Credit Card Acceptance, Product Quality and Merchant Fees

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Abstract

This paper explores a reason why retailers pay such large merchant fees to credit card issuers. Credit cards as media of exchange are introduced to a New Monetarist model in which exchange occurs in alternating centralized and decentralized markets. Sellers who exert high (low) effort produce a good with a high (low) probability of being high quality. The quality of the good is revealed only after trade. Buyers who use credit cards commit to repay the purchase price of the good to the card issuer in the next period. If the good is of low quality, the issuer stops payment to the seller and the buyer is not charged. Sellers who exert high effort, price goods to encourage credit card sales and to establish credibility. Thus a benefit offered to buyers can then be used by the credit card network to support charging sellers high merchant fees.

JEL Classification: E42, D82

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1 Introduction

Retailers who use credit card networks pay merchant discount fees of 1% to 3% of the purchase price.[1] This paper contributes to the literature on why merchants have historically been prepared to pay such large fees. The essential mechanism developed here is that credit card issuers will, to varying extents, act as a buyer's attorney in the case of defective merchandise sales.[2] By accepting credit cards, then, a retailer is demonstrating the extent to which he stands behind his product. That is, the retailer uses credit card acceptance as a signal of product quality. Simulations suggest that the ability to signal quality is worth as much as 4.4% of the product value to the retailer.

Methodologically, the approach here is to develop the simplest model that allows for exchange via credit card that also permits transactions in cash. Then I will show that the hypothesized cause of the merchant fees is sufficiently strong to support the kind of fee structure observed in the market place. To provide an environment in which money is endogenously valued, the basic framework follows that of Rocheteau and Wright [2005] (henceforth RW). To abstract from network effects, buyers are assumed to

[1] According to the May 2014 edition of the *Nilson Report* the average fee charged in 2013 was 2.17%.

[2] The documentation that comes with the Mastercard issued by NatWest Bank provides the following description of this service: "Paying in full with a credit card gives useful protection against faulty goods costing between £100 and £30,000. If you have a complaint about a purchase you have made with your NatWest Credit Card, please contact the retailer first. If the retailer can't resolve the issue or has gone out of business, contact us and we will take up the matter on your behalf."

In the USA the Fair Credit Billing Act provides a similar protection for credit card purchases of over $50 made within the purchaser's home state or within 100 miles of his or her residence. In practice, according to industry studies (see Bankrate.com or CreditCards.com) the larger credit card issuers have voluntarily waived the act's $50 and geographic limits and will help in any dispute. Moreover, the Federal Trade Commission specifically recommends on-line purchasers to use credit cards because of the protection afforded by the Fair Credit Billing Act.

be able to direct their search.[3] That is, buyers observe the cash and credit prices offered by sellers and shop accordingly. Sellers who exert a high degree of effort produce goods that have a low probability of a defect. Those that exert low effort face a high probability of a defect. Prior to receiving it, the buyer cannot observe how much effort the seller put into producing the good. Once exchange occurs, however, the quality of the good becomes common knowledge. If the cost of high effort is not too great, the efficient allocation involves the production only of high-effort goods.

In the market economy without credit cards, buyers have no recourse; only low-effort goods are produced. Anyone attempting to incur high effort and charging a higher price would immediately be imitated by a seller who did not incur high effort. When the buyer uses a credit card to acquire the good, however, the credit card issuer will stop payment in the event that the good turns out to be defective. In this case, as long as the cost of high effort is not too great, the market economy can support the production of high-effort goods. Imitation of high-effort producers will not occur because the increased probability of not getting paid outweighs the cost of high effort.

The point of this paper is to show that the desire to establish reputation through acceptance of credit cards can support the imposition of realistic merchant fees. Suppose the cost of high effort is low enough that all retailers are transacting using credit cards. The question becomes: how large can the transaction fees charged of the retailers become before they switch to using cash or incurring low effort? If the direct transaction costs are similar, the main apparent difference is that the credit card provides a grace period to consumers during which cash depreciates due to inflation. We might expect then, that retailers will switch to cash use when the transaction fees exceed

[3] When matching is random, network effects typically generate multiple equilibria in which a critical mass of credit card holders means that retailers are prepared to undergo the set-up costs for accepting them. And, the propensity for cards to be accepted affects the likelihood that a buyer holds a credit card. Masters and Rodriguez-Reyes [2005] and Markose and Loke [2003] investigate such network effects. For a more up-to-date treatment see Lotz and Zhang [2014].

the consumers' cost of inflation. To do so a retailer has to offer a "viable" cash price. That is, a price low enough to induce credit card buyers to bring cash to the market. Viable cash prices are necessarily lower than credit card prices because cash purchases push the risk of the good's defectiveness onto the buyer. As soon as a retailer makes it clear that he prefers cash to credit transactions, however, any association with having exerted high effort is lost. If high-effort retailers attempt to charge viable cash prices for their goods, low-effort retailers will simply swamp the market. Thus, the threshold value for the transaction fee becomes that at which retailers will switch to low effort. In the simulations section I will argue that this can realistically be as large as 4.4%.

A growing body of work looks at how merchant discount fees are set. A common motivation is the idea that they are large relative to the cost of using other media of exchange and the apparent cost of running the network. Rochet and Tirole [2002] and Wright [2010] provide models of oligopolistic or monopolistically competitive retail markets in which consumers differ in their (unmodelled) benefits from using cards versus cash in transactions. Retailers are prepared to pay high transaction fees to attract those customers who like to use cards, away from their competitors. Rochet and Wright [2010] point out that these papers may just as well have been talking about debit cards. Their paper focuses more on the "credit" aspect of credit cards. That is, retailers are prepared to pay the large fees due to the increased set of sales they are able to make because people can trade using their future incomes. Both of these approaches recognize that allowing the parties involved to act strategically to "steal business" from their competitors can bestow greater benefits to card use than comes from the technological attributes of the card alone.

The current paper argues that the value of one of the technological attributes of credit cards, fraud protection for consumers, has been overlooked in the literature. Shy and Wang [2011] quote industry analyses that have found only 0.08% of credit card transaction value is fraudulent. This figure even includes stolen identity cases that I ignore. This does not, however, mean that the fraud protection is worth less than 0.08% of transaction values. Measured fraud is an equilibrium outcome. Knowing that a consumer has some recourse can prevent unscrupulous retailers from participating in the market at all.

McAndrews and Wang [2012] provide a different theory of high merchant fees. Retailers operate in fully contestable markets so that the business stealing effect does not apply. Credit card use incurs a higher fixed cost of adoption but lower marginal cost in use than cash. With consumers distinguished by income level, the constrained efficient allocation (implemented by a Ramsey regulator) involves some cash-only buyers and others (those with higher incomes) who use cards. The authors impose equal cash and credit prices so the Ramsey regulator recognizes that by charging a low merchant fee the cash users, along with the card users, benefit from the ensuing low retail prices. A monopoly credit card issuer, on the other hand, who is only interested in maximizing his own profit and does not care about cash users, will charge a higher merchant fee. Doing so forces retailers to raise their prices which reduces the number of credit card users but increases the issuer's profits. The up-shot is that whenever the market for credit card issuance is not fully contestable, the merchant fee is inefficiently high. In the context of their model, that of the current paper can be viewed as providing some micro-foundation for the lower marginal cost of cards relative to cash.

The theory developed here has similarities with Delacroix and Shi [2013]. Theirs is a one-shot directed search model with an unobserved quality investment by sellers. They allow the buyers to receive a signal as to the true quality of the good. When the quality differential between the goods is sufficiently high, the market economy implements the first best. In the current model, the seller's investment only affects the probability with which the good is high quality and, other than the propensity of sellers to accept credit cards, there is no additional quality signal. Incentive compatibility among sellers accepting credit cards means that even when cards are in use, the market economy may not generate the first best outcome.

The remainder of the paper progresses as follows. Section 2 describes the general model environment. Section 3 ascertains the efficient allocation. Section 4, that considers market economies, constitutes the main body of the paper. After introducing the concept of money it goes on to analyze the market outcomes without, and then with, credit cards. Section 5 discusses some possible variations to the environment. Section 6 concludes.

2 Model Environment

Time is discrete and continues forever. Every period of time is divided into 2 subperiods. During the first subperiod there is a centralized frictionless market (CM) for an homogeneous and perfectly divisible but non-storable good. In the second subperiod, trade occurs in an anonymous decentralized market (DM) that is characterized by search frictions. There are two types of individual in the model: those who consume the good produced in the DM and those who can produce DM goods. Following RW they will be referred to as buyers and sellers respectively. Both buyers and sellers produce and like to consume the CM good. The the total measure of buyers is normalized to 1 while the measure of sellers is $\bar{n}$.

DM goods can be of high or low quality. Consuming a high quality good yields u utils. Not consuming or consuming a low quality good yields 0 utils. Let x_t be the quantity of the CM good consumed and y_t be the quantity of CM good produced in period t. Then the net instantaneous utility of a buyer at date t is

$$U_t^b = v(x_t) - y_t + \begin{cases} u \text{ if a high quality good is consumed.} \\ 0 \text{ otherwise.} \end{cases}$$

The utility function $v(.)$ is twice differentiable, strictly increasing and strictly concave. I also require that there exists an x^* such that $v'(x^*) = 1$ and that $v(.)$ is normalized so that $v(x^*) = x^*$. (Note that $v(.)$ can be chosen so that x^* is arbitrarily large.)

Sellers have the technology to produce a single DM good each period. At the beginning of the CM, they decide whether or not to manufacture their

DM good. Manufacturing occurs at the end of the CM.[4] If they incur high effort, at a utility cost k, the good they manufacture has a probability λ_h of being high quality. If they incur low effort, at a utility cost of 0, the good they manufacture has a probability $\lambda_l < \lambda_h$ of being high quality. Although market entry is a choice for sellers, the zero cost of low effort means that all sellers will actually participate. Neither buyers nor sellers can identify the quality of the good until it is purchased. Once purchased, the quality of the good is common knowledge. Sellers cannot commit to contingent bilateral contracts such as money-back guarantees. Unconsumed goods rot at the end of the period. The instantaneous utility of a seller at date t is

$$U_t^s = v(x_t) - y_t - \begin{cases} k \text{ if the seller incurs high effort} \\ \quad\quad 0 \text{ otherwise} \end{cases}.$$

There is a common discount factor, $\beta < 1$ between periods so that the lifetime utility of an individual type $i = b, s$ is $\sum_{t=0}^{\infty} \beta^t U_t^i$.

In any DM, buyers and sellers will be able to direct their search toward particular submarkets based on the observable characteristics of their potential trading partners. In any submarket, the probability that a buyer gets a trading opportunity is equal to $\alpha(\theta)$ where θ is the ratio of sellers to buyers who enter the submarket. I assume that $\alpha(\theta) \leq \min\{1, \theta\}$, $\alpha(0) = 0$, $\alpha'(\theta) > 0$, $\alpha''(\theta) < 0$ and $\lim_{\theta\to\infty} \alpha(\theta) = 1$. These largely reflect the requirement that the matching probability of the buyers, $\alpha(\theta)$, cannot exceed 1. The matching probability of the sellers (with goods in hand) is $\alpha(\theta)/\theta$ which will also be less than 1 under these restrictions. Constant returns to scale in matching is a further desirable (and commonly assumed) feature of the underlying matching technology. Here, this amounts to $\alpha(\theta)/\theta$ decreasing in θ or $\alpha(\theta) \geq \alpha'(\theta)\theta$. To rule out corner solutions I also assume that $\alpha'(0) = 1$ and $\alpha''(0) > -\infty$ which will be imposed in the remainder of the paper.

A nonstandard assumption I make is that the ratio of sellers to matches in any market is convex in the ratio of sellers to buyers. That is

$$\frac{d^2}{d\theta^2}\left(\frac{\theta}{\alpha(\theta)}\right) \geq 0. \tag{1}$$

[4] All that matters is that the good is produced prior to meeting any buyer.

I will need this because, unlike in the standard RW model, sellers have to produce their good prior to market entry which introduces a hold-up problem. If the ratio of sellers to matches increases too slowly with the market tightness, equilibrium is likely to entail multiple small markets and price dispersion. While such an outcome might be of interest in its own right, it detracts from the focus of the current paper. There is no reason to believe that the main results of this paper do not apply in environments with price dispersion.

A commonly used matching function for one time or discrete time matching models was devised by Den Haan et al [2000]. Microfoundations for it were established by Stevens [2007]. The general form of this DRWS matching function is

$$\alpha(\theta) = \frac{\theta}{(1+\theta^\rho)^{1/\rho}} \qquad \text{for } 0 < \rho < \infty.$$

This matching function satisfies all of the previous assumptions, along with assumption (1), for $\rho \geq 1$.[5]

3 Efficiency

The model does not feature any transitional dynamics so the Planner can and will always choose a stationary path for consumption. Equal treatment implies that contingent on type (buyer or seller), everyone produces and consumes the same amount. The Planner is subject to the same trading frictions as the market but does not require *quid pro quo* for exchange to occur. As utility functions are strictly increasing and goods are perishable, all output brought to any exchange opportunity will be consumed. But where there is no match, the Planner cannot avoid output going to waste. As the cost of production for low-effort sellers is 0, the Planner will have

[5] The urn-ball matching function,

$$\alpha(\theta) = 1 - e^{-\theta}$$

also satisfies these conditions.

everyone produce. The issue is simply how many of the sellers should exert high effort in their production.

The relevant welfare function is

$$W(\pi, x) = (v(x) - x)(1 + \bar{n}) + \alpha(\bar{n})\left[\pi\lambda_h + (1 - \pi)\lambda_l\right] u - \bar{n}\pi k.$$

where π is the share of sellers who exert high effort. The Planner then obtains

$$(\pi_p, x_p) \in \arg \max_{\pi \in [0,1], x \geq 0} W(\pi, x).$$

Clearly, $x_p = x^*$.

For the problem at hand to be of interest, we need the Planner to require that at least some participating sellers exert high effort ($\pi_p > 0$). This will be true as long as $W(\pi, x^*)$ is weakly increasing in π. That is when,

$$\alpha(\bar{n})\left[\lambda_h - \lambda_l\right] u \geq \bar{n}k. \tag{R1}$$

Moreover, whenever restriction (R1) holds with a strict inequality, the Planner will require every seller to exert effort.

A potential concern here is that when we consider the market economy, effort is unobserved so that the preceding analysis affords the Planner an unfair advantage. The question becomes whether or not there is a mechanism available to the planner that can implement the first best outcome. To this end, suppose the Planner simply makes a transfer, τ, in utility out of that going to the buyer whenever the good turns out to be of high quality. Clearly under restriction (R1) setting $\tau = u$ will generate the efficient level of effort. Furthermore, if (R1) holds with a strict inequality, there is some non-empty range of values for $\tau < u$ for which sellers will make the optimal effort choice.

4 Market Economy

4.1 Money

Due to the lack of a double coincidence of wants, some medium of exchange is essential for trade in the DM. Buyers can use money or a credit card to purchase goods. Money is perfectly divisible and agents can hold any non-negative amount. The aggregate nominal money supply M_t grows at a constant gross rate γ so that $M_{t+1} = \gamma M_t$. New money is injected (or withdrawn if $\gamma < 1$) by lump-sum transfers (taxes) in the CM. Following RW, I assume these transfers go only to buyers, but this is not essential for the results. What matters is that transfers do not depend on the choices individuals make. Also, attention is restricted to policies where $\gamma \geq \beta$. For $\gamma < \beta$ there is no equilibrium.

In the CM, the price of goods is normalized to 1 at each date t, while the relative price of money is denoted ϕ_t. Let $z_t = \phi_t m_t$ be the real value of an amount of money m_t. I will focus throughout on steady-state allocations in which aggregate real variables are constant over time. This means that $\phi_{t+1} = \phi_t/\gamma$. I will use individual real money balances, z_t, as the individual's choice variable for money holding rather than m_t.

4.2 Market Economy without credit cards

At this point it might be obvious to any reader that no seller will exert effort if there is no way for them to be punished for bad behavior. This subsection is included to provide a "dry run" for the solution methodology in this relatively simple context. Introducing credit cards will significantly complicate matters.

4.2.1 Timeline

The timeline for one period in the economy without credit cards is shown in Figure 1.

4.2.2 Buyers

Submarkets to the DM are characterized by the real posted price, q, and the ratio of buyers to sellers in that submarket, θ. After recognizing that buyers will only bring enough money to any submarket to acquire the good, the choice over the real price, q, and real money balances, z, carried into the

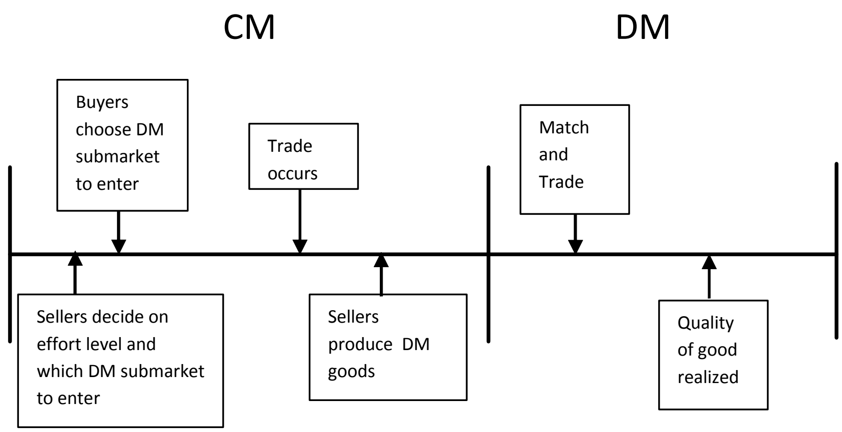

Figure 1: Timeline without credit cards

DM can be condensed to a choice over either. The variable z is, therefore, dropped from the subsequent analysis. This means that for buyers and sellers, q will represent the real balances brought into the CM. Then, as they enter the subsequent submarket (q, θ) of the DM, for buyers q represents the real balances they bring to market, for sellers it represents the real price they post for their good.

Let $V^b(q, \theta)$ be the value to a buyer entering the submarket (q, θ) in the DM for the good he wants to consume. Then,

$$V^b(q,\theta) = \alpha(\theta)\left[\Lambda^b(q,\theta)\,u + \beta W^b(0)\right] + \left[1-\alpha(\theta)\right]\beta W^b\left(\frac{q}{\gamma}\right) \qquad (2)$$

where $\Lambda^b(q, \theta)$ is the buyer's belief as to propensity for goods in market (q, θ) to be of high quality and $W^b(q)$ is the value to carrying the real quantity of money, q, into a CM. Then,

$$\begin{aligned} W^b(q) &= \max_{x,y,\hat{q},\hat{\theta}} \left\{ v(x) - y + V^b(\hat{q},\hat{\theta}) \right\} \\ \text{subject to, } \hat{q} + x &= q + T_b + y \text{ and } x, y \geq 0 \end{aligned} \qquad (3)$$

where T_b is the transfer/tax associated with distribution/withdrawal of new cash. As long as βx^* is bigger than the real amount of cash required to purchase a single high quality good, the non-negativity constraints on x and y will not bind. As x^* can be chosen arbitrarily, the remainder of the paper ignores these constraints. Substituting x out of problem (3) using the equality constraint, implies that $W^b(q) = q + W^b(0)$.

4.2.3 Sellers

In any DM submarket there are potentially two types of seller, those who exerted high effort in the production of their good, type h, and those who exerted low effort, type l. Let $V_i^s(q,\theta)$ be the value to being a type $i = h, l$ seller who makes a good and enters the market (q,θ). Then, as buyers cannot distinguish between seller types,

$$V_i^s(q,\theta) = \frac{\alpha(\theta)}{\theta}\beta W^s\left(\frac{q}{\gamma}\right) + \left(1 - \frac{\alpha(\theta)}{\theta}\right)\beta W^s(0) \tag{4}$$

for $i = h, l$ where $W^s(q)$ is the value to taking real balances q into the next CM. So, $V_h^s(q,\theta) = V_l^s(q,\theta) = V^s(q,\theta)$ and any distinction between high and low-effort producers will be captured by the market, (q,θ), they enter. Then, as sellers have no need for cash in the DM,

$$W^s(q) = \max_{x,y,\hat{\pi}} \left\{ v(x) - y + \hat{\pi} \max_{\hat{q}_h,\hat{\theta}_h} \left[V^s(\hat{q}_h,\hat{\theta}_h) - k\right] + (1-\hat{\pi}) \max_{\hat{q}_l,\hat{\theta}_l} V^s(\hat{q}_l,\hat{\theta}_l) \right\}$$

$$\text{subject to, } x = q + y,\ \hat{\pi} \in [0,1],\ \text{and } x, y \geq 0 \tag{5}$$

where $\hat{\pi}$ represents the choice to exert effort or not. Substituting x out of problem (5) using the equality constraint implies that $W^s(q) = q + W^s(0)$.

4.2.4 Equilibrium

Definition 1 *A symmetric equilibrium is a set of active submarkets, $\Gamma \subset \mathbb{R}_+^2$, to each DM, functions, $\Lambda^b(q,\theta)$ and $\Lambda(q,\theta)$ that specify the buyer beliefs about, and the true proportion of, high quality goods in submarket (q,θ), a function, $n(q,\theta)$, that specifies how many sellers enter submarket (q,θ), and a propensity, π^*, for sellers to exert effort such that:*

1. *Given $\Lambda^b(.,.)$ every $(q^*, \theta^*) \in \Gamma$ solves the sellers' problem (5) for $(\hat{q}_i, \hat{\theta}_i)$ for $i = h$ or l or both.*

2. *Every $(q^*, \theta^*) \in \Gamma$ solves the buyers' problem (3) for $(\hat{q}, \hat{\theta})$.*

3. *Consistency of buyers' equilibrium beliefs: for every $(q^*, \theta^*) \in \Gamma$, $\Lambda^b(q^*, \theta^*) = \Lambda(q^*, \theta^*)$.*

4. *Consistency of seller effort choice:*[6]

$$\pi^* = \int_\Gamma \left(\frac{\Lambda(q,\theta) - \lambda_l}{\lambda_h - \lambda_l} \right) \frac{n(q,\theta)}{\bar{n}} \, dq \, d\theta.$$

5. *The population constraints for sellers and buyers hold:*

$$\int_\Gamma n(q,\theta) \, dq \, d\theta = \bar{n}, \qquad \int_\Gamma \frac{n(q,\theta)}{\theta} \, dq \, d\theta = 1.$$

Symmetry here means that for any given market: all buyers have the same beliefs as to the likelihood that goods are of high quality, buyers have the same propensity to enter with the same amount of cash and, all sellers, contingent on their effort level, have the same propensity to enter. Because production without effort is costless to sellers the population constraint on sellers will bind. This equilibrium corresponds to a perfect Bayesian equilibrium with symmetric buyer beliefs.

4.2.5 Characterization

The first step in identifying the nature of any equilibrium is to recognize that regardless of buyers' beliefs, $\Lambda(q, \theta)$ is either λ_h or λ_l. This is because sellers are ex ante identical and in any market, (q, θ), having exerted effort does not affect the probability with which a seller gets to trade. No seller will ever optimally exert high effort and enter a market that sellers who have exerted low effort will also be optimally entering.

The second step is to recognize that in any active submarket, $\Lambda(q, \theta)$ can only be λ_l. To see why, suppose instead that there is a market (q^*, θ^*) such

[6] The term $\frac{\Lambda(z,\theta)-\lambda_l}{\lambda_h-\lambda_l}$ is the proportion of high effort sellers in submarket (z, θ).

that $\Lambda^b(q^*,\theta^*) = \Lambda(q^*,\theta^*) = \lambda_h$. Then sellers must be sufficiently rewarded by entering (q^*,θ^*) to justify incurring the cost of high effort. But another seller can get the same reward without incurring the cost of effort by simply entering the (q^*,θ^*) market too. Equilibrium must involve only sellers who do not incur effort. Sellers face a moral hazard problem: as soon as they try to incur effort in the production of their good it is impossible for them to charge a higher price for the increased probability of the good being of high quality.

The upshot from this is that in the economy without credit cards the model reverts to that of RW with prior production of (low effort) DM goods. So, any element (q^*,θ^*) of Γ is also a solution to

$$\max_{(q,\theta)} V^s(q,\theta) \quad \text{subject to} \quad V^b(q,\theta) - q = V^b(q^*,\theta^*) - q^* \tag{6}$$

where

$$\begin{aligned} V^s(q,\theta) &= \frac{\alpha(\theta)\beta q}{\theta\gamma} + \beta W^s(0) \\ V^b(q,\theta) &= \alpha(\theta)\lambda_l u + (1-\alpha(\theta))\frac{\beta q}{\gamma} + \beta W^b(0). \end{aligned}$$

For sellers the term $\beta W^s(0)$ represents the value to opting out of the DM. The first term in $V^s(q,\theta)$ represents the probability of making a sale, $\alpha(\theta)/\theta$, times the value of making the sale, $\beta q/\gamma$. Recall that q is the price of the good but that sellers do not reap the benefit of acquiring the cash until the next CM by which time the cash is devalued by the rate of money growth and their utility is discounted. Buyers also automatically get the value, $\beta W^b(0)$, of non-participation in the DM even though they all participate by assumption. The first term in $V^b(q,\theta)$ represents the value to matching in the DM. Since no sellers exert effort, the good they acquire is high quality with probability λ_l. The second term represents the value to failing to match.

Proposition 1 *There exists a unique equilibrium, with a single DM submarket, (q^*,θ^*), that solves problem (6) and $\theta^* = \bar{n}$.*

Proof. See Appendix. ■

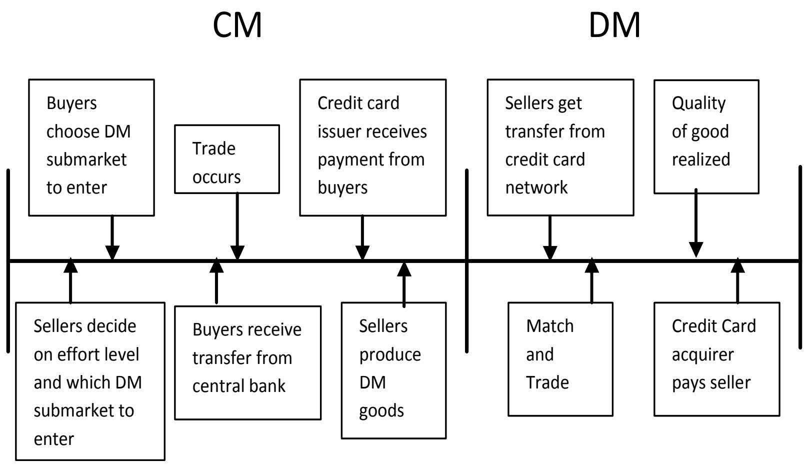

Figure 2: Timeline with credit cards

As anticipated, the absence of any consumer recourse means the only possible outcome is one in which all sellers exert low effort and, under restriction (R1), equilibrium is inefficient.

4.3 Market economy with credit cards

We now suppose that in every DM, sellers have access to a credit card network. At the beginning of the CM subperiod sellers decide on how much effort to exert, the credit price, p, and the cash price, q, for the DM good. The prices become common knowledge and are used by buyers to direct their search. When a good is purchased using a credit card and the quality of the good is verified, the seller receives a cash payment $(1-\omega)p$ where ω is the proportional cost of using the card network (i.e. the merchant fee). The buyer pays p (in cash) to his credit card issuer in the subsequent CM. Buyers are fully committed to the payment. I assume that there is a credit card company that sets ω and runs the network at zero cost.

It is important to think about what happens to the cash in the economy when credit cards are in use. Figure 2 shows a timeline for one period. At the end of the CM the credit card company receives cash in payment from the satisfied buyers who used their card in the preceding DM. To ensure that the company receives at least as much cash at the beginning of the CM as it needs to meet its obligations in the next DM, I will restrict attention to cases where $\gamma(1-\omega) \leq 1$. If this does not hold, the credit card company makes a loss. When this condition holds with a strict inequality the credit card company receives more cash from buyers than it will need to pay off the successful sellers in the next DM. In that case, the company distributes the extra cash lump sum to all sellers (regardless of the prices they post) at the beginning of the DM. Buyers then acquire the cash from the sellers in exchange for CM goods in order to pay off their debts to the company.

An issue here is why do the buyers not settle their debt in the CM good? The credit card company itself does not consume any goods and since they are non-storable, the goods cannot be delivered to the sellers in the DM according to the specified timeline. It would, however, be possible to change the timeline so that all credit card settlement occurs in the CM. From a perspective of realism though, if we view money here as representing M1 then the credit card buyers are simply paying off their debt by check in the period following the purchase. Thus for them, as for real credit card users, money is reduced to a means of settlement rather than a medium of exchange.

All buyers are endowed with credit cards. Using a credit card for purchases means that the payment to the seller can be stopped in the case that the good turns out to be of inferior quality. Buyers decide on whether or not they expect to use their credit card and, based on their beliefs and prices posted by sellers, which submarket to enter. As a seller's effort choice is his private information, buyers cannot direct their search based on that choice. However, I will restrict attention to equilibria in which markets are separated by sellers' effort levels. Of course it will be necessary to ensure that sellers do indeed wish to enter their appropriate market. And, to support consistent buyers' beliefs, it has to be the case that sellers of the wrong type will not want to enter.

Submarkets to each DM are then indexed by the ratio of sellers to buyers in that submarket, θ, the credit price, p, the cash price q, the proportion, ψ, of buyers who are carrying cash, and whether or not the other sellers exerted effort, $i = h, l$.[7]

4.3.1 Buyers

Buyers will either bring a credit card and no cash or, just enough cash to make the purchase. Because of this, cash holding is not a separate decision from the choice of cash transaction price. In what follows, the variable z will be dropped because it will always be identical to q, the real cash price posted by sellers. Buyers entering the DM will be categorized by whether or not they have brought their credit card. Thus, if $V_c^b(p, q, \theta, \psi, i)$ is the value to entering market (p, q, θ, ψ, i) without cash,

$$\begin{aligned} V_c^b(p, q, \theta, \psi, i) &= \alpha(\theta) \left\{ \lambda_i \left[u + \beta W^b \left(-\frac{p}{\gamma} \right) \right] + [1 - \lambda_i] \beta W^b(0) \right\} \\ &\quad + [1 - \alpha(\theta)] \beta W^b(0). \end{aligned}$$

That is, a buyer who intends to use a credit card meets a potential trading partner with probability $\alpha(\theta)$. With probability λ_i the good is of high quality and the buyer enjoys its consumption but has to pay the credit card company in the following CM. With probability, $1 - \lambda_i$, the good is of low quality. The buyer gets zero utility but is not charged for the purchase. With probability $1 - \alpha(\theta)$ there is no encounter.

Then, if $V_m^b(p, q, \theta, \psi, i)$ is the value to carrying real balances q into market (p, q, θ, ψ, i) with intention of spending it,

$$\begin{aligned} V_m^b(p, q, \theta, \psi, i) &= \alpha(\theta) \left\{ \lambda_i \left[u + \beta W^b(0) \right] + [1 - \lambda_i] \beta W^b(0) \right\} \\ &\quad + [1 - \alpha(\theta)] \beta W^b \left(\frac{q}{\gamma} \right). \end{aligned}$$

[7] To be symmetric between buyers and sellers this final element of the index of submarkets could have been π restricted to being either 0 or 1. Using $i = h, l$ reduces the subsequent notation.

In this case the buyer hands over money for the good when an encounter occurs. If it is of high quality he gets u from its consumption. Otherwise he gets zero. With probability $1-\alpha(\theta)$ there is no encounter and he carries the money into the next CM.

Then,

$$W^{b}(q)=\max_{x,y,\tilde{\psi}}\left\{\begin{array}{c} v(x)-y+(1-\tilde{\psi})\max_{\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}}V_{c}^{b}(\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}) \\ +\tilde{\psi}\max_{\hat{p},\hat{q},\hat{\theta}\hat{\psi},\hat{\imath}}V_{m}^{b}(\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}) \end{array}\right\} \tag{7}$$

$$\text{subject to, } \hat{q}+x=q+T_{b}+y.$$

Substituting x out of problem (7) using the equality constraint implies that $W^{b}(q)=q+W^{b}(0)$. The variable $\tilde{\psi}$ represents the probability with which an individual carries cash. This will determine which market he will enter in the next DM. The variable $\hat{\psi}$ therefore represents the proportion of other buyers who carry cash in the DM submarket he enters.

4.3.2 Sellers

In the CM, sellers choose whether or not to exert effort in the production of their goods. Let $V_{j}^{s}(p,q,\theta,\psi,i)$ be the value to being a type $j=h,l$ seller who makes a good and enters the market (p,q,θ,ψ,i). Then, as buyers cannot distinguish between seller types,

$$\begin{aligned} V_{j}^{s}(p,q,\theta,\psi,i) &= \frac{\alpha(\theta)}{\theta}\left\{\begin{array}{c} (1-\psi)\left[\lambda_{j}\beta W^{s}\left(\frac{(1-\omega)p}{\gamma}\right)+(1-\lambda_{j})\beta W^{s}(0)\right] \\ +\psi\beta W^{s}\left(\frac{q}{\gamma}\right) \end{array}\right\} \\ &\quad +\left(1-\frac{\alpha(\theta)}{\theta}\right)\beta W^{s}(0) \quad \text{for } i=h,l \text{ and } j=h,l. \end{aligned}$$

That is, with probability $\alpha(\theta)/\theta$ a seller meets a trading partner. With probability $1-\psi$ the trading partner has no cash and will trade with a credit card. With probability λ_{j} the seller's good is of high quality, a credit card sale is completed and payment is received. With probability $1-\lambda_{j}$ the seller's good is of low quality, and the credit card sale does not go through leaving the seller with nothing. With probability ψ the buyer has cash and a cash price transaction occurs regardless of the type of seller. It is clear from these equations that the propensity with which other sellers in a given market are of type h does not directly affect the returns to being of type h for an individual entrant. Moreover, in the type of equilibrium sought here (defined below) only high (low) type sellers enter high (low) type markets so that either $i = j = h$ or $i = j = l$.

As sellers have no need for cash in the DM,

$$W^s(q) = \max_{x,y,\tilde{\pi}\in[0,1]} \left\{ \begin{array}{c} v(x) - y + \tilde{\pi}\left(\max_{\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}} V_h^s(\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}) - k\right) \\ +(1-\tilde{\pi})\max_{\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}} V_l^s(\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}) \end{array} \right\} \tag{8}$$

$$\text{subject to, } x = q + T_s + y,$$

where T_s is the current period real value of the cash transfer received from the credit card company at the beginning of the previous DM. So, sellers choose CM production based on how much cash they have, they choose the propensity, $\tilde{\pi}$, with which they will exert effort in the production of their DM good and choose the DM submarket to enter. Clearly all buyers and sellers set $x = x^*$. And, $W^s(q) = q + W^s(0)$.

Definition 2 *A symmetric seller separated equilibrium is a set of active submarkets, $\Gamma \subset \mathbb{R}_+^3 \times [0,1] \times \{h,l\}$, to each DM, a function, $n(p,q,\theta,\psi,i)$, that specifies how many sellers enter submarket (p,q,θ,ψ,i), the aggregate propensity for sellers to exert high effort $\tilde{\pi}^* \in [0,1]$, and $\tilde{\psi}^* \in [0,1]$, the aggregate propensity for buyers to bring cash to the DM, such that we have:*

1. *Separation of sellers: $(p,q,\theta,\psi,h) \in \Gamma \Rightarrow (p,q,\theta,\psi,l) \notin \Gamma$*

2. *Buyers' optimality: every $(p^*,q^*,\theta^*,\psi^*,i^*) \in \Gamma$ maximizes either V_c^b, or V_m^b or both.*

3. *Sellers' optimality: Every $(p^*,q^*,\theta^*,\psi^*,i^*) \in \Gamma$ maximizes either V_h^s, or V_l^s or both.*

4. *Buyer population constraint: $1 = \int_\Gamma \frac{n(p,q,\theta,\psi,i)}{\theta}\, dp\, dq\, d\theta\, d\psi\, di$*

5. *Seller population constraint:* $\bar{n} = \int_\Gamma n(p, q, \theta, \psi, i)\, dp\, dq\, d\theta\, d\psi\, di$

6. *Consistency of sellers' effort choice:* $\tilde{\pi}^* = \frac{1}{\bar{n}} \int_\Gamma n(p, q, \theta, \psi, h)\, dp\, dq\, d\theta\, d\psi$,

7. *Consistency of buyers cash versus credit choice:*

$$\tilde{\psi}^* = \int_\Gamma \psi\theta n(p, q, \theta, \psi, i)\, dp\, dq\, d\theta\, d\psi\, di.$$

This definition is central to the paper. The first condition is required because buyers do not directly observe the effort level of the sellers they will impute it from the other elements of the submarket index. If those elements are identical, buyers cannot expect to direct their search appropriately. The second condition requires that each element of Γ solves the buyers' problem. Only when $V_c^b = V_m^b$ and $\tilde{\psi} \in (0, 1)$ will it need to maximize both V_c^b and V_m^b. Condition 3 is symmetric to Condition 2 for sellers.

This equilibrium corresponds to the perfect Bayesian equilibrium that is best for the sellers. If, for instance, all buyers believed that when they observed some arbitrary credit price, p, that the sellers had incurred high effort and for any other price they had not, then, subject to incentive compatibility, high-effort sellers would offer the price p. Instead, here we will allow the sellers to maximize their profits subject to buyers being indifferent across any submarkets that sellers choose to open. That is, buyers have diffuse prior beliefs and use the posted prices and what they know about model parameters to determine their expectations as to which sellers have exerted high effort. I am, therefore, focussing on one of potentially a continuum of perfect Bayesian equilibria. Recall, though that the point of the paper is to quantify the extent to which equilibria with credit cards are robust to realistic values of the merchant fee, ω. The results based on this equilibrium will then provide an upper bound on those values that correspond to any other perfect Bayesian equilibrium.

4.3.3 Characterization

Equilibrium with seller separation does not mean that I impose separation ex ante. Rather, it will simply be the case that no low-effort seller will strictly prefer to enter the high-effort sellers' market and vice versa. I will focus on equilibria with at most two submarkets to each DM. In the most general case Γ contains one submarket, $(p_h, q_h, \theta_h, \psi_h, h)$, for high-effort sellers (market h) and one submarket, $(p_l, q_l, \theta_l, \psi_l, l)$, for low-effort sellers (market l). Thus,

$$\begin{aligned}(p_h, q_h, \theta_h, \psi_h, h) &= \arg\max_{\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}} V_h^s(\hat{p}, \hat{q}, \hat{\theta}, \hat{\psi}, \hat{\imath}) \\ \text{subject to } V_m^b(\hat{p}, q, \hat{\theta}, \hat{\psi}, \hat{\imath}) - \hat{q} &= V_m^b(p_h, q_h, \theta_h, \psi_h, h) - q_h, \\ V_c^b(\hat{p}, \hat{q}, \hat{\theta}, \hat{\psi}, \hat{\imath}) &= V_c^b(p_h, q_h, \theta_h, \psi_h, h) \\ \text{and } V_h^s(p_h, q_h, \theta_h, \psi_h, h) &\geq V_h^s(p_l, q_l, \theta_l, \psi_l, l)\end{aligned} \tag{9}$$

$$\begin{aligned}(p_l, q_l, \theta_l, \psi_l, l) &= \arg\max_{\hat{p},\hat{q},\hat{\theta},\hat{\psi},\hat{\imath}} V_l^s(\hat{p}, \hat{q}, \hat{\theta}, \hat{\psi}, \hat{\imath}) \\ \text{subject to } V_m^b(\hat{p}, \hat{q}, \hat{\theta}, \hat{\psi}, \hat{\imath}) - \hat{q} &= V_m^b(p_l, q_l, \theta_l, \psi_l, l) - q_l \\ V_c^b(\hat{p}, \hat{q}, \hat{\theta}, \hat{\psi}, \hat{\imath}) &= V_c^b(p_l, q_l, \theta_l, \psi_l, l) \\ V_l^s(p_l, q_l, \theta_l, \psi_l, l) &\geq V_l^s(p_h, q_h, \theta_h, \psi_h, h).\end{aligned} \tag{10}$$

The inequality constraints in problems (9) and (10) reflect the requirements that having chosen whether or not to incur effort, sellers at least weakly prefer to enter the appropriate market. For sellers to participate in both requires

$$V_h^s(p_h, q_h, \theta_h, \psi_h, h) - k = V_l^s(p_l, q_l, \theta_l, \psi_l, l) \tag{11}$$

For buyers with and without cash to enter both markets requires,

$$V_m^b(p_i, q_i, \theta_i, \psi_i, i) - q_i = V_c^b(p_i, q_i, \theta_i, \psi_i, i), \quad i = h, l \tag{12}$$

$$V_m^b(p_h, q_h, \theta_h, \psi_h, h) - q_h = V_m^b(p_l, q_l, \theta_l, \psi_l, l) - q_l. \tag{13}$$

After substitution for the value functions, requirements (12) imply that for both cards and cash to be in use in market $i = h, l$

$$\alpha(\theta_i)\lambda_i p_i = q_i \left[\frac{\gamma}{\beta} - 1 + \alpha(\theta_i)\right]. \tag{14}$$

Similarly for sellers to be indifferent between cash and card transactions in market i requires

$$\lambda_i(1 - \omega)p_i = q_i. \tag{15}$$

Suppose, for now, that there were no cost to using the system (i.e. $\omega = 0$). If in addition $\gamma = \beta$, equations (14) and (15) imply that both buyers and sellers would be indifferent between cards and cash. For $\gamma > \beta$, however, the credit card price required to induce buyer indifference would exceed that at which sellers are indifferent. By shaving the credit card price down slightly, sellers could get all buyers to use cards and make everyone better off. With $\omega = 0$ then, credit cards have an advantage over cash because they avoid buyers having to acquire cash that they may not use. More generally, using equations (14) and (15) we can define

$$\omega(\gamma, \theta) = \frac{\left(\frac{\gamma}{\beta} - 1\right)}{\left(\alpha(\theta) + \frac{\gamma}{\beta} - 1\right)}. \tag{16}$$

Then it should be clear that as long as $\omega < \omega(\gamma, \theta_i)$ no buyers will bring cash to market $i = h, l$. This also means that as long as $q_i > \lambda_i(1 - \omega)p_i$ the cash price offered is moot.

4.3.4 Low-effort equilibrium

If $k > u$, we know that no sellers will exert effort and only the l market will exist. In that case, if $\omega < \omega(\gamma, \bar{n})$ all transactions will be by credit card. Buyers will be completely indifferent over cash prices as long as $q_l > \lambda_l(1 - \omega(\gamma, \bar{n}))p_l$. Cash prices that satisfy this constraint are non-viable in that such prices do not lead buyers to switch from credit to cash use. So, under this restriction and after substituting for the value functions, a seller's problem reduces to

$$\begin{aligned} V_l^s(p_l, q_l, \bar{n}, \psi_l &= 0, l) = \max_{\hat{\theta}, \hat{p}} \frac{\alpha(\hat{\theta})}{\hat{\theta}} \lambda_l \frac{\beta \hat{p}(1 - \omega)}{\gamma} + \beta W^s(0) \qquad (17) \\ \text{subject to:} \qquad & \alpha(\hat{\theta})\lambda_l \left(u - \frac{\beta \hat{p}}{\gamma}\right) = B_l \end{aligned}$$

where

$$B_l \equiv \alpha(\bar{n})\lambda_l \left(u - \frac{\beta p_l}{\gamma}\right). \tag{18}$$

Substituting $\hat{p}$ out of problem (17) reduces it to

$$\theta_l = \arg\max_{\hat{\theta}} \frac{\alpha(\hat{\theta})\lambda_l u - B_l}{\hat{\theta}}.$$

Under the assumed restrictions on $\alpha(.)$, the objective function is strictly concave so the solution is unique. As we seek a low-effort only equilibrium, that unique solution for θ_l has to be $\bar{n}$. Substituting back in for B_l implies

$$p_l = \frac{u\gamma\eta(\bar{n})}{\beta}. \tag{19}$$

where η is the elasticity of α. Notice that p_l does not vary with ω or λ_l. As both enter V_l^s proportionally and sellers have constant marginal valuation of cash, they drop out of the analysis. While variations in λ_l do impact buyers (through the probability of receiving a high quality good), variations in the merchant fee, ω, do not.

If $\omega > \omega(\gamma, \bar{n})$ all buyers will bring cash and the equilibrium outcome is identical to the cash only economy. Only if $\omega = \omega(\gamma, \bar{n})$ can the model support the use of both cash and credit in the same market.

A concern here is that in reality this threshold might be much smaller than the charges made of retailers by credit card companies. Suppose a period of time in the model represents a week where the CM is the work week and the DM is the weekend. Then with an annual discount factor of 0.96 and an annual money growth rate of 2%, $\gamma/\beta \approx 1.0012$. The value of $\omega(\gamma, \bar{n})$ depends on $\alpha(\bar{n})$, the chance that a buyer meets a potential trading partner. If trades are infrequent, then using a credit card becomes more advantageous for buyers who can thereby avoid the inflation tax. For very low values of $\alpha(\bar{n})$, therefore, $\omega(\gamma, \bar{n})$ can be arbitrarily close to 1 but if we suppose that a buyer has even a 50% chance of making the purchase over a given weekend we get $\omega(\gamma, \bar{n}) \approx 0.24\%$. Credit card fees are typically an order of magnitude larger than this. To support, say, a 2% system fee, a buyer's probability of making a purchase would have to be no more than 6%.

4.3.5 High-effort credit-only equilibrium

When k is small enough, sellers will consider exerting high effort. The objective here is to understand the circumstances under which all sellers exert high effort and all buyers use credit cards. Of particular interest is the robustness of the equilibrium to values of the merchant fee ω that exceed $\omega(\gamma, \bar{n})$.

In this case buyers will be completely indifferent over cash prices as long as $q_h > \lambda_h(1 - \omega(\gamma, \bar{n}))p_h$. So, under this restriction and after substituting for the value functions, a sellers's problem reduces to

$$
\begin{aligned}
V_h^s(p_h, q_h, \bar{n}, \psi_h &= 0, h) = \max_{\hat{\theta}, \hat{p}} \frac{\alpha(\hat{\theta})}{\hat{\theta}} \lambda_h \frac{\beta \hat{p}(1-\omega)}{\gamma} + \beta W^s(0) \qquad (20) \\
\text{subject to:} \qquad & \alpha(\hat{\theta})\lambda_h \left(u - \frac{\beta \hat{p}}{\gamma} \right) = B_h
\end{aligned}
$$

where

$$
B_h \equiv \alpha(\bar{n})\lambda_h \left(u - \frac{\beta p_h}{\gamma} \right). \qquad (21)
$$

By the same logic as for the low-effort equilibrium, $\theta_h = \bar{n}$ and

$$
p_h = \frac{u\gamma\eta(\bar{n})}{\beta}.
$$

Notice that p_h is the same as p_l from equation (19). When credit cards are in use, the probability of a defect is isomorphic to a proportional decline in the matching function, $\alpha(.)$. What matters for the price level is the elasticity of matching, $\eta(.)$, which is unaffected by a proportional shift in $\alpha(.)$. The distinction between the equilibria comes from the effort choices of the sellers as a consequence of the parameter values.

We now turn to identifying what parameter restrictions are required to ensure existence of the high-effort credit-only equilibrium. For the above solution to represent an equilibrium, we have to check that the appropriate incentive constraints hold. These are that:

A. Sellers would not prefer to offer a viable cash price,

$$\begin{aligned} & V_h^s(p_h, q_h > \lambda_h(1-\omega(\gamma,\bar{n}))p_h, \bar{n}, \psi_h = 0, h) \\ & \geq V_h^s(p_h, q_h \leq \lambda_h(1-\omega(\gamma,\bar{n}))p_h, \bar{n}, \psi_h = 0, h) \end{aligned} \tag{22}$$

B. Sellers will not choose low effort and enter the high-effort market,

$$V_h^s(p_h, q_h, \bar{n}, \psi_h, h) - k \geq V_l^s(p_h, q_h, \bar{n}, \psi_h, h) \tag{23}$$

C. No seller will strictly prefer to open a low-effort market,

$$V_h^s(p_h, q_h, \bar{n}, \psi_h, h) - k \geq V_l^s(p_l, q_l, \theta_l, \psi_l, l). \tag{24}$$

Here I argue that restriction (22) is not relevant for this equilibrium. Based on what happens in the low-effort market when $k > u$, we should expect that for $\omega > \omega(\gamma, \bar{n})$ both parties would prefer to trade using cash. However, it cannot be optimal for sellers to exert high effort and drop their cash price so that

$$q_h \leq \lambda_h(1-\omega(\gamma,\bar{n}))p_h.$$

This is because buyers recognize that any seller offering such a price had no reason to exert high effort. Buyers will treat any high-effort seller who offers such a cash price as a low-effort seller. Offering a viable cash price is synonymous with exerting low effort; a new submarket will open which will attract buyers in the same numbers that low-effort sellers offering that price would attract.

Restriction (23) requires

$$\frac{\alpha(\bar{n})\lambda_h\beta p_h(1-\omega)}{\bar{n}\gamma} - k \geq \frac{\alpha(\bar{n})\lambda_l\beta p_h(1-\omega)}{\bar{n}\gamma}.$$

After substituting for p_h this becomes

$$\alpha(\bar{n})\eta(\bar{n})(1-\omega)\left[\lambda_h - \lambda_l\right] u \geq \bar{n}k. \tag{25}$$

This can be compared to restriction (R1). As $\eta(\bar{n}) < 1$, the incentive constraint (23) means that, even with credit cards, the market economy cannot lead to high effort in all parameter arrangements for which it is efficient.

Restriction (24) requires calculating $V_l^s(p_l, q_l, \theta_l, \psi_l, l)$. While $\omega < \omega(\gamma, \theta_l)$, however, we know that the deviant seller would want to sell using credit cards. He has to offer a credit price that maximizes his own utility subject to the constraint that any buyer can equally enter the high-effort market. Thus,

$$\begin{aligned} V_l^s(p_l, q_l, \theta_l, \psi_l &= 0, l) = \max_{\hat{p}, \hat{\theta}} \frac{\alpha(\hat{\theta})\lambda_l \beta(1-\omega)\hat{p}}{\hat{\theta}\gamma} + \beta W^s(0) \qquad (26) \\ \text{subject to} &: \quad \alpha(\hat{\theta})\lambda_l \left(u - \frac{\beta \hat{p}}{\gamma} \right) = B_h \end{aligned}$$

Solving this implies

$$p_l = \frac{u\gamma\eta(\theta_l)}{\beta}.$$

Substituting this and B_h into the constraint in problem (26) implies

$$\frac{\alpha(\theta_l) - \alpha'(\theta_l)\theta_l}{\alpha(\bar{n}) - \alpha'(\bar{n})\bar{n}} = \frac{\lambda_h}{\lambda_l}.$$

As $\lambda_h > \lambda_l$, this means that $\theta_l > \bar{n}$. Given this, it is simple to show that $V_l^s(p_h, q_h, \bar{n}, \psi_h = 0, h) > V_l^s(p_l, q_l, \theta_l, \psi_l = 0, l)$. This means that at values of ω for which any low-effort deviant, setting up a low-effort market would sell for credit, he would in fact, prefer to enter the high-effort market. For low values of ω, then, restriction (24) is implied by restriction (23) and hence by (25).

The up shot is that as soon as a retailer makes it clear that he prefers cash to credit transactions any association with having exerted high effort is lost. If high-effort retailers attempt to charge viable cash prices for their goods, low-effort retailers will imitate them. Thus, the threshold value for the transaction fee becomes that at which retailers will switch to low effort.

4.3.6 Robustness to realistic values of ω?

We have established existence of the high-effort credit-only equilibrium only for low values of ω. We have not addressed the quantitative issue as to whether realistic values of ω (say 2%) can be sustained. For larger values of ω the preceding analysis remains relevant except that in implementing restriction (24) the deviant seller will accept cash in the low-effort market. Thus,

$$V_l^s(p_l, q_l, \theta_l, \psi_l = 1, l) = \max_{\hat{q},\hat{\theta}} \frac{\alpha(\hat{\theta})\beta\hat{q}}{\hat{\theta}\gamma} + \beta W^s(0) \tag{27}$$

$$\text{subject to} \quad : \quad \alpha(\hat{\theta})\lambda_l u - \frac{\left[\gamma - \beta(1-\alpha(\hat{\theta}))\right]\hat{q}}{\gamma} = B_h$$

where B_h is still given by equation (21).

Lemma 1 *The problem (27) has a (generically) unique solution.*

Proof. See Appendix ■

The result is only generic because it can fail in the case that $\alpha''(0) = -1$.[8] The restrictions on ω and k that are consistent with the high-effort credit-only equilibrium are obtained from (24) and (25) using the solution to problem (27) to obtain (q_l, θ_l).

Since the issue as to the robustness of the equilibrium to larger values of ω is a quantitative question I will answer it quantitatively. The only functional form to be determined is that of $\alpha(.)$. From the previous discussion as to the magnitude of ω, it would be appropriate to have $\alpha(\bar{n}) = 0.5$, which implies a 50% chance of making a purchase on any given weekend. If we then set $\bar{n} = 1$, it implies the DRWS matching function parameter, $\rho = 1$. Under this restriction, $\eta(\theta) = 1/(1+\theta)$ and $\eta(\bar{n}) = 0.5$. The remaining parameters are in Table 1.

ρ	$\bar{n}$	u	λ_h	λ_l	k	γ	β
1	1	1	0.99	0.95	0.0095	1.0004	0.9992

Table 1: Parameter values for leading example.

The value of u is a normalization. The choice of λ_h and λ_l are clearly arbitrary. The value of λ_h reflects the idea that credit card users rarely have

[8] In the case of the DRWS matching function, $\alpha''(0) = 2$ when $\rho = 1$ and 0 when $\rho > 1$. In the case of the urn-ball matching function, $\alpha''(0) = -1$ but $\lim_{\theta\to 0}\eta'(\theta) = -\frac{1}{2}$ so a solution exists. See the proof for more details.

to stop payments on their purchases. Making λ_l close to λ_h avoids giving credit cards too much of an advantage. Lower values of λ_l do not affect $\omega(\gamma, \bar{n})$ but do increase the size of ω that can be sustained in the high-effort credit-only equilibrium. The cost of effort, k, is constrained by restriction (23) as characterized by equation (25). As ω also enters equation (25) we need to guess a value for k and check that, at the maximal sustainable value of the merchant fee, $\omega_{\max}$, it is satisfied. Given these values for λ_h and λ_l it should be clear that k has to be smaller than 0.01. If the cost of effort is too high, it will always be preferable to exert low effort.[9] The values of γ and β are based on a target annual inflation rate of 2% and a discount rate of 4% adapted to the time unit of one week.

Under these functional form and parameter restrictions, the solution to problem (27) is $\theta_l = 1.044$ and $q_l = 0.465$ which implies $V_l^s - \beta W^s(0) = 0.227$. Now given $p_h = 0.5006$ and $\theta_h = \bar{n} = 1$, $V_h^s - \beta W^s(0) = 0.99(1-\omega)/4$. The largest sustainable value for ω such that $V_h^s - V_l^s - k \geq 0$ is then $\omega_{\max} = 4.37\%$.

4.3.7 Sensitivity analysis

As the parameters used to calculate the maximal supported value of ω are based on an arbitrary set of outcomes I consider some sensitivity analysis to ascertain whether the results are robust to changes in those outcomes. If we change $\bar{n}$ to $\frac{1}{2}$, then $\alpha(\bar{n}) = 1/3$. Maintaining the same value, 0.0095, for k we find that p_h rises to 0.6674 and the maximal value for ω drops to 4.20%. There are two opposing forces at play here. Reducing the number of sellers means they have a higher chance of trading which will increase the value to exerting both high and low effort by similar proportions. This should increase $\omega_{\max}$. However, the reduced matching rate for buyers means that their benefit from entering a high-effort market is also reduced. As the sellers' benefit from high effort is dependent on the buyers' desire to enter high-effort markets, this effect tends to lower $\omega_{\max}$. Here, the latter effect

[9] The maximal value of k for which high effort is efficient is 0.02.

dominates.

Maintaining the other parameters from the leading example and lowering λ_l to 0.9 increases $\omega_{\max}$ to 14.1%; increasing λ_h to 0.999 increases $\omega_{\max}$ to 6.09%; doubling the net rate of money growth so that $\gamma = 1.0008$ increases $\omega_{\max}$ to 4.45%; doubling the discount rate so that $\beta = 0.9984$ increases $\omega_{\max}$ to 4.51%. Each of these changes makes credit card use increasingly preferable to cash.

5 Discussion

5.1 Coexistence of cash and credit

In all of the equilibria of the model that are considered here, cash and credit do not coexist. One way to ensure coexistence would be to introduce cash-only buyers who do not hold a credit card. In that case the counterpart to the high-effort credit-only equilibrium would be one in which all sellers exert high effort and all buyers who have credit cards use them. As long as the cash price, q_h, offered is such that $q_h \geq \lambda_h(1 - \omega(\gamma, \bar{n}))p_h$, credit card holders will be happy to use them. If the cash price drops below that threshold, every buyer will presume that the seller did not incur high effort. In such an equilibrium, the cash buyers would be free-riding on the credit card users because they are able to purchase high-effort goods that would not be available to them in a cash-only economy.

It is also possible for intermediate values of k, that both the high-effort credit-only and low-effort cash-only equilibria coexist. When this happens, there will also be a mixed strategy equilibrium featuring two separate markets one in which cash alone is used and one in which credit cards alone are used. Such equilibria, however, are typically dynamically unstable and exhibit pathological comparative statics (see Markose and Loke [2003] for an example of such an equilibrium).

5.2 Equal price rule

Historically, the main credit card networks, Visa and Mastercard, have required that retailers wishing to access the networks charge the same price for cash and credit. That is, they have prevented retailers from charging a credit card surcharge. A class action antitrust law suit brought by retailers, which began in 2005, is still working its way through the US Federal courts (see Segal [2017] and Official Court-authorized settlement website [2017]). In this context, then, it is of interest to consider how that rule would play out in the model described above.

In the high-effort credit-only equilibrium, forcing retailers to offer the same price for cash and credit has no effect other than to possibly coordinate the posted cash price. This is because the credit price is not a viable cash price. If the transaction cost, ω, exceeds that for which the equilibrium exists, retailers will switch to a cash-only low-effort equilibrium. The same-price rule does not affect the robustness of the equilibrium.

Suppose now that k is too large for the high-effort equilibrium to exist for any value of ω. Then, for $\omega < \omega(\gamma, \bar{n})$ sellers exert low effort and trade occurs exclusively using credit cards. Again imposing the same price for cash as card transactions will make no difference. While ω is low enough, the benefit to buyers of avoiding the holding of idle cash balances sustains the use of cards. Once $\omega > \omega(\gamma, \bar{n})$, all transactions move to cash and sellers opt out of the network.

If some buyers did not have credit cards, an equal price rule would have the cash-only buyers paying more than the retailers might prefer to charge them. In this case, then, there is a sense that credit card users free-ride on the cash users as emerges in McAndrews and Wang [2012]. For low enough values of k, this would imply a kind of symbiosis in which cash buyers benefit from the quality guarantee that arises from the presence of sufficiently many credit card buyers but the latter group will face lower prices by virtue of the presence of cash buyers.

The results are consistent with the expressed ambivalence from retailers toward the removal of the same-price rule. Cohen and Hamlin [2012] report that retailers may not want to charge different prices even when the networks are no longer able to impose the rule.

In states where differential pricing is permitted, the practice has become prevalent in automobile filling stations. In that market, though, the product is highly regulated so acceptance cannot signal any quality assurance. The model would predict that merchant fees would have to be quite small anyway. What appears to sustain the use the credit cards at filling stations are rewards. Many cards now give 3% or more in cash rewards for fuel purchases. While buyers still show up expecting to use a card, the retailer will continue to offer viable card prices even in the face of high merchant fees.

6 Conclusion

This paper develops a new theory of large merchant fees based on the surety that credit card issuers provide consumers as to the quality of the good they are buying. Cash transactions do not provide any customer recourse in the case of mis-sold items while credit cards do. Retailers who accept credit cards therefore send a message to potential customers that they stand behind their product. Simulations indicate that retailers should be prepared to pay in the order of 4.4% of the product value to distance themselves from unscrupulous traders.

7 Appendix

7.1 Proof of Proposition 1

First it is helpful to recognize that problem (6) can be re written as

$$\max_{(q,\theta)} V^b(q,\theta) - q \quad \text{subject to} \quad V^s(q,\theta) = V^s(q^*,\theta^*). \tag{28}$$

In standard directed search (see Rogerson et al [2005]) this can be interpreted as: it does not matter which side sets wages (i.e. moves first). In the current context there is no such interpretation. Allowing buyers to move first would change the model from one of signalling to one of screening. Still, the mathematics remains. To see why let λ_i, $i = s, b$ be the co-state variable associated with the constraint in problems (6) and (28) respectively. Taking first order conditions with respect to q and θ indicate that the requirements for optimality are identical with $\lambda_s = 1/\lambda_b$. Of course, the pair, (q^*, θ^*) that solve (28) could represent a minimum in problem (6). But, as long as (q^*, θ^*) is unique this cannot be the case. This is because putting $q = u$ implies $V^i(q, \theta) = W^i(0)$ for $i = s, b$ for all θ while for any $q \in (0, u)$, $V^i(q, \theta) > W^i(0)$. Solving problem (28) reduces the amount of algebra.

Substituting the value functions into problem (28) and eliminating q leads to

$$\begin{aligned} \theta^* &= \arg\max \Omega(\theta) \\ \text{where, } \Omega(\theta) &\equiv \alpha(\theta) \lambda_l u - A\theta - \frac{(\gamma - \beta) A\theta}{\beta \alpha(\theta)} \\ \text{and } A &\equiv \frac{\alpha(\theta^*) \beta q^*}{\gamma \theta^*}. \end{aligned} \tag{29}$$

The first order condition is

$$\alpha'(\theta) \lambda_l u - A - \frac{(\gamma - \beta) A(1 - \eta(\theta))}{\beta \alpha(\theta)} = 0 \tag{30}$$

where $\eta(.)$ is the elasticity of $\alpha(.)$. By virtue of the assumption that $\theta/\alpha(\theta)$ is convex, the second order condition is negative. Now, using L'Hospital's rule $\lim_{\theta \to 0} \Omega(\theta) = -\frac{(\gamma-\beta)A}{\beta} \leq 0$. And, $\lim_{\theta \to \infty} \Omega(\theta) = -\infty$. However,

$$\lim_{\theta \to 0} \Omega'(\theta) = \lambda_l u - A - \frac{(\gamma - \beta) A}{\beta} \lim_{\theta \to 0} \frac{1 - \eta(\theta)}{\alpha(\theta)}.$$

Using L'Hospital's rule twice

$$\lim_{\theta \to 0} \frac{1 - \eta(\theta)}{\alpha(\theta)} = - \lim_{\theta \to 0} \frac{\alpha''(\theta) \theta}{2\alpha'(\theta)} = -\frac{1}{2} \lim_{\theta \to 0} \alpha''(\theta) \theta$$

which is 0 as, by assumption, $\lim_{\theta \to 0} \alpha''(\theta)$ is finite. So $\Omega'(0) > 0$ as long as $\lambda_l u > A$. Under this last restriction, then, there exists a strictly positive solution to problem (29) at which (30) must hold.

Substituting A back into (30) provides a condition for any solution to problem (28),

$$\alpha'(\theta^*)\,\lambda_l u - \frac{q^*}{\gamma\theta^*}\left[\alpha(\theta^*)\,\beta + (\gamma-\beta)(1-\eta(\theta^*))\right] = 0. \tag{31}$$

As the solution to (30) is unique, and low-effort production is costless, $\theta^* = \bar{n}$ and q^* is the unique solution to (31). It, then, follows from $\alpha'(\bar{n}) < 1$ and that the second term in (31) is larger than A, that $\lambda_l u > A$.

7.2 Proof of Lemma 1

The necessary conditions for a solution, (q_l, θ_l), to problem (27) are

$$\alpha(\theta_l)\lambda_l u(1-\eta(\theta_l)) - \frac{\eta(\theta_l)\,[\gamma-\beta]\,q_l}{\gamma} = B_h \tag{32}$$

$$\alpha(\theta_l)\lambda_l u - \frac{[\gamma-\beta(1-\alpha(\theta_l))]\,q_l}{\gamma} = B_h. \tag{33}$$

Eliminating B_h leads to

$$q_l = \frac{\alpha(\theta_l)\lambda_l u\eta(\theta_l)\gamma}{(\gamma-\beta)\,(1-\eta(\theta_l)) + \beta\alpha(\theta_l)}. \tag{34}$$

Substitution into (33) implies that $\Psi(\theta_l) = B_h$ where

$$\Psi(\theta) \equiv \frac{\alpha(\theta)\lambda_l u\left[(\gamma-\beta)\,(1-2\eta(\theta)) + \beta\alpha(\theta)\,(1-\eta(\theta))\right]}{(\gamma-\beta)\,(1-\eta(\theta)) + \beta\alpha(\theta)}.$$

After dropping arguments and using primes to denote derivatives, we get

$$\begin{aligned} \Psi' \quad = \quad & -\frac{\lambda_l u\,[\gamma-\beta(1-\alpha)]}{[(\gamma-\beta)\,(1-\eta)+\beta\alpha]^2}\times \\ & \left\{\beta\alpha\left[\alpha\eta' - \alpha'(1-\eta)\right] + (\gamma-\beta)\left[\alpha\eta' + \alpha'(2\eta-1)(1-\eta)\right]\right\} \end{aligned} \tag{35}$$

As the fraction is positive, the sign of Ψ' will be determined by that of the curly brackets. Now, the convexity of $\theta/\alpha(\theta)$ implies

$$\alpha(\theta)\eta'(\theta) + \alpha'(\theta)\,(1-\eta(\theta)) \leq 0.$$

The inequality is strict if $\theta > 0$. It means that the first term in the curly brackets in (35) is negative and that $\eta'(\theta) \leq 0$. Moreover, as for $\theta > 0$, $\eta < 1$, $2\eta - 1 < 1$ and the second term is also strictly negative. Thus $\Psi' > 0$ for $\theta > 0$.

We need to consider what happens to $\Psi(\theta)$ as θ approaches both 0 and ∞. As $\theta \to 0$, $\eta(\theta) \to 1$ so

$$\begin{aligned}\Psi \quad &\to \quad \frac{-\lambda_l u\,(\gamma-\beta)\,(\lim_{\theta\to 0}\alpha(\theta))}{(\gamma-\beta)\,(\lim_{\theta\to 0}(1-\eta(\theta)) + \beta\,(\lim_{\theta\to 0}\alpha(\theta))} \\ &= \quad \frac{-\lambda_l u\,(\gamma-\beta)\left(\lim_{\theta\to 0}\frac{\alpha(\theta)}{1-\eta(\theta)}\right)}{(\gamma-\beta)+\beta\left(\lim_{\theta\to 0}\frac{\alpha(\theta)}{1-\eta(\theta)}\right)}\end{aligned}$$

Consider

$$\lim_{\theta\to 0}\frac{\alpha(\theta)}{1-\eta(\theta)} = \frac{\lim_{\theta\to 0}\alpha'(\theta)}{-\lim_{\theta\to 0}\eta'(\theta)} = -\frac{1}{\lim_{\theta\to 0}\eta'(\theta)}.$$

Now,

$$\eta'(\theta) = \frac{\alpha''(\theta)\theta}{\alpha^2(\theta)} + \frac{\alpha'(\theta)\eta(\theta)}{\alpha(\theta)}.$$

So,

$$\begin{aligned}\lim_{\theta\to 0}\eta'(\theta) \quad &= \quad \left(\lim_{\theta\to 0}\alpha''(\theta)\right)\left(\lim_{\theta\to 0}\frac{\theta}{\alpha(\theta)}\right)\left(\lim_{\theta\to 0}\frac{1}{\alpha(\theta)}\right) \\ &\quad + \left(\lim_{\theta\to 0}\alpha'(\theta)\right)\left(\lim_{\theta\to 0}\eta(\theta)\right)\left(\lim_{\theta\to 0}\frac{1}{\alpha(\theta)}\right) \\ &= \quad \left[\begin{array}{c}(\lim_{\theta\to 0}\alpha''(\theta))\left(\lim_{\theta\to 0}\frac{\theta}{\alpha(\theta)}\right) \\ +(\lim_{\theta\to 0}\alpha'(\theta))\,(\lim_{\theta\to 0}\alpha'(\theta))\left(\lim_{\theta\to 0}\frac{\theta}{\alpha(\theta)}\right)\end{array}\right]\left(\lim_{\theta\to 0}\frac{1}{\alpha(\theta)}\right) \\ &= \quad \left[\left(\lim_{\theta\to 0}\alpha''(\theta)\right) + \left(\lim_{\theta\to 0}\alpha'(\theta)\right)^2\right]\left(\lim_{\theta\to 0}\frac{1}{\alpha'(\theta)}\right)\left(\lim_{\theta\to 0}\frac{1}{\alpha(\theta)}\right) \\ &= \quad \left[\left(\lim_{\theta\to 0}\alpha''(\theta)\right)+1\right]\left(\lim_{\theta\to 0}\frac{1}{\alpha(\theta)}\right) \\ &= \quad \left\{\begin{array}{c}-\infty \text{ when } \alpha''(0) < -1 \\ \infty \text{ when } \alpha''(0) > -1 \\ \text{indeterminate when } \alpha''(0) = -1\end{array}\right. .\end{aligned}$$

Which means that generically, $\Psi(0) = 0$. Also, notice that when $\alpha''(0) = -1$, as long as $\lim_{\theta\to 0}\eta'(\theta) < 0$ $\lim_{\theta\to 0}\Psi(\theta) < 0$.

Now as θ gets large,

$$\lim_{\theta\to\infty}\eta(\theta)=\lim_{\theta\to\infty}\frac{\alpha'(\theta)\theta}{\alpha(\theta)}=\frac{\lim_{\theta\to\infty}\alpha'(\theta)}{\lim_{\theta\to\infty}\frac{1}{\theta}}=\lim_{\theta\to\infty}\frac{\alpha(\theta)+C}{\ln(\theta)+D}=0$$

where C and D are constants of integration.[10] This means that $\lim_{\theta\to\infty}\Psi(\theta)=\lambda_l u$. As $0<B_h<\lambda_l u$ the system (32) and (33) has a generically unique solution which is also a solution to problem (27). This is because at $\theta_l=0$, for any value of q_l, the constraint in problem (27) is violated. And, if $\theta_l=\infty$, $V_l^s=\beta W^s(0)$ while for the interior value of (q_l,θ_l) that solves the system (32) and (33), $V_l^s>\beta W^s(0)$.

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[10] The second to last equality uses L'Hospital's rule in "reverse". This cannot usually be done. But, recall that the rule applies whenever the denominator in the ratio of the original functions is infinite (see Rudin [1976] p.109). Then the rule can be reversed up to arbitrary constants of integration. The final equality holds for any values of C and D.

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