective job-search time typically cannot be credibly revealed. Some discussion of the implications of relaxing this assumption is provided in Section 5.

cussion of the consequences of relaxing this restriction is provided in section 5.

2.2 Interviews

Following Coate and Loury [1993], when a firm and worker meet, the firm will interview (or test) the worker which generates a noisy signal θ as to her ability. If the worker is productive, the probability distribution function over θ is H. If the worker is unproductive, the probability distribution function over θ is G. Sufficient conditions on H and G for making meaningful inferences from the signal as to the worker's true ability (see Milgrom [1981]) are that they have everywhere differentiable densities with the same connected support and exhibit the Monotone Likelihood Property (MLP). The MLP requires that the ratio of the densities $h(\theta)/g(\theta)$ is increasing in θ. Under the MLP it is straightforward to show (Milgrom [1981]) that we can normalize G to be uniform on $(0, 1]$. Then, $H(\theta) < \theta$ and h, the density of H, is strictly increasing.[5]

Given a worker's type, each score is an independent draw from the appropriate distribution. Interview scores are assumed to be observable to the worker and are used as a basis for wage negotiation.[6] The other firms do not observe the outcome of the interview, the worker cannot credibly reveal it to them and (as is typically restricted by law) such information may not be shared among firms.[7]

From Bayes' rule, the expected productivity of a worker given interview

[5] The normalization follows from the fact that the random variable $x = G(\theta)$ is uniformly distributed. So what ever the original distribution of θ, we can replace it by x. If $\hat{H}$ is the implied distribution of test scores for productive workers it can be obtained from the fundamental distributions H and G by defining $\hat{H}(x) \equiv H(G^{-1}(x))$ where G^{-1} is the inverse of G.

[6] In the literature on employment interviews, has established that the perceptions of interview performance by interviewers and interviewees are positively correlated; see Liden *et al* [1993].

[7] This assumption is consistent with the approach of Morris and Shin [2002] where the possibility of pooling private information that would benefit everyone is prohibited.

score, θ, and model consistent prior μ, is

$$\pi(\theta, \mu) = \frac{\mu h(\theta)}{\mu h(\theta) + (1 - \mu)} \tag{1}$$

This forms the basis for the wage negotiation. Straightforward differentiation shows that π is increasing in both arguments.

2.3 Bargaining

Following Mailath *et al* [2000], I assume that wages are determined by a single round of strategic bargaining. Once a worker has met a firm with a vacancy, the worker is interviewed and nature chooses either party to make a take-it-or-leave-it wage offer. Defining, V_f as the continuation value for the firm and, V_p and V_u as the continuation value for the productive worker and unproductive worker respectively, we define the effective match surplus as $\pi(\theta, \mu) - rV_f - rV_p$. It should be clear that who ever makes the offer, there will be no match formation if the effective match surplus is negative. There can be no wage that is simultaneously acceptable to the firm and productive workers and, the firm will avoid matching with unproductive workers. If the effective match surplus, is non-negative,

Lemma 1 *When the firm gets to make the offer, every sub-game perfect equilibrium has the form that: there is match formation and the wage paid to the worker is* rV_p.

Proof. See Appendix ■

Lemma 2 *When the worker gets to make the offer, Bayesian perfect equilibria have the form that: the wage paid to the worker is lies in* $[rV_p, \pi(\theta, \mu) - rV_f]$.

Proof. See Appendix ■

When the worker makes an offer, the firm can use this as information on which to up date its beliefs as to the productivity of the worker. As Bayesian perfect equilibrium places no restrictions on out-of-equilibrium beliefs, any wage in $[rV_p, \pi(\theta,\mu) - rV_f]$ can be supported. The firm simply has to believe that wage offers higher than that called for in equilibrium would only be made by low productivity workers. This dissuades the high productivity workers from making anything other than the equilibrium wage offer. There is no standard restriction on beliefs that can help in this regard.

As nature picks either side to make the offer with equal probability, Lemmas 1 and 2 imply that workers get between a zero and a half share of the effective expected match surplus. When the worker's share is zero, so that $w = rV_p$, the Diamond Paradox outcome will apply and in the (general) equilibrium all wages will drop to zero.[8] The set of equilibria to the bargaining game that provide zero surplus to the productive workers, however, is measure zero in the set of possible equilibria. As long as workers get some of the surplus, $w > rV_p$ and $V_p \geq V_u$. This is all we need to generate the mechanisms that drive the results of the paper.

In the sequel I impose that when the worker makes the offer, $w = \pi(\theta,\mu) - rV_f$. This means that in expectation the agreement divides the effective surplus equally between firms and productive workers. This particular solution is chosen for a number of reasons. First, is allows for direct comparison with the full information environment; Mailath *et al* [2000] established that, in the absence of private information, the expected wage divides the surplus equally. Second, it is consistent with the standard Nash bargaining outcome.[9] Third, it simplifies notation.

[8] This is because V_p comes entirely from the expectation of eventual employment. Discounting implies that whenever $w > 0$, rV_p must be less than the expected wage. The only solution is $rV_p = rV_u = w = 0$.

[9] The matching literature has historically used generalized Nash bargaining for the determination of the terms of trade. In that context the "bargaining power" of the firm is chosen arbitrarily.

2.4 Search

As free-entry drives the continuation value, V_f, of a vacancy to zero,[10] a productive worker and a firm will match if the realized value of θ means that $\pi(\theta,\mu) \geq rV_p$. As π is increasing in θ and any worker and firm who meet each other take V_p and μ as given, then, as along as $\pi(1,\mu) \geq rV_p$ there exists a unique threshold interview score, $\theta^* \in [0,1]$, such that $\pi(\theta^*,\mu) = rV_p$. If the realized value of θ exceeds θ^* the worker is hired. (The possibility that $\pi(1,\mu) < rV_p$ is addressed below.)

Let $w(\theta)$ represent the expected wage agreement when the interview score is $\theta \geq \theta^*$. As $w(\theta)$ divides the effective surplus equally between the worker and the firm,

$$w(\theta) = \frac{1}{2}\left(\pi(\theta,\mu) - rV_p\right) + rV_p = \frac{1}{2}\left(\pi(\theta,\mu) + rV_p\right) \tag{2}$$

Now, the standard continuous time asset value equation for V_p is

$$rV_p = \alpha\left[1 - H(\theta^*)\right]\left(\mathbb{E}^p_{\{\theta\geq\theta^*\}}\left[\frac{w(\theta)}{r}\right] - V_p\right)$$

where $\mathbb{E}^p_{\{\theta\geq\theta^*\}}$ is the expectation with respect to $H(.)$ given $\theta \geq \theta^*$. Then,

$$rV_p = \alpha\left[1 - H(\theta^*)\right]\left(\mathbb{E}^p_{\{\theta\geq\theta^*\}}\frac{\pi(\theta,\mu) - rV_p}{2r}\right) \tag{3}$$

Replacing rV_p by $\pi(\theta^*,\mu)$ implies

$$\pi(\theta^*,\mu) = \frac{\alpha}{2r}\int_{\theta^*}^{1}\left(\pi(\theta,\mu) - \pi(\theta^*,\mu)\right)dH(\theta) \tag{4}$$

Condition (4) is the threshold interview performance equation and represents constrained efficient match formation. That is, given all the information that can credibly be revealed, matches always form when they should. The previous analysis assumed that $\pi(1,\mu) \geq rV_p$. We need to establish that it is indeed true.

[10] While setting $V_f = 0$ at this stage constitutes imposing equilibrium conditions before equilibrium has been defined, it is standard within the search and matching literature. Here, it is done for notational brevity.

Lemma 3 $\pi(1, \mu) \geq rV_p$

Proof. As $\pi(.,.)$ is increasing in both arguments and $\mu > 0$ implies $\pi(1, \mu) > 0$, for any threshold $t \in (0, 1)$, we have $\pi(1, \mu) > \mathbb{E}^p_{\{\theta \geq t\}}\pi(\theta, \mu) \geq \mathbb{E}^p\pi(\theta, \mu) > 0$. This means that from (3)

$$rV_p \leq \frac{\alpha\pi(1, \mu)}{2r + \alpha} < \pi(1, \mu)$$

■

Essentially, the only reason that $V_p > 0$ is through the possibility of match formation. If rV_p were bigger than $\pi(1, \mu)$ there would never be any match surplus and matching would not happen driving V_p to zero. Lemma 3 establishes the existence and uniqueness of θ^*.

Depending on parameters, it is possible that

$$rV_p = \alpha \left(\mathbb{E}^p_{\{\theta \geq 0\}} \frac{\pi(\theta, \mu) - rV_p}{2r} \right) < \pi(0, \mu)$$

In which case, $\theta^* = 0$; the probability that a worker is productive, even with the worst possible interview performance, is still sufficient to warrant match formation. It should be clear that as long as $h(0) = 0$, equation (4) implies $\theta^* > 0$ for all μ.

The upshot from this is that there exists a function $\theta^*(\mu)$ which provides the threshold value of interview performance, θ^* in $[0, 1]$ for each value of $\mu \in [0, \eta]$.

Claim 4 $\theta^*(0) < 1$, $\theta^*(.)$ *is downward sloping.*

Proof. Substituting for $\pi(\theta, \mu)$ into (4) and dividing through by $h(\theta^*)$ yields

$$\Psi(\theta^*, \mu) \equiv 2r - \alpha(1 - \mu) \int_{\theta^*}^{1} \frac{h(\theta) - h(\theta^*)}{[\mu h(\theta) + (1 - \mu)]\, h(\theta^*)} dH(\theta) = 0 \quad (5)$$

then

$$\frac{d\theta^*}{d\mu} = -\frac{\frac{\partial \Psi}{d\mu}}{\frac{\partial \Psi}{d\theta^*}}$$

where
$$\frac{\partial \Psi}{d\mu} = \alpha \int_{\theta^*}^{1} \frac{h(\theta)}{\left[\mu h(\theta) + (1-\mu)\right]^2 h(\theta^*)} dH(\theta) > 0 \tag{6}$$
$$\frac{\partial \Psi}{d\theta^*} = \alpha(1-\mu) \int_{\theta^*}^{1} \frac{h'(\theta^*) h(\theta)}{\left[\mu h(\theta) + (1-\mu)\right] h^2(\theta^*)} dH(\theta) > 0 \tag{7}$$

Simple inspection of (5) indicates that $\theta^*(0) < 1$. ■

As the quality of the unemployment pool improves so does the firm's prior probability that any worker who applies is productive. This means the firms become less reliant on the interview to protect them from hiring an unproductive worker.

Claim 5 *The productivity of the marginal worker, $\pi(\theta^*, \mu)$, increases with μ*

Proof. We have
$$\frac{d\pi(\theta^*, \mu)}{d\mu} = \frac{\partial \pi}{\partial \theta^*} \frac{d\theta^*}{d\mu} + \frac{\partial \pi}{\partial \mu}$$
From (1), (6) and (7) the sign of $d\pi^*/d\mu$ is the same as that of
$$\int_{\theta^*}^{1} \frac{h^2(\theta)\left[\mu(h(\theta)-1) + 1 - \mu\right]}{\left[\mu h(\theta) + (1-\mu)\right]^2} d\theta > \mu \int_{\theta^*}^{1} \pi^2(\theta, \mu)(h(\theta) - 1) d\theta$$
We need to show that the last integral is positive. Because $h(.)$ is an increasing density function on $[0, 1]$, there exist a unique $\tilde{\theta}$ such that $h(\tilde{\theta}) = 1$. Above $\tilde{\theta}$, $h(\theta) > 1$. Now, if $\theta^* \geq \tilde{\theta}$ the integral is positive and we are finished. If $\theta^* < \tilde{\theta}$ we have
$$\int_{\theta^*}^{1} \pi^2(\theta, \mu)(h(\theta)-1) d\theta = \int_{\theta^*}^{\tilde{\theta}} \pi^2(\theta, \mu)(h(\theta)-1) d\theta + \int_{\tilde{\theta}}^{1} \pi^2(\theta, \mu)(h(\theta)-1) d\theta$$
as $\pi^2(\theta, \mu)$ is increasing in θ
$$\int_{\theta^*}^{\tilde{\theta}} \pi^2(\theta, \mu)(h(\theta) - 1) d\theta > \int_{\theta^*}^{\tilde{\theta}} \pi^2(\tilde{\theta}, \mu)(h(\theta) - 1) d\theta$$
and
$$\int_{\tilde{\theta}}^{1} \pi^2(\theta, \mu)(h(\theta) - 1) d\theta > \int_{\tilde{\theta}}^{1} \pi^2(\tilde{\theta}, \mu)(h(\theta) - 1) d\theta$$

so

$$\int_{\theta^*}^{1} \pi^2(\theta,\mu)(h(\theta)-1)d\theta > \pi^2(\tilde{\theta},\mu)\left[(1-H(\theta^*))-(1-\theta^*)\right] > 0$$

■

That is, the direct effect of an increase in μ on π outweighs the indirect effect through θ^*. So, while the threshold interview performance falls with μ, the expected productivity of the marginal worker increases. As $rV_p = \pi(\theta^*,\mu)$ productive workers are better off when μ is higher. Unproductive workers are better off too - their matching rate $\alpha(1-\theta^*)$ improves as does their implied productivity at every interview score along with the expected productivity of the marginal worker.

Lemma 6 *For $\theta^* > 0$, the unconditional expected productivity of those workers hired exceeds μ.*

Proof. For any given value of μ, the unconditional distribution of the interview scores is $\bar{H}(\theta) \equiv (1-\mu)\theta + \mu H(\theta)$ so that the distribution conditional on $\theta > \theta^*$ is

$$\frac{\bar{H}(\theta)}{1-\bar{H}(\theta^*)} = \frac{(1-\mu)\theta + \mu H(\theta)}{(1-\mu)(1-\theta^*) + \mu(1-H(\theta^*))}$$

unconditional expectation of productivity given $\theta > \theta^*$ is

$$\mathbb{E}_{\theta>\theta^*}\pi(\theta,\mu) = \int_{\theta^*}^{1} \pi(\theta,\mu)\frac{d}{d\theta}\left[\frac{(1-\mu)\theta + \mu H(\theta)}{(1-\mu)(1-\theta^*) + \mu(1-H(\theta^*))}\right]d\theta$$

substituting for π from (1) we have

$$\mathbb{E}_{\theta>\theta^*}\pi(\theta,\mu) = \frac{\mu(1-H(\theta^*))}{(1-\mu)(1-\theta^*) + \mu(1-H(\theta^*))} > \mu \tag{8}$$

■

This implies that whenever workers are being rejected, the interview is doing its job - those getting hired are (on average) more productive than the unemployment pool. It is worth noting, though, that $\pi^* < \mu$ is quite possible. That is, the expected productivity of the marginal worker hired may be lower than the prior the firm had on the worker's productivity.

2.5 Steady-state population flows

The proportion of productive people, μ, in the unemployment pool is obtained from the steady-state population flow equations. For any given threshold interview score, θ^*, equating the inflow of productive people to the rate at which they acquire jobs implies

$$\eta = \alpha\left[1 - H(\theta^*)\right]\mu u \tag{9}$$

Similarly for unproductive people,

$$(1-\eta) = \alpha\left[1-\theta^*\right](1-\mu)u$$

Eliminating u also eliminates α and yields

$$\mu = \mu(\theta^*) \equiv \frac{\eta\,(1-\theta^*)}{(1-\eta)\,(1-H(\theta^*)) + \eta\,(1-\theta^*)} \tag{10}$$

Because $H(\theta^*) < \theta^*$, we know that $\mu < \eta$; the qualification rate is lower in the steady-state market population than it is among the new entrants. Continuity of $\mu(.)$ follows from continuity of H.

Now,

$$\mu'(\theta^*) = \frac{(1-\eta)\eta\left[(1-\theta^*)h(\theta^*) - (1-H(\theta^*))\right]}{\left[\eta(1-\theta^*) + (1-\eta)\,(1-H(\theta^*))\right]^2}$$

and

$$(1-\theta^*)h(\theta^*) - (1-H(\theta^*)) = \int_{\theta^*}^{1}\left[h(\theta^*) - h(\theta)\right]d\theta < 0$$

So $\mu(.)$ is always downward sloping. This is because raising θ^* causes a higher proportion of productive workers to leave the unemployment pool and reduces the average quality of the unemployed workers.

Furthermore, in (θ^*, μ) space, the graph of $\mu(.)$ passes through the points $(0, \eta)$ and $(1, \hat{\mu})$ where

$$\hat{\mu} = \lim_{\theta\to 1}\frac{\eta}{(1-\eta)\,h(\theta) + \eta}$$

If $\lim_{\theta\to 1} h(\theta) = \infty$, $\hat{\mu} = 0$, otherwise, $\hat{\mu} > 0$.

2.6 Vacancies

Let $J_i(\theta)$ represent the expected asset value of a job occupied by a worker of type $i = p, u$ whose interview score was θ. Then

$$rJ_p(\theta) = 1 - \frac{1}{2}\left(\pi(\theta,\mu) + \pi(\theta^*,\mu)\right)$$

and

$$rJ_u(\theta) = -\frac{1}{2}\left(\pi(\theta,\mu) + \pi(\theta^*,\mu)\right)$$

If V_f represents the asset value of a vacancy,

$$rV_f = \frac{\alpha u}{v}\left\{\mu\int_{\theta^*}^{1} J_p(\theta)dH(\theta) - (1-\mu)\int_{\theta^*}^{1} J_u(\theta)d\theta - V_f\right\} - a$$

so,

$$\begin{aligned}\left(r + \frac{\alpha u}{v}\right)V_f = \frac{\alpha u}{2rv}&\left\{2\mu\left[1 - H(\theta^*)\right]\right.\\ &\left.-\int_{\theta^*}^{1}\left[\mu h(\theta) + (1-\mu)\right]\left[\pi(\theta,\mu) + \pi(\theta^*,\mu)\right]d\theta\right\} - a\end{aligned}$$

where v is the mass of vacancies.

Substituting from (1) and integrating yields,

$$(r + \alpha u)V_f = \frac{\alpha u}{2r}\left\{\mu\left[1 - \pi(\theta^*,\mu)\right]\left[1 - H(\theta^*)\right] - (1-\mu)\,\pi(\theta^*,\mu)\left[1 - \theta^*\right]\right\} - va \tag{11}$$

The contents of the curly brackets is the "pre-interview" expected match surplus. To see why, notice that a match with a productive worker provides surplus $1 - \pi(\theta^*,\mu)$ and the probability that such a match forms is $\mu\left[1 - H(\theta^*)\right]$. Similarly, a match with an unproductive worker provides surplus $-\pi(\theta^*,\mu)$. The probability that such a match forms is $(1-\mu)\left[1-\theta^*\right]$. The 2 in equation (11) comes from the fact that firms get half of the surplus in any match.

2.7 Equilibrium

Definition 7 *A Steady-State Equilibrium is a list* $\{\theta^*, \mu, v, u\}$ *that satisfies:*

free-entry, $V_f = 0$*:*

$$2rva = \alpha u \left\{ \mu \left[1 - \pi(\theta^*, \mu) \right] \left[1 - H(\theta^*) \right] - (1 - \mu)\, \pi(\theta^*, \mu) \left[1 - \theta^* \right] \right\} \qquad (12)$$

efficient match formation:

$$2r\pi(\theta^*, \mu) = \alpha \int_{\theta^*}^{1} \left(\pi(\theta, \mu) - \pi(\theta^*, \mu) \right) dH(\theta) \qquad (13)$$

steady-state conditions:

$$\mu = \frac{\eta\,(1 - \theta^*)}{(1 - \eta)\,(1 - H(\theta^*)) + \eta\,(1 - \theta^*)} \qquad (14)$$

$$u = \frac{\eta}{\alpha \left[1 - H(\theta^*) \right] \mu} \qquad (15)$$

where

$$\pi(\theta, \mu) = \frac{\mu h(\theta)}{\mu h(\theta) + (1 - \mu)}.$$

The system is block recursive. Equations (13) and (14) jointly determine θ^* and μ. Then, given θ^* and μ, as long as RHS of (12) is positive (verified below), the implied values of u and v are positive and unique.

Claim 8 *A steady-state equilibrium always exists.*

Proof. First I show the existence of a solution, (θ^*, μ), to (13) and (14). From earlier analysis we say that from equation (13) and (14), the implied function $\Gamma(\theta^*, \mu) \equiv (\theta^*(.), \mu(.))$ maps $[0, 1] \times [0, \eta]$ into itself. Continuity of Γ comes from the continuity of $\mu(.)$, and $\theta^*(.)$. This implies the existence of a fixed point by Brouwer's theorem.

As u is positive, non-negativity of v, follows from (12). Substituting for μ from (14) and for u from (15) we have

$$2rva = \eta - \pi(\theta^*, \mu) \qquad (16)$$

Substituting from (10) into (8) reveals that the *ex ante* expected productivity of a worker who gets hired,

$$\mathbb{E}_{\theta > \theta^*} \pi(\theta, \mu) = \eta. \qquad (17)$$

As $\theta^* < 1$ we have

$$\pi(\theta^*, \mu) < \eta \tag{18}$$

and any solution to (13) and (14) will induce market entry by firms. ■

Equation (17) confirms that in steady-state, the average productivity of those hired has to equal that of the inflow. By implication, equation (18) tells us that in equilibrium, the marginal worker hired has expected productivity that is below the mean of those flowing into the market.

In the sequel, as equilibrium values of u and v follow directly from efficient matching and steady-state, equilibria will be referred to simply as a pair, (θ^*, μ) that solves (13) and (14). Notice that if α were endogenized through the introduction of a more general matching function, the system would not be block recursive making equilibrium much harder to characterize.

2.8 Welfare

I will use the following measure of welfare,

$$W = \eta V_p + (1 - \eta) V_u$$

The variable W, therefore, measures the *ex ante* expected welfare contribution of a new entrant. While this measure necessarily ignores firms, the justification for doing so is that free entry drives expected profits to zero.[11]

3 Analysis

3.1 The economy without interviews

When firms have no means of distinguishing workers, every meeting leads to a match. Let V represent the value to being a worker and w the expected

[11] Any snap shot of the economy finds firms making net profits but those go entirely to paying for the advertising costs incurred prior to match formation.

wage (formed by the usual bargaining protocol). Then

$$rV = \alpha\left(\frac{w}{r} - V\right)$$

and

$$w = \frac{1}{2}(\eta + rV)$$

so

$$w = \frac{(r+\alpha)\eta}{2r+\alpha}, \quad rV = \frac{\alpha\eta}{2r+\alpha} \tag{19}$$

This outcome is identical to the equilibrium of the model with interviews in which the threshold interview performance, $\theta^* = 0$. This means that $(0, \eta)$ is an equilibrium for any interview technology for which $\pi(0, \eta) > rV$. From the definition of $\pi(.,.)$ and the solution for rV, a necessary and sufficient condition on the interview technology, $h(.)$ for the existence of a $(\theta^*, \mu) = (0, \eta)$ equilibrium is

$$h(0) \geq \frac{\alpha(1-\eta)}{2r+\alpha(1-\eta)} \tag{20}$$

It follows from (19) that in any economy without interviews or in a $(0, \eta)$ equilibrium,

$$W = \frac{\alpha\eta}{r(2r+\alpha)}$$

and the size of the unemployment pool, $u = 1/\alpha$.

3.2 Complete Information

If firms can perfectly identify the workers' productivities there is no private information and productive workers get hired by the first firm they meet. If V_p represents the value to being productive in such a world, standard analysis reveals

$$rV_p = \frac{\alpha}{2r+\alpha}$$

As for the unproductive workers, firms would hire them at a wage of 0; $V_u^p = 0$. Here,

$$W = \frac{\alpha\eta}{r(2r+\alpha)}$$

An important point to note is that the complete information outcome will not be the limiting allocation as the interview technology approaches perfection. This is because whenever there is some chance of confusion as to the productivity of a worker the unproductive will not work for nothing. They will necessarily mount up in the unemployment pool and impact the prior probability that any firm has as to whether any worker is productive. That is, as realized values of θ for productive workers approach 1, μ will approach 0 so what happens to the distribution of $\pi(\theta, \mu)$ is truly ambiguous. This will be demonstrated in the examples that follow.[12]

3.3 Multiplicity of Equilibrium

It has been shown above that both $\theta^*(.)$ and $\mu(.)$ are downward sloping in (θ^*, μ) space. It is well known that such outcomes lead to "multiplier" effects - equilibrium outcomes are very sensitive to small changes in parameters. When the slopes of the "reaction functions" are sufficiently similar multiplicity of equilibrium can occur.[13] Figure 1 depicts such a case. The curve marked SS represents the steady-state relation $\mu(.)$ from (14). The curve marked EM is the efficient matching relation $\theta^*(.)$ from (13)

While no general condition for the existence of multiple equilibria have been found, they are straightforward to construct if h is piecewise linear.[14]

[12] A more general formulation incorporates a common death rate, δ, for all workers. (The model analyzed in this paper has $\delta = 0$.) When $\delta > 0$, the total population is stationary but the monotonicity of the steady-state relation $\mu(\theta^*)$ is lost. This is because as θ^* approaches 1, the death rate dominates the matching rate and the share of high productivity workers in the unemployment pool converges back to η.

With $\delta > 0$, as the population no longer grows without bound, the discontinuity pointed out here does not occur. In general, however, the results developed here are still valid as long as δ is small enough relative to α.

[13] A similar discussion can be found in Smith and Wright [1992]

[14] For example if $r = 0.1$, $\alpha = 0.31$, $\eta = 0.5$, and

$$h(\theta) = \begin{cases} 0.5 \text{ for } 0 < \theta \leq 0.4 \\ \dfrac{10\theta - 3}{2} \text{ for } 0.4 < \theta \leq 0.6 \\ 1.5 \text{ for } 0.6 < \theta \leq 1 \end{cases}$$

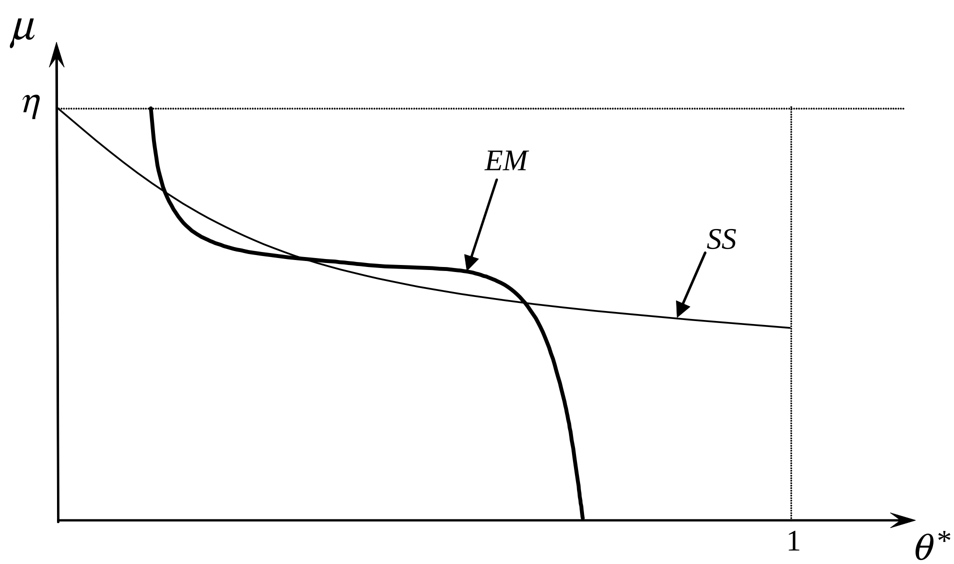

Figure 1: Multiple steady-state equilibria

Multiplicity occurs because a high interview score threshold means that productive workers get hired more quickly than unproductive workers. When this happens the average quality of the unemployed workers, μ, is low. Whenever μ is low, firms require high interview performance. Conversely, a low interview threshold score causes the quality of the unemployment pool to be high and firms need not be very picky. Multiple equilibria of this type provide a purely statistical basis for discrimination. For example, if for historical reasons employers believe that the average unemployed black worker is less productive than the average unemployed white worker, the former group will face a more exacting matching standard. In this model, when there are multiple equilibria, this higher matching standard generates a difference in average unemployed worker quality sufficient to sustain the higher matching standard.

From Claim 5 we know that, under efficient matching, V_p, and V_u decrease as μ decreases. Both productive and unproductive workers are worse-off in equilibria with a higher threshold interview performance. This means that W is lower for equilibria with lower values of μ. To see why this happens, notice that higher θ^* and lower μ both raise u, the size of the unemployment pool. Even though a higher threshold means that productive workers get hired relatively more quickly, it also means that in absolute terms their matching rate, $\alpha(1 - H(\theta^*))$, is lower. The up-shot is that the bad equilibria are characterized by longer average spells of unemployment and lower hiring wages.

3.4 Comparative statics

The possibility of multiple steady-state equilibria raises the issue of what is meant by comparative statics. The smoothness of $\theta^*(.)$ and $\mu(.)$ imply (through Sard's theorem) that (generically) at any equilibrium either

$$\mu'(\theta^*) > \frac{d\theta^{*-1}(\theta^*)}{d\theta^*} \text{ or } \mu'(\theta^*) < \frac{d\theta^{*-1}(\theta^*)}{d\theta^*}. \tag{21}$$

There are 3 steady-state equilibria: $(0, 0.5)$, $(0.2689, 0.4581)$, $(0.4043, 0.4275)$. The Matlab$^{\mathrm{TM}}$ code is available from the author.

where $\theta^{*-1}(.)$ is the inverse function of $\theta^*(.)$. While true dynamics are beyond the scope of this analysis, heuristically, when the former inequality applies, a slight deviation from equilibrium tends to lead the economy away from steady-state. To see this, consider a deviation to a value of θ^* below the equilibrium value. The implied value of μ consistent with steady-state, from equation (14), will be larger than that associated with efficient matching. Consequently, μ increases and θ^* falls moving the economy further from the equilibrium. Here, comparative statics will be provided on the basis of equilibrium being stable in this sense (i.e. the latter inequality in equation (21) applies). It is well known that in such models all stable equilibria have qualitatively the same comparative statics and that the comparative static results for the unstable equilibria will be opposite to those for the stable equilibria. Of course some equilibria can disappear as parameters change and so the results are necessarily local in nature.

An increase in α (or a decrease in r) shifts the EM curve to the right at every value of μ. Matching becomes more selective (θ^* rises) which lowers the steady-state proportion of productive workers (μ falls). An increase in η shifts the SS curve upwards for every value of θ^*. With more productive workers around, matching becomes less selective which leads to a further increase in μ.

In each case, the fact that the efficient matching relation and the steady-state condition imply downward sloping loci in (θ^*, μ) space lead to multiplier effects. Outcomes are very sensitive to changes in the environment. This means that whenever workers can be distinguished in to groups, due to say ethnicity, exogenous differences between the groups are exaggerated by the firms' rational choices to use the group average as a prior on individual productivity.

4 Changes in the Interview technology, H.

Here we consider a change in the accuracy of the interview technology. Attention is restricted to variations in H that satisfy the monotone likelihood property (MLP). That is, interview technology 2 is deemed more *accurate* than interview technology 1 if the likelihood ratio, $h_2(\theta)/h_1(\theta)$, is increasing in θ. This implies that H_2 first-order stochastically dominates H_1 (see Milgrom [1981]) which means that for any given performance threshold, the probability that a productive worker gets hired on the basis of interview 1 is lower than the probability that a productive worker gets hired on the basis of interview 2.

As $H_2(\theta) < H_1(\theta)$ for any θ in $(0, 1)$, it should be clear from equation (14) that moving to a more accurate interview decreases steady-state μ for every given value of θ^* – the *SS* curve shifts down. Essentially as the interviewing gets more precise, firms are more able to distinguish the productive workers leaving less of them among the unemployed.

Claim 9 *Efficient matching, equation (13), implies that for any given value of μ, an increase in interview accuracy makes matching more selective - the EM curve shifts to the right.*

Proof. See Appendix ■

These shifts are demonstrated in Figure 2 for a unique equilibrium. As discussed earlier, when there are multiple equilibria this analysis will be relevant for those that are stable. The solid lines represent the *EM* and *SS* curves prior to the change in H. The dashed lines represent the *EM* and *SS* curves after the change in H. The combined effect of the increased accuracy (Point 1 to Point 2) leads to a fall in μ and increase in θ^*.

Also of interest is the effect of increased accuracy on π^*, the productivity of the marginal worker. In Figure 2, point X represents the immediate impact of the universal firm adoption of the more accurate interview technology (*i.e.* what happens when μ is held constant).

Claim 10 *With μ held constant, efficient matching implies that an increase in accuracy leads to an increase in $\pi(\theta^*, \mu)$.*

Proof. See Appendix ■

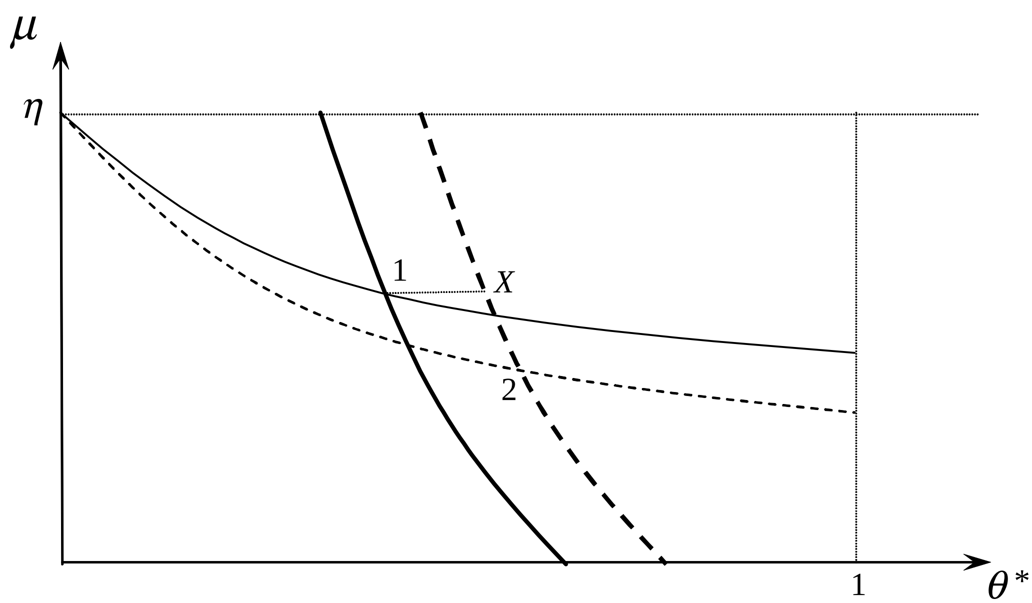

Figure 2: Changes in $H(.)$

In the absence of any quality adjustment in the unemployment pool, improved accuracy necessarily makes the productive workers better off. The effect on their matching rate $\alpha(1 - H(\theta^*))$ can go either way but the distribution of wages they receive on match formation improves enough to make them better off.

Point X however, is above the SS curve which means that productive workers would be getting jobs in greater proportions than in the inflow - μ has to fall. From Claim 5 the implied South Easterly movements along the EM curve (from point X to point 2) makes $\pi(\theta^*, \mu)$ fall. This is because the implied reduction in the matching rate from the worsening prior out-weighs the benefits from increased selectivity even for the productive workers. So, in general, the overall impact of increased accuracy has an ambiguous effect on π^*. More simply put, more accurate interviews are initially good for productive workers but the ensuing dilution of the unemployment pool tends to reduce the rate at which they get jobs and reduce wages in jobs they do get. This ambiguity is explored further in the examples.

4.1 Firm adoption of interview technologies

To reduce the complexity of the preceding analysis we have so far imposed the interview technology on the environment. Yet, a positive analysis should ask whether the adoption of a more accurate interview technique is in a firm's private interest.

Consider what happens if a firm unilaterally adopts technology H_n such that $H_n < H$ for all θ. Let θ_n be the performance threshold implied by the unilateral adoption of H_n. That is

$$\pi_n(\theta_n, \mu) = rV_p = \pi(\theta^*, \mu)$$

where

$$\pi_n(\theta, \mu) \equiv \frac{\mu h_n(\theta)}{\mu h_n(\theta) + 1 - \mu}$$

and h_n is the density of H_n. So

$$\frac{\mu h_n(\theta_n)}{\mu h_n(\theta_n) + 1 - \mu} = \frac{\mu h(\theta^*)}{\mu h(\theta^*) + 1 - \mu} \Longleftrightarrow h_n(\theta_n) = h(\theta^*)$$

As the individual vacancy is measure zero in the set of all open vacancies, neither V_p nor the rate at which the firm encounters unemployed workers are affected by adoption of the new technology. The question becomes: knowing threshold θ_n will apply if interview technology H_n is adopted, is it in a firm's interest to adopt the new technology?

We saw from equation (11) that firms get a fixed share of the *pre-interview* expected match surplus. Given rV_p is unaffected by the individual firm choice, for any threshold interview performance, t, the *pre-interview* expected match surplus would be

$$S_n(t) \equiv \mu\,[1 - rV_p]\,[1 - H_n(t)] - (1-\mu)\,rV_p\,[1-t]$$

Lemma 11 $\theta_n = \arg\max_t S_n(t)$

Proof. The first-order condition implies that $\hat{t}$, the turning point value of t, is given by

$$-\mu\,[1 - rV_p]\,h_n(\hat{t}) + (1-\mu)\,rV_p = 0$$

The second order condition is clearly negative. Substituting for $rV_p = \pi(\theta^*, \mu)$ and from the definition of $\pi(.,.)$ we get $h_n(\hat{t}) = h(\theta^*)$ ■

So holding V_p and μ fixed, the threshold value of interview performance that emerges under unilateral adoption of technology H_n is also the threshold that is *ex ante* optimal for the firm adopting that technology. So,

$$\begin{aligned} S_n(\theta_n) &\geq \mu\,[1 - rV_p]\,[1 - H_n(\theta^*)] - (1-\mu)\,rV_p\,[1-\theta^*] \\ &> \mu\,[1 - rV_p]\,[1 - H(\theta^*)] - (1-\mu)\,rV_p\,[1-\theta^*] \end{aligned}$$

The last inequality comes from the change in the interview technology.

As firms do not take account of the impact of their adoption choice on other market participants, they will always adopt the most accurate interview technique even when it is not in their collective, long-term, interest to do so.

4.2 Example

To demonstrate further how changes in the accuracy of the interview technology can affect outcomes it is necessary to turn to an example.

The example is based on the underlying signal distributions being normally distributed. Thus $H = \mathcal{N}(1, \sigma^2)$ and $G = \mathcal{N}(0, \sigma^2)$ respectively. To see that the MLP exists between H and G recall that we require $h(\theta)/g(\theta)$ to be increasing. From the definition of the normal density we have

$$\frac{h(\theta)}{g(\theta)} = \exp\left(\frac{2\theta - 1}{2\sigma^2}\right) \tag{22}$$

which is clearly increasing in θ. The measure of accuracy here will be $1/\sigma$. We also require therefore, that a change in σ induces the MLP in the implied change in the functional form of $\hat{h}(\theta)$ which is the density of $\hat{H}$, defined as

$$\hat{H}(y) \equiv H(G^{-1}(y))$$

where $G^{-1}(.)$ is the inverse function of $G(.)$. So,

$$\hat{h}(y) = h(G^{-1}(y))\frac{dG^{-1}(y)}{dy} = \frac{h(G^{-1}(y))}{g(G^{-1}(y))} \tag{23}$$

In the case at hand, for the MLP to hold with respect to changes in $\hat{H}(.)$, we require that $\sigma_1^2 < \sigma_2^2$ implies

$$\mathcal{A} \equiv \hat{h}_1'(y)\hat{h}_2(y) - \hat{h}_1(y)\hat{h}_2'(y) > 0$$

where $\hat{h}_i$ is the density induced by σ_i^2. Using equation (23) (and suppressing arguments for notational brevity) we have

$$\mathcal{A} \equiv \left[\frac{h_1'g_1 - h_1g_1'}{g_1^2}\right]\left(\frac{1}{g_1}\right)\left(\frac{h_2}{g_2}\right) - \left[\frac{h_2'g_2 - h_2g_2'}{g_2^2}\right]\left(\frac{1}{g_2}\right)\left(\frac{h_1}{g_1}\right)$$

So the sign of $\mathcal{A}$ is the same as that of

$$\left[\frac{d}{d\theta}\left(\frac{h_1(\theta)}{g_1(\theta)}\right)\right]\left(\frac{1}{h_1}\right) - \left[\frac{d}{d\theta}\left(\frac{h_2(\theta)}{g_2(\theta)}\right)\right]\left(\frac{1}{h_2}\right)$$

For normal distributions, from (22), this is equal to

$$\frac{\exp(\theta^2/2\sigma_1^2)}{\sigma_1} - \frac{\exp(\theta^2/2\sigma_2^2)}{\sigma_2}$$

$1/\sigma$	θ^*	μ	V_u	V_p	Accept	W	u
0	0.0000	0.5000	3.571	3.571	1	3.571	2
0.2	0.0029	0.4997	3.563	3.605	0.9985	3.584	2.004
0.25	0.0170	0.4979	3.549	3.613	0.9911	3.581	2.026
0.4	0.1480	0.4793	3.422	3.587	0.9258	3.504	2.254
1.0	0.7628	0.2793	2.137	3.246	0.6120	2.692	5.851
2.5	0.9986	0.0045	0.023	2.571	0.3127	1.297	715.6
Full	1	0.5	0	7.143	1	3.571	2

Table 1: Simulation results for model with underlying normal signal distributions

which is strictly positive as $\sigma_1 < \sigma_2$.

Providing general results on how the important variables change with accuracy $(1/\sigma)$ has proven too difficult so parameters were chosen to best demonstrate the general patterns that emerge. I used $r = 0.1$, $\alpha = 0.5$ and $\eta = 0.5$. A wide range of other parameterizations have been used with similar qualitative results. No examples of multiple steady-state equilibria have been found using normal distributions.[15] Table 1 summarizes the results for selected values of $(1/\sigma)$. Each column heading corresponds to previous definitions. The column headed "Accept" reports the acceptance rate of productive workers. The first row refers to when there is no interviewing. The last row refers to the economy with complete information.

The trend that emerges is that whenever the acceptance rate of productive workers is relatively high, they do better with interviews than without and interviewing can improve overall welfare. But, this only happens when the interview technology is relatively imprecise. As accuracy improves productive workers find work harder and harder to get and the improved conditions of employment do not sufficiently compensate them for the longer durations of unemployment. Figure 3 further demonstrates this effect by

[15] Qualitatively similar results have been found (with no multiplicity) if G is uniform on [0,1] and $F(\theta) = \theta^n$ for $n > 0$.

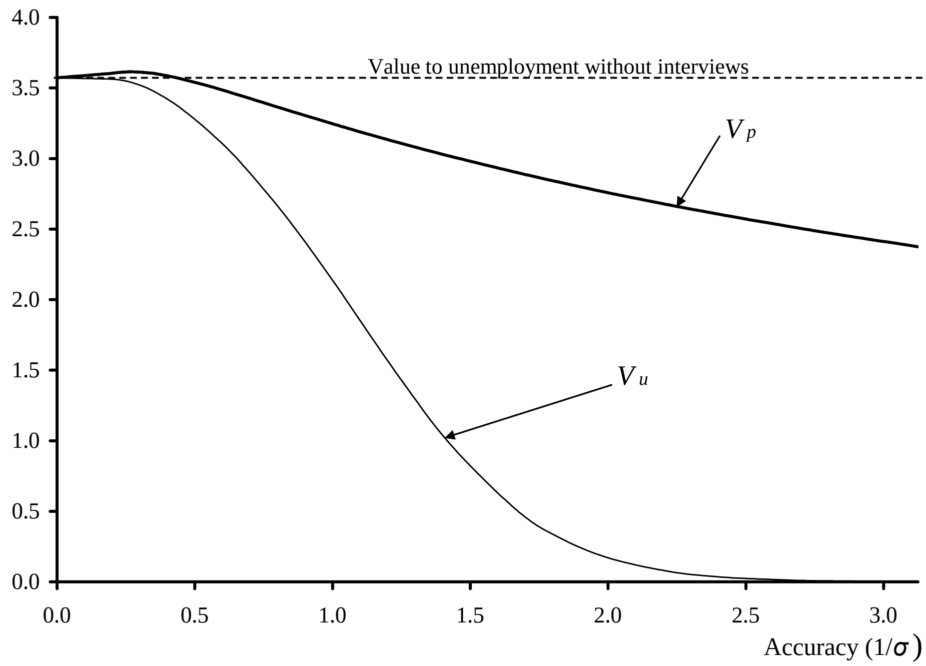

Figure 3: Plot of Value Functions against Accuracy for Numerical Example.

plotting V_u and V_p against accuracy $(1/\sigma)$ for the same parameter values.[16]

5 Extensions

As it stands, in the model there is no opportunity cost to matching for workers or firms so that matching is always weakly efficient. Alternative

[16] To get the average acceptance rate for all unemployed workers down to the 1 in 9 found by Autor and Scarborough [2005] along with an average unemployment duration of 13 weeks requires a more realistic ratio α/r of around 200. At those values the turning point in the graph of V_p with respect to $1/\sigma$ is imperceptibly close to the vertical axis. Other than that, the graphs look very similar. The current parameters were chosen to demonstrate the possibility that interviewing can be beneficial relative to no interviewing at all. Matlab™ code is available from the author.

formulations that avoid this would be to incorporate an uninsurable up-front capitalization cost, k, of vacancy creation or a flow value, b, of leisure. Both $k > 0$ and $b > 0$ have been explored numerically. Actually, $k > 0$ is very simple to accommodate. As long as $rk < 1$ it does not qualitatively affect anything. It is equivalent to reducing the productivity of the workers. The threshold level of interview performance increases with k but as long as there is an expected match surplus after the interview, the worker gets hired. Similarly, for b between 0 and η, steady-state equilibria like those described above can be supported. What happens if $b \geq \eta$ is left for future work.

An important simplification made here has been that α is exogenous. To the extent that interviewing interferes with the matching process, a more standard matching function would lead to workers matching even more slowly, in the example above the the steady-state number of vacancies falls along with value functions for the workers as the accuracy of the interview technology improves.

Another important simplification was that the duration of unemployment is private information to the worker. If the firm knew the worker's duration of unemployment this could be used to infer a more accurate prior on the worker's productivity. This would feed back into the value functions for workers so that V_p and V_u would depend on duration of unemployment. This would provide a model of long-term unemployment which (unlike Blanchard and Diamond [1994]) would be robust to the terms of engagement depending on the duration of unemployment. The composition effects studied here would continue to matter but would play out over a worker's unemployment spell. While this is a potentially interesting extension it goes beyond the scope of the current paper.

An alternative formulation would be to allow for directed search. With all the jobs being *ex ante* identical this would result in equilibria with random meetings. Firms would however have to commit to some aspects of their hiring policies. The most obvious possibilities are the wage and interview performance threshold, or a whole wage function which maps interview performance into pay. The former option requires a strong level of commitment from the firm as it will inevitably involve turning down workers with whom there is expected gains from trade. The latter option, posting a wage function, just does not ring true. Firms might say "salary dependent on qualifications" but job postings do not usually specify a mapping between qualifications and salaries that covers all contingencies. More likely, the people involved in hiring are given some "wiggle room" in which to negotiate with candidates. In any case it is not obvious that directed search would change the results much - the steady-state requirement that whoever gets hired have to be similar to those that enter unemployment would still hold.

An important issue intoned to above but ignored in this paper is the effects of statistical discrimination on incentives to invest in human capital. Compared with a world in which there is no interviewing at all, interviews necessarily increase incentives to become productive. However, examples indicate that increasing accuracy of the interview technology does not necessarily improve incentives to invest. Figure 3 shows that $V_p - V_u$ can decrease as interviews get more accurate.

One direction to take this research would be include firm heterogeneity either in terms of the job productivity or in terms of the industrial specialization. Match specific heterogeneity could also be incorporated. This would generate a more general role for the interview and potentially reverse some of the efficiency results. Still, depending on the relative magnitude of the components to match output, the effects highlighted in this paper should continue to be relevant.

6 Conclusion

This paper provides a market based model of interviews where worker productivity is private information. An individual's productivity is the same at every firm. The use of interviews generates composition effects which can be used to shed light on how labor market outcomes can differ widely across social groups with very similar or even identical employment related attributes. Because of these composition effects the long-run impact of the adoption of more accurate interview techniques may simply lead to a slow-down in the rate of matching without any welfare gain - even for the more productive workers.

7 Appendix

7.1 Proof of Lemma 1

Proof. For the purpose of the proof $\pi = \pi(\theta, \mu)$, and V_i, $i = p, u, f$ will be taken as a parameter of the game.

Firm action: wage offer $w_f \in [0,1]$

Worker action: picks probability of acceptance, $a_i \in [0,1]$, $i = p, u$

Then,

Firm pay-offs: $\begin{cases} a_p(1-w_f) + (1-a_p)rV_f \text{ if worker type } p \\ -a_u w_f + (1-a_u)rV_f \text{ if worker type } u \end{cases}$

Type i worker pay-offs: $a_i w_f + (1-a_i)rV_i$, $i = p, u$

Firm strategies: $w_f \in [0,1]$

Type i worker strategies: $a_i : [0,1] \to [0,1]$

A sub-game perfect equilibrium is a triple $\{w_f^*, a_p^*, a_u^*\}$ such that

$$\begin{aligned} a_i^*(w) &= \arg\max_{a \in [0,1]} \{aw + (1-a)rV_i\} \\ w_f^* &= \arg\max_{w \in [0,1]} \left\{ \begin{array}{c} \pi a_p^*(w)(1-w) - (1-\pi)a_u^*(w)w + \\ \left[\pi(1-a_p^*(w)) + (1-\pi)(1-a_u^*(w))\right] rV_f \end{array} \right\} \end{aligned}$$

Equilibria have the form of

$$w_f^* = \begin{cases} rV_p \text{ for } \pi \geq r(V_f + V_p) \\ \omega < rV_u \text{ otherwise} \end{cases}$$

$$a_i^* = \begin{cases} 1 \text{ if } w \geq rV_i \\ 0 \text{ otherwise} \end{cases} \text{ for } i = p, u$$

There is a continuum of equilibria; one for every $\omega \in [0, rV_u]$ but they are all pay-off equivalent ■

7.2 Proof of Lemma 2

Proof. Type i worker action: wage offer, $w_i \in [0,1]$

Firm action: picks probability of acceptance, $a_f \in [0,1]$,

Firm pay-offs: $\begin{cases} a_f(1-w_p) + (1-a_f)rV_f \text{ if worker type } p \\ -a_f w_u + (1-a_f)rV_f \text{ if worker type } u \end{cases}$

Type i worker pay-offs: $a_f w_i + (1-a_f)rV_i$, $i = p, u$

Type i worker strategies: $w_i \in [0,1]$

In formulating their strategies firms will up-date their beliefs, $\tilde{\pi} : [0,1] \to [0,1]$ as to the productivity of the worker based on the wage the worker offers.

Firm strategies: $a_f : [0,1] \to [0,1]$

A perfect Bayesian equilibrium is a triple $\{w_p^*, w_u^*, a_f^*, \tilde{\pi}\}$ such that

$$a_f^*(w) = \arg\max_{a\in[0,1]} \{a[\tilde{\pi}(w)(1-w) - (1-\tilde{\pi}(w))w] + (1-a)rV_f\} \quad (24)$$

$$w_i^* = \arg\max_{w\in[rV_p,\pi-rV_f]} \{a_f^*(w)w + (1-a_f^*(w))rV_i\} \quad (25)$$

As perfect Bayesian equilibrium does not restrict out-of-equilibrium beliefs, whenever $\pi > r(V_f + V_p)$ every $w \in [rV_p, \pi - rV_f]$ can be supported as an equilibrium by the belief of the firm that offers other than the equilibrium wage will only be made by unproductive workers. ■

7.3 Proof of Claim 9

This analysis makes use of the $\Psi(.,.)$ function as defined in (5). As $\Psi(\theta^*, \mu)$ is increasing in θ^* the effect of increased accuracy on θ^* will be the negative of its effect on $\Psi(\theta^*, \mu)$. I use the variational method, an alternative approach to the same result is to use the Voltara Derivative as in Ryder and Heal [1976].

Suppose, $\hat{\theta} > 0$ is any threshold value of the interview score and θ is any interview score such that $\theta > \hat{\theta}$. Define $\Delta(.)$ to be any small change in h such that $\Delta + h$ is a density function, continuously differentiable and more accurate than h in the sense of MLP. The implied restrictions on Δ are

$$\int_0^1 \Delta d\theta = 0 \quad \text{and for any } \theta > \hat{\theta}, \quad \frac{\Delta(\theta)}{h(\theta)} > \frac{\hat{\Delta}}{\hat{h}} \quad (26)$$

where $\hat{\Delta} \equiv \Delta(\hat{\theta})$ and $\hat{h} \equiv h(\hat{\theta})$. Also MLP implies that there is a unique $\tilde{\theta}$ such that $\Delta(\tilde{\theta}) = 0$. To see this, recall that MLP requires

$$\frac{d}{d\theta}\left(\frac{\Delta(\theta) + h(\theta)}{h(\theta)}\right) > 0$$

so (using the prime to represent differentiation with respect to θ), for any θ we have $\Delta' h - \Delta h' > 0$. This implies that $\Delta'(\tilde{\theta}) > 0$ which precludes multiple crossings. Notationally, we use $\tilde{h} \equiv h(\tilde{\theta})$.

We wish to obtain the effect of the change $\varepsilon\Delta$ to h on $\Psi(\hat{\theta}, \mu)$ where ε is any scalar such that $\varepsilon > 0$. By the way Δ was defined, it is clear that $\varepsilon\Delta + h$ is also a density function and represents a more accurate interview than h.

Defining the functional $I(h)$ by

$$I(h) \equiv \int_{\hat{\theta}}^{1} \frac{(h - \hat{h})h}{(\mu h + 1 - \mu)\hat{h}} d\theta$$

We have

$$I(\varepsilon\Delta + h) - I(h) \equiv \int_{\hat{\theta}}^{1} \frac{(\varepsilon\Delta + h - \varepsilon\hat{\Delta} - \hat{h})(\varepsilon\Delta + h)}{[\mu(\varepsilon\Delta + h) + 1 - \mu]\left(\varepsilon\hat{\Delta} + \hat{h}\right)} d\theta - \int_{\hat{\theta}}^{1} \frac{(h - \hat{h})h}{(\mu h + 1 - \mu)\hat{h}} d\theta \tag{27}$$

Define

$$I'(h|\Delta) \equiv \lim_{\varepsilon \to 0} \frac{I(\varepsilon\Delta + h) - I(h)}{\varepsilon}$$

as the derivative of $I(h)$ with respect to Δ. $I'(h|\Delta)$ is how the functional $I(h)$ changes as h moves infinitesimally toward $\Delta + h$ under the restrictions imposed by (26).

A first order Taylor series expansion of (27) around $\varepsilon = 0$ implies

$$I'(h|\Delta) = \int_{\hat{\theta}}^{1} \left[\frac{\mu h^2 + (1-\mu)(2h - \hat{h})}{(\mu h + 1 - \mu)^2 \hat{h}}\right] \Delta d\theta - \hat{\Delta} \int_{\hat{\theta}}^{1} \frac{h^2}{(\mu h + 1 - \mu)\hat{h}^2} d\theta. \tag{28}$$

The sign of $I'(h|\Delta)$ is not yet obvious as Δ is negative for $\theta < \tilde{\theta}$.

Now suppose Δ is chosen so that so that $\tilde{\theta} > \hat{\theta}$. In this case $\hat{\Delta}$ is negative and the second term in (28) is positive. Furthermore, as

$$\frac{d}{d\theta}\left[\frac{\mu h^2 + (1-\mu)(2h - \hat{h})}{(\mu h + 1 - \mu)^2 \hat{h}}\right] = \frac{2(1-\mu)(\mu\hat{h} + 1 - \mu)}{(\mu h + 1 - \mu)^3 \hat{h}} > 0,$$

the integrand in the first term of (28) is positive and increasing in θ for all $\theta > \hat{\theta}$. So

$$\frac{\mu h^2 + (1-\mu)(2h - \hat{h})}{(\mu h + 1 - \mu)^2 \hat{h}} \left\{ \begin{array}{c} \geq \\ \leq \end{array} \right\} \frac{\mu \tilde{h}^2 + (1-\mu)(2\tilde{h} - \hat{h})}{(\mu \tilde{h} + 1 - \mu)^2 \hat{h}} \left\{ \begin{array}{c} \theta \geq \tilde{\theta} \\ \hat{\theta} \leq \theta \leq \tilde{\theta} \end{array} \right.$$

As $\Delta(\theta) < 0$ for $\theta < \tilde{\theta}$

$$\int_{\hat{\theta}}^{1} \left[\frac{\mu h^2 + (1-\mu)(2h - \hat{h})}{(\mu h + 1 - \mu)^2 \hat{h}} \right] \Delta d\theta > \frac{\mu \tilde{h}^2 + (1-\mu)(2\tilde{h} - \hat{h})}{(\mu \tilde{h} + 1 - \mu)^2 \hat{h}} \int_{\hat{\theta}}^{1} \Delta d\theta > 0.$$

For $\tilde{\theta} > \hat{\theta}$ both terms in (28) are positive.

If Δ is chosen so that so that $\tilde{\theta} \leq \hat{\theta}$, $\hat{\Delta} \geq 0$ and the second term of (28) is negative. However, as indicated above in (26), in this case $\Delta \geq \hat{\Delta} h / \hat{h}$. So

$$\begin{aligned} I'(h|\Delta) &> \int_{\hat{\theta}}^{1} \left\{ \left[\frac{\mu h^2 + (1-\mu)(2h - \hat{h})}{(\mu h + 1 - \mu)^2 \hat{h}} \right] \hat{\Delta} \frac{h}{\hat{h}} - \frac{\hat{\Delta} h^2}{(\mu h + 1 - \mu) \hat{h}^2} \right\} d\theta \\ &= \hat{\Delta} \int_{\hat{\theta}}^{1} \frac{(1-\mu) h (h - \hat{h})}{(\mu h + 1 - \mu)^2 \hat{h}^2} d\theta > 0 \end{aligned}$$

Consequently, $I(h)$ increases with the accuracy of h and so $\Psi(\hat{\theta}, \mu)$ is decreasing with accuracy of h for every $\hat{\theta}$. This means that $\Psi(\theta^*, \mu)$ must also fall with accuracy and θ^* therefore rises at every value of μ.

7.4 Proof of Claim 10

Maintaining the notation from the Proof of Claim 9, we examine the impact of a change in the density function of the form $\varepsilon \Delta(\theta)$ where ε is an infinitesimal scalar and Δ is subject to restrictions (26). We define θ_ε^* to be the value of θ^* that solves (13). Also define π_ε^* to be the value of $\pi(\theta_\varepsilon^*, \mu)$ associated with the change in H while holding μ fixed. Then

$$\pi_\varepsilon^* - \pi^* = \frac{\mu \left[\varepsilon \Delta(\theta_\varepsilon^*) - h(\theta_\varepsilon^*) \right]}{\mu \left[\varepsilon \Delta(\theta_\varepsilon^*) - h(\theta_\varepsilon^*) \right] + 1 - \mu} - \frac{\mu h(\theta^*)}{\mu h(\theta^*) + 1 - \mu}$$

A first-order Taylor series expansion around $\varepsilon = 0$, using (28) implies

$$\lim_{\varepsilon \to 0} \frac{\pi_\varepsilon^* - \pi^*}{\varepsilon} = \left[\frac{\mu (1-\mu)}{(\mu h(\theta^*) + 1 - \mu)^2} \right] \left\{ h'(\theta^*) \frac{\alpha(1-\mu) I'(h|\Delta)}{\frac{\partial \Psi}{\partial \theta^*}} + \Delta(\theta^*) \right\} \tag{29}$$

The contents of the square brackets are clearly positive. The first term in the curly brackets represents the indirect effect of the change in H on θ^* and then on $\pi(\theta^*, \mu)$. The second term represents the direct effect of Δ on $\pi(.,.)$. Simple substitution from (28) and (7) into (29) implies that the sign of the curly brackets is that of

$$\int_{\theta^*}^{1} \left[\frac{\mu h^2 + (1-\mu)(2h - h^*)}{(\mu h + 1 - \mu)^2 h^*} \right] \Delta d\theta$$

which the analysis following (28) has shown to be positive. Thus, under efficient matching with μ fixed, increased accuracy increases $\pi(\theta^*, \mu)$.

8 References

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