EX-ANTE PRICE COMMITMENT WITH RENEGOTIATION IN A DYNAMIC MARKET

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Abstract. This paper studies the endogenous determination of the price formation procedure in markets characterized by match-specific heterogeneity. We study a model of a market in which, in each time period, agents on one side (e.g., sellers) choose whether or not to post a price before they encounter agents of the opposite type. After a pair of agents have encountered each other, their match-specific values from trading with each other are realized. If a price was not posted, then the terms of trade (and whether or not it occurs) are determined by bargaining. Otherwise, depending upon the agents' match-specific trading values, trade occurs (if it does) either on the posted price or at a renegotiated price. We analyze the symmetric Markov subgame perfect equilibria of this market game, and address a variety of issues such as the impact of market frictions on the equilibrium proportion of trades that occur at a posted price rather than at a negotiated price.

1. Introduction

The procedure of price determination varies not only across markets but often also within the same market. For example, in housing markets, trade sometimes occurs at prices posted by sellers and at other times at prices determined by negotiations; some houses are even sold at auctions. In labour markets, firms often post wages, and, depending upon the nature of the "match" between a firm and a worker, employment may occur either at the posted wage or at a renegotiated wage (when renegotiation is mutually beneficial — perhaps because the posted wage is too low while the worker has turned out to be a good match for the firm). What factors determine the procedure of price formation in any particular market? Under what circumstances can two or more pricing mechanisms co-exist in the same market? What role do frictions play in determining the pricing mechanism?

This paper aims to address these and other issues in the context of frictional markets with match-specific heterogeneity. Such heterogeneity is meant to capture, for example, markets in which sellers own differentiated commodities and buyers have heterogeneous preferences. When embedded

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in the context of endogenous price determination, it leads us to develop and explore a model that is different from the other models in the relatively small literature that studies the endogenous determination of the pricing mechanism.

The three main price formation procedures that are typically observed in real-life, and that have received the most attention from economic theorists are auctions, bargaining and price posting. A common feature of the enormous literature on models of decentralized markets, however, is that it takes the price formation procedure as exogenously given. Following Vickrey (1961) there is a large literature on models in which prices are determined via auctions; while following Diamond (1981), Mortensen (1982), and, Rubinstein and Wolinsky (1985) there is vast literature on models in which prices are determined by negotiations; and furthermore, following Diamond (1971) there is a literature on models in which prices are determined by price posting.

In models that allow for a price posting mechanism there is a potential, exogenously built-in ex-post inefficiency that arises from the fact that when a pair of agents meet they have to either trade at some convex combination of the two posted prices or not trade at all.[1] This implies that in an environment characterized by match-specific heterogeneity, it is possible that trade may not occur (since it might not be individually rational for at least one of the two agents to trade at such a price) although it might be mutually beneficial for the agents to trade (but at some other price). Thus, ex-post renegotiation of the terms of trade can be mutually beneficial. This is not surprising, since the posted price is an incomplete (or, to be precise, non-comprehensive) "contract" — in that the posted price is not conditioned on the potential match-specific realizations of the agents' respective values from trading with each other. A key novel feature of our market model is that we allow for such mutually beneficial renegotiation to take place. This price formation procedure may be called the *price posting cum renegotiation mechanism*.[2]

Another novel feature of our model is that we allow one side of the market (e.g., sellers) to choose whether to determine the terms of trade ex-post via bargaining, or to determine the terms of trade via the price posting cum renegotiation mechanism. As indicated above, a main aim of this paper is to endogenously determine the pricing mechanism as part of the market equilibrium.

As mentioned above, there is a relatively small literature that studies the endogenous determination of the pricing mechanism. Specifically, this

[1]This point also applies to models in the literature following Diamond (1971) in which prices are exogenously assumed to be determined via a price posting mechanism — for two recent models in this literature, see Burdett and Mortensen (1998) and Masters (1999).

[2]Interestingly, over fifteen years ago, Hart and Moore (1988) established the crucial role of mutually beneficial renegotiation but in the context of an incomplete bilateral contracting model.

literature studies market models that allow for two of the three potential pricing mechanisms mentioned above. For example, while Wang (1993), and, Bulow and Klemperer (1996) study models in which the allowable pricing mechanisms are auctions and bargaining, Peters (1991), Bester (1993), Wang (1995) and, Ellingsen and Rosen (2003) — like us — allow for price posting and bargaining. They differ from ours in that they view these as distinct mechanisms to which participants have to commit ex ante. Under our price posting cum renegotiation mechanism, whether the good is sold at a negotiated or a posted price will depend on the realized value of trade to the participants.

After laying down our model in the next section, we then, in section 3, derive some results concerning the characteristics of an arbitrary market equilibrium, and establish its existence. In particular, we show that in any market equilibrium, the pricing mechanism will be the price posting cum renegotiation mechanism. Several results concerning the impact of frictions will also be derived here. One key insight obtained is that when the matching rates of the two sides of the market are unequal, then aggregate market welfare would be maximized either when the agents with the relatively higher matching rate post prices or when the agents with the relatively lower matching rate post comprehensive price contracts. In particular, the posting of comprehensive price contracts by agents on the short side of the market adversely affects aggregate market welfare.

Then, in sections 4 and 5, we derive — under various, alternative additional assumptions — some further results concerning the properties of a market equilibrium such as the impact of market frictions on the equilibrium proportion of trades that occur at a posted price. A main insight obtained here is that trade in markets with small frictions is likely to occur at negotiated prices, while in markets with large frictions it is more likely to occur at posted prices. An implication of this result — which appears to be consistent with real-life retail markets — is that in retail markets in which buyers search intensively (such as in housing markets) trade is more likely to occur at negotiated prices, while in retail markets in which their intensity of search is negligible (such as in the market for eggs) trade is more likely to occur at prices posted by the sellers. Section 6 summarizes, and discusses some of our key modelling assumptions. We relegate formal proofs to the Appendix.

2. The Model

The market considered in the model operates over an infinite number of discrete points in time with two types of agents, namely, "buyers" and "sellers", who are respectively denoted by type b and type s; there are a large number (formally, a continuum) of each type of agent. The market is in a steady state; that is, the numbers of buyers and sellers in the market are constant over time.

An important feature of this market is the existence of match-specific, payoff-relevant heterogeneity: the value to an agent from trading with an agent of the opposite type depends on the nature of their specific match. Agents of the opposite types encounter each other through a random, pairwise matching process.[3] After they meet, their match-specific values are realized. The buyer's and the seller's values v_b and v_s from trading with each other are randomly (and independently) drawn from the distributions F_b and F_s respectively. Thus, if a pair of agents agree to form a match, and trade at price p, then the buyer's and the seller's payoffs are respectively $v_b - p$ and $p + v_s$. Agents discount future payoffs according to a common discount rate $r > 0$.

We assume that F_k $(k = b, s)$ has a bounded support, where the infimum and supremum of this support are respectively denoted by $\underline{v}_k$ and $\bar{v}_k$. It is assumed that $\bar{v}_b + \bar{v}_s > 0$; for otherwise no gains to trade exist between any pair of agents. The sequence of events that occur at each point in time t, where $t = 0, \Delta, 2\Delta, \ldots$, with $\Delta > 0$ (but small), is described by the following four-stage process.

- **Stage 1: Post a Price?** Each agent on one (and only one) side of the market — sellers, for example — simultaneously posts a price.[4] We denote the type of agents who have this option to post a price by i, where $i = b$ or $i = s$; the other type of agents is denoted by j $(j \neq i)$.
- **Stage 2: Random Matching.** Each seller meets a buyer according to a Poisson process with parameter $\lambda_s > 0$; and each buyer meets a seller according to an independent Poisson process with parameter $\lambda_b > 0$. After a pair of agents of the opposite types encounter each other, their match-specific values are realized.
- **Stage 3: Renegotiation?** A pair of matched agents decide whether or not to renegotiate the posted price. They will renegotiate if and only if both agree to do so — the decision to renegotiate is made simultaneously. If at least one agent refuses to renegotiate, then the process moves to stage 4. However, if both agents choose to renegotiate, then they engage in the following bargaining process. With equal probability, Mother Nature picks either agent to make an offer of a (new) price, which the other agent can accept or reject. In either case the process then moves to stage 4.
- **Stage 4: Trade?** The two agents simultaneously decide whether or not to form a match and trade. If both of them choose to match and trade,

[3]An alternative to the random matching framework that we could use is "directed" search, in which the location of sellers and their prices are known but how many buyers show up at any establishment is random. While some progress has been made in the use of directed search in a retail context (see for example Corbae *et al*, 2003), in our view it is rarely the case that one knows exactly what prices are being offered by whom when one goes out shopping (even for relatively large items like refrigerators).

[4]A posted price is a single number, independent of the match-specific pairs of agents' values. Notice that posting a sufficiently high price or a sufficiently low price is formally equivalent to not posting a price.

then they exit the market and trade. Otherwise they split up and wait until time $t + \Delta$ when the four-stage process recurs.

We analyze the symmetric Markov subgame perfect equilibria (Market Equilibrium or ME, henceforth) of this stochastic, dynamic game. For any ME, V_b and V_s respectively denote the associated equilibrium expected payoffs to a buyer and a seller at the beginning of any time period t.

Before proceeding with the analysis, we characterize the unique ME under the (alternative) assumption that agents can post comprehensive price contracts (i.e., contracts in which the price can depend on the match-specific pairs of agents' values). In any ME under this assumption, a type i agent extracts all of the surplus from any match, which implies that $V_j = 0$. Consequently, the Bellman equation for V_i is:

$$rV_i = \lambda_i \iint\limits_{v_i+v_j \geq V_i} [v_i + v_j - V_i] dF_i dF_j.$$

Since the right-hand side of this equation is decreasing in V_i, it follows that there exists a unique solution to it in V_i. Hence there exists a unique ME; the equilibrium posted price contract is $p = v_b$ if $i = s$ and $p = -v_s$ if $i = b$.

3. Market Equilibrium

3.1. **Characterization and Existence.**

The lemma stated below describes the circumstances under which in any ME a pair of agents will and will not renegotiate, will and will not form a match and trade, and, if they match, the terms of trade. The proof of Lemma 1 is straightforward, and hence omitted.

Lemma 1. *Fix an arbitrary ME, and consider an arbitrary pair of agents who have met (at any time t) such that at stage 1 the posted price is p and such that the realized match-specific pair of values is (v_b, v_s). Then, in equilibrium:*
(a) if $v_b + v_s < V_b + V_s$, then the agents do not trade;
(b) if $v_b - p \geq V_b$ and $p + v_s \geq V_s$, then trade occurs at price p; and
(c) if $v_b + v_s \geq V_b + V_s$ and either (i) $v_b - p < V_b$, or (ii) $p + v_s < V_s$, then the agents renegotiate, and trade at price $V_s - v_s$ with probability one-half and at price $v_b - V_b$ with probability one-half.

Notice that Lemma 1(c) implies that if the posted price is either arbitrarily high or arbitrarily low, then, for any realization of the pair (v_b, v_s), the terms of trade are determined by bargaining. Figure 1, which divides the (v_b, v_s) space according to the four possible equilibrium outcomes described in Lemma 1, may be useful when understanding the Bellman equations for the players' equilibrium expected payoffs, to which we now turn.[5] For

[5]It may be noted that there is a discontinuity in the players' payoffs at the boundaries between the areas marked Part (b) and Part (c); this is due in part to our assumption that the players' types are perfectly observable.

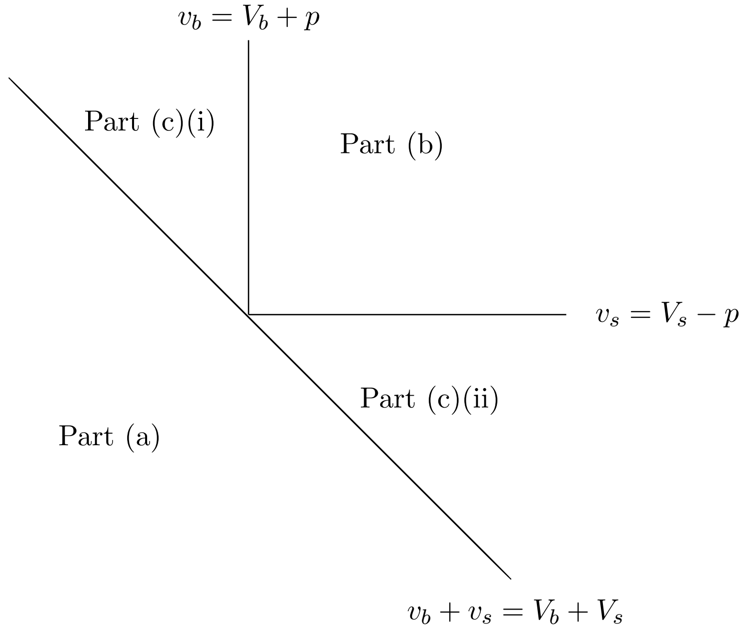

Figure 1. An illustration of Lemma 1.

convenience, we denote the subsets of all (v_b, v_s) that respectively lie in the regions marked Part (b) and Part (c) by $\Omega_R(p)$ and $\Omega_P(p)$.

Fix an arbitrary ME, and consider an arbitrary agent of type i (at any time t) who has posted a price p. Using Lemma 1, it is straightforward to show that his equilibrium expected payoff at the beginning of stage 2 (before the random meeting process occurs) is

$$Z_i(p, V_b, V_s) = \frac{V_i}{1+r\Delta} + \frac{\lambda_i \Delta}{1+r\Delta}\Bigg[\iint\limits_{(v_b,v_s)\in\Omega_P(p)} (v_i - \mathcal{I}p - V_i)dF_b dF_s + \iint\limits_{(v_b,v_s)\in\Omega_R(p)} \left(\frac{v_b + v_s - V_b - V_s}{2}\right) dF_b dF_s\Bigg],$$

with $$\mathcal{I} = \begin{cases} 1 & \text{if } i = b \\ -1 & \text{if } i = s. \end{cases}$$

Letting p_i^* denote the equilibrium posted price, it follows (by definition) that $V_i = Z_i(p_i^*, V_b, V_s)$. Hence, it follows that V_i satisfies the following Bellman equation:

(1)
$$\frac{rV_i}{\lambda_i} = \iint\limits_{(v_b,v_s)\in\Omega_P(p_i^*)} (v_i - \mathcal{I}p_i^* - V_i)dF_b dF_s + \iint\limits_{(v_b,v_s)\in\Omega_R(p_i^*)} \left(\frac{v_b + v_s - V_b - V_s}{2}\right) dF_b dF_s.$$

Furthermore, optimality requires that

$$p_i^* = \arg\max_p Z_i(p, V_b, V_s). \tag{2}$$

Finally, V_j (where $j \neq i$) satisfies the following Bellman equation:

(3)

$$\frac{rV_j}{\lambda_j} = \iint\limits_{(v_b,v_s)\in\Omega_P(p_i^*)} (v_j + \mathcal{J}p_i^* - V_j)dF_b dF_s + \iint\limits_{(v_b,v_s)\in\Omega_R(p_i^*)} \left(\frac{v_b + v_s - V_b - V_s}{2}\right) dF_b dF_s,$$

$$\text{with} \qquad \mathcal{J} = \begin{cases} 1 & \text{if } j = s \\ -1 & \text{if } j = b. \end{cases}$$

We have thus established that for any solution (V_i, V_j, p_i^*) to $(1) - (3)$ there exists a unique ME in which the equilibrium posted price is p_i^* and equilibrium expected payoffs to any agent of type i and any agent of type j are respectively V_i and V_j. There exist no other ME.

The following lemma establishes the existence of a ME under various alternative assumptions on the distribution functions:

Lemma 2. *(a) If F_i and F_j are continuous, then there exists a mixed-strategy ME.*
(b) If F_j is differentiable, $1 - F_j$ is log-concave, and F_i is degenerate, then there exists a unique pure-strategy ME.
(c) If both F_i and F_j are uniformly distributed, then there exists a unique pure-strategy ME.

3.2. **Equilibrium Pricing Mechanism.** Recall that type i agents (effectively) choose the mechanism through which the terms of trade are determined. In particular, a ME can be one of the following two types. A ME in which the posted price is arbitrarily high (or arbitrarily low) has the property that the terms of trade are always determined by ex-post bargaining. On the other hand, a ME in which the posted price is, what may be termed, "serious" (in the sense that it is neither too low nor too high) has the property that at least some trades are executed at the ex-ante posted price while others at an ex-post renegotiated price. Proposition 1 below establishes that any ME is of the latter type.

It is useful to first introduce the concept of the agents' reservation values. In any ME, R_b and R_s — where $R_b = V_b + p_i^*$ and $R_s = V_s - p_i^*$ — may be respectively interpreted as a buyer's and a seller's reservation values. A buyer would like to trade with a seller at the equilibrium posted price p_i^* if and only if his realized value from trading $v_b \geq R_b$. Similarly, a seller would like to trade with a buyer at the equilibrium posted price p_i^* if and only if his realized value from trading $v_s \geq R_s$. Indeed, trade occurs at the equilibrium posted price p_i^* if and only if $v_b \geq R_b$ and $v_s \geq R_s$. Otherwise trade occurs at a renegotiated price or trade does not occur, depending on whether $v_b + v_s \geq V_b + V_s$ or $v_b + v_s < V_b + V_s$.

Proposition 1 (Equilibrium Pricing Mechanism)**.** *In any ME, $\bar{v}_b > R_b$ and $\bar{v}_s > R_s$, where $R_b = V_b + p_i^*$ and $R_s = V_s - p_i^*$. In words, in any ME some trades are executed at the equilibrium posted price while others at an equilibrium negotiated price.*

To capture the essence of the proof, suppose that $i = b$, and, contrary to Proposition 1, that all buyers (such as firms in a labour market) are posting an arbitrarily low price (wage) which is unacceptable to all sellers (workers). In that case a firm and a worker split equally the match surplus (when gains to trade exist) via bargaining. Now suppose a firm unilaterally deviates and posts a wage equal to $V_s - \bar{v}_s + \epsilon$, where $\epsilon > 0$ but sufficiently small. It follows that there exist realizations of v_s such that workers with such realizations would (like the firm) be willing to trade at such a wage. Since such a wage forces the worker down to (almost) his continuation payoff V_s, the firm would (for such realizations) now get *all* the surplus — rather than have to split it with the worker. The deviation is, therefore, profitable for the firm.[6]

At first blush, this result may seem trivial. After all, by allowing one side of the market to post a price is like granting them market power which they are bound to use. To see why this result is interesting, consider once more Figure 1. For realizations of (v_b, v_s) in the regions marked Part (c), the agents bargain and any match surplus is divided equally. Within the region marked Part (b), trade occurs at the posted price and the division of the surplus depends on the actual realization of the match-specific values v_b and v_s. In particular, given a posted price p, realizations of (v_b, v_s) toward the south-east of this region generates trade such that a seller would ex-post regret having posted that price — he would receive less than half the match surplus. The price posting decision amounts to picking a location for the south-west corner of the Part (b) region on the $v_b + v_s = V_b + V_s$ line. The type i agent would like to minimize the probability of outcomes that he would (ex-post) regret vis-a-vis those which he would welcome. Proposition 1 implies that he can always find a (serious) price at which the benefits from posting it outweigh the opportunity cost of not bargaining.

Proposition 1 implies that in any ME, the equilibrium *proportion* of trades which occur at the posted price (rather than at a renegotiated price) is strictly positive, but (in general) it will be strictly less than one. It is therefore interesting to study how various parameters (such as those which capture market frictions) affect this equilibrium proportion. Such an analysis may, in particular, shed some light on the question of why in some markets trade typically occurs at posted prices (such as in some retail markets), while in other markets it typically occurs at negotiated prices (such

[6]Thus, the price posting cum renegotiation mechanism will be preferred by the firms over the ex-post bargaining mechanism, precisely because it allows them to extract a greater amount of surplus from some workers without affecting the surplus that they obtain from the others. Without the possibility of mutually beneficial renegotiation, this result may not hold.

as in bazaars and some labour markets). We address this issue in sections 4 and 5.

3.3. **Role of Market Frictions.** We now derive some results concerning the role of the main parameters (namely, r, λ_b and λ_s) on various aspects of an arbitrary ME. Our first result concerns the role of the matching rates on *aggregate* market welfare. We define aggregate market welfare to be the sum of the payoffs of *all* the agents in the market (in a steady state). That is, aggregate market welfare is $W = V_b N_b + V_s N_s$, where N_b and N_s are respectively the (steady state) measures of buyers and sellers in the market. Since agents meet in pairs, it must be the case that $\lambda_b N_b = \lambda_s N_s$. Hence, after normalizing the measures so that $N_b + N_s = 1$, it follows that aggregate market welfare

$$W = \frac{\lambda_s V_b + \lambda_b V_s}{\lambda_b + \lambda_s}.$$

We first state our result concerning the role of the matching rates on aggregate market welfare, and then discuss it.

Proposition 2 (Matching Rates and Market Welfare)**.** *Let W_i^C denote the aggregate market welfare in the unique ME under the assumption that type i agents can post comprehensive price contracts, and W_i^I the aggregate market welfare in an arbitrary ME of our market model (in which they post a single price).*
(a) If $\lambda_b = \lambda_s$, then $W_b^C = W_b^I = W_s^C = W_s^I$.
(b) If $\lambda_b < \lambda_s$, then $W_b^C > W_b^I$ and $W_s^I > W_s^C$.

Thus, when the matching rates are identical, aggregate market welfare is the same whether agents can post comprehensive price contracts or just prices. Not surprisingly, it does not matter which side of market gets to post prices. It should be noted, however, that the distribution of welfare between the two sides of the market will crucially depend on whether contracts are incomplete or comprehensive.

Another important result contained in Proposition 2 is that when matching rates are not identical, the posting of comprehensive price contracts by agents with the higher matching rate adversely affects aggregate market welfare. The insight contained here is that when one side of the market has too much bargaining power — by not only being able to post comprehensive price contracts but also by having a relatively higher matching rate — aggregate market welfare and market efficiency are compromised. By making such agents post a single price (with the option to engage in mutually beneficial renegotiation), on the other hand, gives some bargaining power to the other side of the market which has a lower matching rate.

A more general insight that one may extract from Proposition 2 is that the distribution of bargaining power amongst market traders has efficiency consequences. In particular, a social planner with an objective to maximize aggregate market welfare should not allow agents on the *short*-side of the market (i.e., agents who have the relatively higher matching rate) to post comprehensive price contracts. It should be noted that Proposition 2 implies that aggregate market welfare would be maximized *either* (i) when the agents with the relatively higher matching rate post a single price *or* (ii) when the agents with the relatively lower matching rate post comprehensive price contracts. The next proposition concerns the role of the matching rates on *each* agent's equilibrium payoff.

Proposition 3 (Matching Rates and Equilibrium Payoffs)**.** *In any ME, if* $\lambda_i \geq \lambda_j$ *then* $V_i > V_j$*, and, if* $\lambda_i < \lambda_j$ *and the difference* $\lambda_j - \lambda_i$ *is not too large then (also)* $V_i > V_j$*.*

The results stated in this proposition follow since a type i agent extracts a relatively greater amount of surplus from *some* type j agents (i.e., for some realizations of v_j) by trading at the equilibrium posted price. Not surprisingly, the ability to post a price (even with the possibility of mutually beneficial renegotiation) gives a type i agent a relatively greater equilibrium payoff. And, this is valid even when there are far more agents of that type than agents of the opposite type in the market.

When the discount rate is arbitrarily small (close to zero), the market contains negligible frictions. The result contained in the following proposition addresses, in particular, this limiting case of negligible market frictions.

Proposition 4 (Discount Rates and Sum of Equilibrium Payoffs)**.** *In any ME, the sum* R *of the equilibrium payoffs to a pair of agents of the opposite types is strictly decreasing in* r*. Furthermore,* $R \to \bar{v}_b + \bar{v}_s$ *as* $r \to 0$*, and,* $R \to 0$ *as* $r \to \infty$*, where* $R = V_b + V_s$*.*

As would be expected, aggregate market welfare increases as the degree of market frictions decreases. In particular, Proposition 4 implies that when market frictions are negligible, in any ME, trade occurs between a buyer and a seller if and only if the realized match-specific pair of values (v_b, v_s) are arbitrarily close to the pair $(\bar{v}_b, \bar{v}_s)$. This makes intuitive sense; when frictions are negligible, the cost to each agent of locating the almost perfect match is negligible — and therefore, each agent waits to match with an agent of the opposite type who will generate almost maximal value for him.

4. The Case of Uniform Distributions

In order to obtain some additional insights concerning the properties of Market Equilibria, over and above those obtained in section 3 above, we now study the unique ME when both F_i and F_j are uniformly distributed. Although the results discussed here are specific to this case, the intuition behind them suggest that some (but certainly not all) of these results may actually hold more generally. In particular, we explore the impact of market frictions on (i) the equilibrium proportion of trades that occur at a posted price, and (ii) the equilibrium payoffs to type i and type j agents. In this section (only) we assume, without loss of generality, that $i = b$ (i.e., buyers post prices); this assumption fits, for example, labour markets in which firms post wages.

Proposition 5 (The Case of Uniform Distributions)**.** *If $i = b$, and, both F_b and F_s are uniformly distributed, then the unique ME possesses the following properties.*
(a) There exists an $\bar{r}$ such that over the interval $(0, \bar{r})$, a change in the discount rate r has in general an ambiguous effect on the equilibrium posted price. However, if $\lambda_b = \lambda_s$, then the equilibrium posted price is strictly decreasing in r; but if λ_s is sufficiently larger than λ_b, then the equilibrium posted price is strictly increasing in r.
(b) The equilibrium proportion of trades which occur at the posted price — we denote this proportion by τ — equals 1/3 for any $r \in (0, \bar{r})$. For any $r > \bar{r}$, τ is strictly increasing in r.

Before we discuss the results stated in this proposition, we first report the results of a simulation that reveals a few other interesting properties of the unique ME for larger values of the discount rate, but when the matching rates are identical.[7] The results of the simulation in question are stated in Table 1. As is evident, the results of this simulation are consistent with the appropriate results stated in Proposition 5 above. There are, however, some other revealing results in this simulation that we have not been able to establish analytically. For example, as can be seen from Table 1, the difference $V_b - V_s$ does *not* vanish as r becomes negligible. This result is particularly interesting as it implies that our ME outcome does not approximate the "competitive" equilibrium outcome even as market frictions become negligible.[8] The message here is that even under frictionless conditions, the option to post prices confers a strategic advantage.

A key intuition that underlies all of these results runs as follows. When the matching rates are identical, then the only asymmetry between a firm and a worker is that the firm has the option to post a wage, which works to its advantage; in particular, the *greater* the degree of market frictions (i.e., the higher the value of r) the *bigger* is that advantage. This is because the worker's "outside option" — which is to wait and find an alternative trading partner — is less attractive the greater the degree of market frictions. However, when a worker's matching rate is sufficiently large relative to a firm's matching rate, the worker's outside option is relatively more attractive than the firm's outside option, and this works to the worker's advantage. Hence when λ_s is sufficiently larger than λ_b, the equilibrium posted wage is strictly increasing in r (Proposition 5(a)).

[7]The model was simulated, with most of the results to be shortly discussed captured also in the several other simulations that we conducted but which we shall not report here.

[8]In a "competitive" equilibrium outcome of our market, all agents would earn the same expected payoff.

r	R	p_b	V_b	V_s	τ
.001	14.2	4.22	9.15	5.05	0.33
0.01	13.3	3.93	8.71	4.59	0.41
0.1	10.9	2.45	8.22	2.68	0.78
0.2	9.69	1.87	7.82	1.87	0.9
0.3	8.86	1.47	7.39	1.47	0.91
0.4	8.21	1.22	6.99	1.22	0.97
0.5	7.67	1.00	6.67	1	1

Table 1. Results of a simulation: Equilibrium when F_b and F_s are uniformly distributed, $\lambda_b = \lambda_s = 1$, $\bar{v}_s = 1$, $\underline{v}_s = 0$, $\bar{v}_b = 14$ and $\underline{v}_b = 8$.

As can be seen from Table 1, when the matching rates are identical, both V_b and V_s are strictly decreasing in r. Furthermore, the ratio V_b/V_s increases with the discount rate. This result indicates that the *relative* advantage that type i agents have over type j agents increases with the degree of market frictions.

Although the result established in Proposition 5(b) that τ, the equilibrium proportion of trades at the posted price, is constant for small values of r may not be robust to other specifications of the distributions, the result that τ is (in general) increasing in r will be shown (in the next section) to hold under a more general class of distributions. Thus, a key insight from our model may be put as follows: *trade in markets with small frictions is likely to occur at negotiated prices, while in markets with large frictions it is more likely to occur at posted prices.*

To illustrate the intuition for this insight in a fairly transparent manner, let us consider the two extreme cases of very large and very small degrees of market friction. In the former case, waiting to find an alternative trading partner is quite costly, and thus, the agents' outside options are pretty unattractive. This immediately implies that trade at the posted price will tend to be individually rational for any pair of agents. In the latter case, on the other hand, the reverse holds: the outside options of both agents are pretty attractive, and thus, trade at the posted price will tend not to be individually rational for at least one them. Hence, trade is more likely to occur at an ex-post negotiated price.

5. One-Sided Heterogeneity

We now study the unique ME under the assumption that F_i is degenerate (i.e., $\bar{v}_i = \underline{v}_i = v_i^*$); this assumption captures many kinds of markets such as some retail markets (or bazaars) in which sellers post prices, and each seller does not care as to which particular buyer she trades with. Our objective here is to explore the robustness or otherwise of the results obtained above in section 4 concerning the role of the main parameters on the equilibrium proportion of trades which occur at a posted price.

We first examine how the parameters affect the equilibrium probability of trade between an arbitrary pair of agents. This probability is

$$\gamma = 1 - F_j(R - v_i^*),$$

where $R = V_i + V_j$. For simplicity of calculation, we assume that $\lambda_b = \lambda_s = \lambda$; and denote r/λ by $\widehat{r}$.

Lemma 3 (Equilibrium Probability of Trade)**.** *Assume that F_j is differentiable, $1-F_j$ is log-concave, F_i is degenerate ($\bar{v}_i = \underline{v}_i = v_i^*$), and $\lambda_b = \lambda_s = \lambda$. For any parameter values such that $R > v_i^* + \underline{v}_j$ (where $R = V_i + V_j$), the equilibrium probability of trade γ is strictly increasing in $\widehat{r}$ (where $\widehat{r} = r/\lambda$), and it is also strictly increasing in v_i^*.*

Thus, the equilibrium probability of trade between an arbitrary pair of agents increases with the degree of market frictions and with the expected total value of a match.[9] An implication of this result is that a *proportional* mean preserving spread of F_j generates a reduced rate of trading.

Proposition 4 implies that $R(\equiv V_b + V_s)$ is strictly decreasing in $\widehat{r}$, with $R \to 0$ as $\widehat{r} \to \infty$, and $R \to v_i^* + \bar{v}_j$ as $\widehat{r} \to 0$. These results imply that the comparative-static results stated in Lemma 3 are valid for *any* parameter values when $v_i^* + \underline{v}_j \leq 0$. But if $v_i^* + \underline{v}_j > 0$, then, when $\widehat{r}$ is sufficiently large, $R < v_i^* + \underline{v}_j$ (i.e., $R < v_i^* + v_j$, for any v_j). Hence, when $\widehat{r}$ is sufficiently large, the equilibrium probability of trade equals one. Indeed, this makes intuitive sense: when the degree of market frictions is sufficiently large, the equilibrium payoffs to any pair of agents from not trading (and thus, waiting to find an alternative trading partner) will be so small that they will (for any v_j) find it mutually beneficial to trade with each other.

We now explore how the parameters affect the equilibrium proportion of trades which occur at a posted price (rather than at a renegotiated price). Let τ denote this proportion; thus, $1 - \tau$ denotes the proportion of trades that occur at a renegotiated price. In the unique ME, trade occurs at the posted price for any $v_j \in [R_j, \bar{v}_j]$, and at a renegotiated price for any $v_j \in [R - v_i^*, R_j]$, where R_j is a type j agent's equilibrium reservation value and $R = V_i + V_j$. Furthermore, for any $v_j \in [\underline{v}_j, R - v_i^*]$ trade does not occur (since it is not mutually beneficial to do so). Thus,

$$\tau = \frac{1 - F_j(R_j)}{1 - F_j(R - v_i^*)} = \frac{1 - F_j(R_j)}{\gamma}. \tag{4}$$

Proposition 6 (Equilibrium Trade at the Posted Price)**.** *Assume that F_j is differentiable, $1 - F_j$ is log-concave, F_i is degenerate ($\bar{v}_i = \underline{v}_i = v_i^*$), and $\lambda_b = \lambda_s = \lambda$.*
(a) The derivative of τ with respect to $\widehat{r}$ is in general ambiguous, where

[9]It may be noted that in terms of the pattern of trade, changes in v_i^* are identical to changes in the expected value of v_j.

$\widehat{r} = r/\lambda$. If, however, $v_i^* + \underline{v}_j > 0$, then there exists a $\widehat{r}^*$ and $\widehat{r}^{**}$ where $\widehat{r}^* > \widehat{r}^{**} > 0$ such that over the interval $(\widehat{r}^{**}, \widehat{r}^*)$, τ is strictly increasing in $\widehat{r}$, and $\tau = 1$ for all $\widehat{r} > \widehat{r}^*$.

(b) The derivative of τ with respect to v_i^* is in general ambiguous. However, there exists a $\widehat{v}_i^*$ and $\tilde{v}_i^*$ where $\widehat{v}_i^* > \tilde{v}_i^* > 0$ such that over the interval $(\tilde{v}_i^*, \widehat{v}_i^*)$, τ is strictly increasing in v_i^*, and, $\tau = 1$ for all $v_i^* > \widehat{v}_i^*$.

The intuition for why the comparative-static results reported in the proposition above are in general ambiguous is because changes in the appropriate parameters have the same qualitative effect on both the denominator of the RHS of (4) — which is the equilibrium probability of trade — and on the numerator of the RHS of (4) — which is the equilibrium probability of trade at the posted price. We now discuss the other results contained in Proposition 6.

An implication of the result in part (a) of the proposition is — as we also discovered in the context of the case of uniform distributions in section 4 — that trade in markets with relatively small frictions is more likely to occur at negotiated prices, while in markets with relatively large frictions it is more likely to occur at posted prices. This result makes sense, and the intuition for it was provided in section 4. On the other hand, the appropriate result in part (b) of the proposition is at first blush seemingly inconsistent with real-life markets. The result implies that trade in retail markets for expensive items (such as cars and houses) should be more likely to occur at posted prices, while in retail markets for relatively cheap items (such as foodstuffs) trade is more likely to occur at negotiated prices.

As we now explain in the context of retail markets, the reason for why our model generates this counter-intuitive relationship between the equilibrium proportion of trades at the posted price and the expected total value of a match is because in our model the matching rate (or equivalently the intensity of search) is exogenously given.

Notice that an implication of the result (contained in Proposition 6(a)) is that (when the matching rate is sufficiently small) the equilibrium proportion of trades at the posted price is strictly decreasing in the matching rate. Since the matching rate is determined by the intensity with which buyers search for sellers, this result implies that the more intensively buyers engage in search the more likely it is that trade occurs at negotiated prices. But casual observation suggests that in real-life, retail market buyers tend to search intensively when buying an expensive item (such as a car or a house) and not when buying cheap items (such as foodstuffs). This is because it is hardly more costly to find another auto dealership than it is to find another supermarket. Accordingly, a simple extension of the model (which goes beyond the scope of the current paper) would be to endogenize search intensity. With search costs fixed, agents will search more intensively for larger ticket items inducing a higher proportion of trades at bargained prices.

We now draw attention to another implication of the appropriate result contained in Proposition 6(b). In the context of labour markets, suppose that firms are posting wages, and that the variance of the distribution of a worker's match-specific value decreases. Since a decrease in the variance of F_j is formally equivalent to an increase in v_i^*,[10] it follows immediately from Proposition 6(b) that as the variance of F_j decreases, the equilibrium proportion of trades at the posted price will increase. This makes intuitive sense. That is partly because as the variability in v_j decreases, the likelihood that trade at the posted wage is individually rational for both parties increases. And partly because as the variability in v_j decreases, after having encountered some firm, a worker's incentive to search for an alternative firm decreases, since the likelihood of her obtaining a better match-specific value has decreased. In the context of retail market, this result suggests that when comparing two items of similar expected value, such as painting and computer, the item over which the idiosyncratic component of value has greater variance is more likely to be sold at a bargained price (*i.e.* the painting).

6. Summary and Concluding Remarks

The two most fundamental results obtained in this paper — concerning markets characterized by match-specific heterogeneity — are as follows:

• In such markets, some trades are executed at posted prices, while others at negotiated prices — with the exact proportions depending on the fundamentals such as the degree of market frictions. In particular, trade in markets with small frictions is likely to occur at negotiated prices, while in markets with large frictions it is more likely to occur at posted prices. This implies, for example, that in retail markets in which buyers search intensively (such as in housing markets) trade is more likely to occur at negotiated prices, while in retail markets in which their intensity of search is negligible (such as in the food market) trade is more likely to occur at posted prices.

• In general (when the numbers of sellers and buyers are unequal, or equivalently when the matching rates are unequal) the posting of comprehensive price contracts by agents on the short-side of the market adversely affects aggregate market welfare. To put it differently, when one side of the market has too much bargaining power — by not only being able to post comprehensive price contracts but also by having a relatively higher matching rate — aggregate market welfare is compromised.

Although our market model is the first to combine in a single framework (of endogenous price determination) the triple features of (i) match-specific heterogeneity, (ii) the option to post prices, and (iii) the option to engage

[10] This is because an increase in v_i^* is equivalent to an increase in the expected value of v_j, which, in turn, is equivalent (after normalization) to keeping the expected value of v_j unchanged but decreasing the variance of v_j.

in mutually beneficial renegotiation, the model contains some restrictive, simplifying assumptions. We conclude by briefly considering some of them.

One such is the assumption that when a pair agents encounter each other, their match-specific trading values become common knowledge between them. This assumption ought to be relaxed in future research, since it is far more plausible that after a pair of agents encounter each other, the realization of type k agent's $(k = b, s)$ match-specific value v_k is his private information.

Second, although the assumption that only one side of the market has the option to post prices may have merit from an applicability point of view — since it is consistent with several real-life markets — it is interesting and important from a theoretical perspective to give both sides of the market this option; and thus, to endogenously determine (as part of a market equilibrium) the conditions under which only one side exercises such an option.

Third, it would be interesting to extend our model by allowing the trading value to be partly match-specific, but also partly endogenous, as a function of some investment decision. That is, the value v_k to a type k agent from trading with some particular type m agent $(m \neq k)$ is a function $f_k(\theta_k, I_k)$, where θ_k is the match-specific component (randomly realized after encountering this particular agent), while I_k is his investment level made upfront before encountering any agent. Such an extended model could also be interpreted from the perspective of the incomplete *bilateral* contracting literature: unlike in that literature, in such a model the parties' "outside options" would now be endogenously determined as part of a market equilibrium.

Fourth, the assumption that the intensities with which agents search is exogenous should be relaxed. As we informally discussed in section 5 above, such an extended model is necessary in order to obtain the plausible relationship between the equilibrium proportion of trades that occur at posted prices (vis-a-vis negotiated prices) and the expected total value of a match. This is because as the expected total value of a match increases, it is intuitive that agents in real-life markets increase their intensity of search — which (using the results obtained in this paper) would imply an increase in the likelihood of trade occuring at negotiated prices (rather than at posted prices).

Appendix

[Note: The proof of Lemma 2 is stated after the proof of Proposition 4.]

Proof of Proposition 1. Fix an arbitrary ME. We first establish, by contradiction, that $\bar{v}_b + \bar{v}_s > V_b + V_s$. Thus suppose that $\bar{v}_b + \bar{v}_s \leq V_b + V_s$. This implies that trade never takes place (i.e., $\Omega_P(p_i^*) = \Omega_R(p_i^*) = \emptyset$). Hence, it follows from (1) and (3) that $V_b = V_s = 0$. But this, in turn, implies that $\bar{v}_b + \bar{v}_s \leq 0$, which is a contradiction. We note that $\bar{v}_b + \bar{v}_s > R$ implies that $V_s - \bar{v}_s < \bar{v}_b - V_b$, where $R = V_b + V_s$. Define, for each $\epsilon \in [0, \bar{v}_b + \bar{v}_s - R]$,

$$p_i(\epsilon) = \begin{cases} V_s - \bar{v}_s + \epsilon & \text{if } i = b \\ \bar{v}_b - V_b - \epsilon & \text{if } i = s. \end{cases}$$

The following Claim implies that $V_s - \bar{v}_s < p_i^* < \bar{v}_b - V_b$, which, in turn (and as required), implies Proposition 1.

Claim A.1. *There exists an $\epsilon \in (0, \bar{v}_b + \bar{v}_s - R)$ such that $Z_i(p_i(\epsilon), V_b, V_s) > Z_i(p, V_b, V_s)$ for any $p \geq \bar{v}_b - V_b$ and for any $p \leq V_s - \bar{v}_s$.*

Proof of Claim A.1. If p is such that either $p \geq \bar{v}_b - V_b$ or $p \leq V_s - \bar{v}_s$, then $\Omega_P(p) = \emptyset$, which implies that for any such p,

$$Z_i(p, V_b, V_s) = \int_{v_i=R-\bar{v}_j}^{\bar{v}_i} \int_{v_j=R-v_i}^{\bar{v}_j} \left(\frac{v_b + v_s - R}{2} \right) dF_b dF_s. \tag{5}$$

Notice that for any such p, $Z_i(p, V_b, V_s)$ is independent of p — since for any such p the terms of trade are determined (for any possible realization of v_b and v_s) ex-post bargaining. Furthermore,

$$Z_i(p_i(\epsilon), V_b, V_s) = \int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=\bar{v}_j-\epsilon}^{\bar{v}_j} (v_i + \bar{v}_j - R - \epsilon) dF_b dF_s + C + D, \text{ where}$$

$$C = \int_{v_i=R-\bar{v}_j}^{v_i=R-\bar{v}_j+\epsilon} \int_{v_j=R-v_i}^{\bar{v}_j} \left(\frac{v_b + v_s - R}{2} \right) dF_b dF_s \quad \text{and}$$

$$D = \int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=R-v_i}^{\bar{v}_j-\epsilon} \left(\frac{v_b + v_s - R}{2} \right) dF_b dF_s.$$

For notational convenience, define $G_i(\epsilon) = 2[Z_i(p_i(\epsilon), V_b, V_s) - Z_i(p, V_b, V_s)]$. Since the right-hand side of (5) equals

$$\int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=\bar{v}_j-\epsilon}^{\bar{v}_j} \left(\frac{v_b + v_s - R}{2} \right) dF_b dF_s + C + D, \qquad \text{it follows that}$$

$$G_i(\epsilon) = \int_{v_i=R-\bar{v}_j+\epsilon}^{\bar{v}_i} \int_{v_j=\bar{v}_j-\epsilon}^{\bar{v}_j} \Bigg[(v_i + \bar{v}_j - R - \epsilon) - [v_j - (\bar{v}_j - \epsilon)] \Bigg] dF_b dF_s.$$

Thus, for any $\epsilon \in [0, \bar{v}_b + \bar{v}_s - R]$,

$$G_i(\epsilon) = -\Psi_j'(\bar{v}_j - \epsilon)\Psi_i(R - \bar{v}_j + \epsilon) + \Psi_j(\bar{v}_j - \epsilon)\Psi_i'(R - \bar{v}_j + \epsilon),$$

where for each $x = b, s$ and for any $z \in \Re$,

$$\Psi_x(z) = \int_{v_x=z}^{v_x=\bar{v}_x} [1 - F_x(v_x)] dv_x = \int_{v_x=z}^{v_x=\bar{v}_x} (v_x - z) dF_x. \tag{6}$$

It may be noted that $\Psi_x'(z) = -[1 - F_x(z)]$.

Since $\Psi_x'(z) = -[1 - F_x(z)]$, and for any $\epsilon > 0$, $\Psi_j(\bar{v}_j - \epsilon) < \epsilon[1 - F_j(\bar{v}_j - \epsilon)]$, it follows that for any $\epsilon \in (0, \bar{v}_b + \bar{v}_s - R]$,

$$G_i(\epsilon) > [1 - F_j(\bar{v}_j - \epsilon)][\Psi_i(R - \bar{v}_j + \epsilon) + \epsilon\Psi_i'(R - \bar{v}_j + \epsilon)].$$

We now show that there exists an $\bar{\epsilon}_i$, where $0 < \bar{\epsilon}_i < \bar{v}_b + \bar{v}_s - R$, such that for any $\epsilon \in (0, \bar{\epsilon}_i)$ the right-hand side of the above inequality is strictly positive — which implies that $G_i(\epsilon) > 0$, as required.

Let $h_i(\epsilon) = \Psi_i(R - \bar{v}_j + \epsilon) + \epsilon \Psi_i'(R - \bar{v}_j + \epsilon)$. Since (by definition) F_i is right-continuous, it follows that h_i is right-continuous at $\epsilon = 0$. Now notice (since $\bar{v}_b + \bar{v}_s > R$) that $h_i(0) > 0$. It thus follows that there exists an $\bar{\epsilon}_i$ (where $0 < \bar{\epsilon}_i < \bar{v}_b + \bar{v}_s - R$) such that for any $\epsilon \in (0, \bar{\epsilon}_i)$, $h_i(\epsilon) > 0$. The desired result follows immediately, since for any $\epsilon > 0$, $1 - F_j(\bar{v}_j - \epsilon) > 0$.

Proof of Proposition 2. For each $i = b, s$, let R_i^C denote the sum of the equilibrium payoffs to a pair of agents of the opposite types in the unique ME when the type i agents post comprehensive price contracts, and, let R_i^I denote the sum of the equilibrium payoffs to a pair of agents of the opposite types in an arbitrary ME when the type i agents just post prices.

First suppose that $\lambda_b = \lambda_s$. It follows from Claim A.2 below that $R_b^I = R_b^C$ and $R_s^I = R_s^C$. Part (a) of the proposition now follows immediately since $R_b^C = R_s^C$, and (when the matching rates are identical) aggregate market welfare W equals one-half of the sum of the payoffs to a pair of agents of the opposite types.

Now we establish part (b) of the proposition. Thus, now suppose that $\lambda_b < \lambda_s$. It follows from Claim A.2 that $R_b^I > R_b^C$ and $R_s^I < R_s^C$. Furthermore,

$$W_b^C = \frac{\lambda_s}{\lambda_b + \lambda_s} R_b^C \quad \text{and} \quad W_b^I = \frac{\lambda_s}{\lambda_b + \lambda_s} \Big[V_b^I + \Big(\frac{\lambda_b}{\lambda_s}\Big) V_s^I \Big].$$

Now

$$R_b^C = \frac{\lambda_b}{r} \iint\limits_{v_b + v_s \geq R_b^C} [v_b + v_s - R_b^C] dF_b dF_s,$$

and equations 1 and 3 imply that

$$V_b^I + \Big(\frac{\lambda_b}{\lambda_s}\Big) V_s^I = \frac{\lambda_b}{r} \iint\limits_{v_b + v_s \geq R_b^I} [v_b + v_s - R_b^I] dF_b dF_s.$$

Hence, since the integral

$$\iint\limits_{v_b + v_s \geq R} [v_b + v_s - R] dF_b dF_s$$

is strictly decreasing in R, and since Claim A.2 below implies that $R_b^I > R_b^C$, it therefore follows that $W_b^I < W_b^C$. By a symmetric argument, since Claim A.2 implies that $R_s^I < R_s^C$, it follows that $W_s^I > W_s^C$.

Claim A.2.

$$R_i^I \gtreqqless R_i^C \quad \textit{if} \quad \lambda_i \lesseqqgtr \lambda_j.$$

Proof of Claim A.2. It follows from equations 1 and 3 that

$$\frac{rV_i}{\lambda_i} + \frac{rV_j}{\lambda_j} = \iint\limits_{v_b + v_s \geq R} [v_b + v_s - R] dF_b dF_s,$$

where $R = V_b + V_s$. Now define $k = \lambda_j / \lambda_i$. After substituting for λ_j (using this latter expression) in the above equation, and re-arranging, we obtain that

$$\frac{r(1-k)V_j}{k} = \xi(R), \tag{7}$$

where

$$\xi(R) = \lambda_i \iint\limits_{v_b+v_s\geq R} [v_b + v_s - R]dF_b dF_s - rR.$$

In any ME, V_j and R must satisfy (7). Now notice that R_i^C is the unique value of R such that $\xi(R) = 0$. Furthermore, note that since the first term of $\xi(.)$ is strictly decreasing in R, it follows that $\xi(.)$ is strictly decreasing in R. Given these properties of ξ, it is now trivial to establish Claim A.2. First note that if $k = 1$ then it follows from (7) that in any ME, $\xi(R) = 0$; hence, $R_i^I = R_i^C$. Secondly, note that if $k < 1$ then — since in any ME, $V_j > 0$ — it follows from (7) that $\xi(R) > 0$; hence, $R_i^I < R_i^C$. And finally, by a similar argument, if $k > 1$ then $R_i^I > R_i^C$.

Proof of Proposition 3. In order to establish this proposition, we first need to derive Claim A.3 (stated below) that provides an alternative characterization of the set of ME. It then follows by subtracting (9) from (8) — stated below in Claim A.3 — that the difference between the equilibrium payoffs (in any ME)

$$V_i - V_j = \frac{(\lambda_i - \lambda_j)R}{\lambda_b + \lambda_s} + \frac{2\lambda_b\lambda_s\widehat{G}_i(R)}{r(\lambda_b + \lambda_s)},$$

where R and $\widehat{G}_i(R)$ are defined below in Claim A.3. Proposition 3 is now an immediate consequence of the observation that $\widehat{G}_i(R) > 0$ and $R > 0$.

Claim A.3. *Fix i and j, where $i, j = b, s$ with $i \neq j$. (V_i, V_j, p_i^*) defines a ME if and only if*

$$V_i = \frac{\lambda_i R}{\lambda_b + \lambda_s} + \frac{\lambda_b\lambda_s\widehat{G}_i(R)}{r(\lambda_b + \lambda_s)} \tag{8}$$

$$V_j = \frac{\lambda_j R}{\lambda_b + \lambda_s} - \frac{\lambda_b\lambda_s\widehat{G}_i(R)}{r(\lambda_b + \lambda_s)} \tag{9}$$

$$p_i^* = \begin{cases} V_s - \bar{v}_s + \epsilon_b^* & \text{if } i = b \\ \bar{v}_b - V_b - \epsilon_s^* & \text{if } i = s, \end{cases}$$

where R is a fixed point of $\widehat{\Gamma}_i$, $\epsilon_i^* \in \Phi_i(R)$ such that $R = \Gamma_i(R, \epsilon_i^*)$, and

$$\widehat{G}_i(R) = \max_{\epsilon\in[0,\bar{q}+\bar{e}-R]} G_i(\epsilon; R), \quad \text{with}$$

$$\Phi_i(R) = \{\epsilon_i^* : G_i(\epsilon_i^*; R) \geq G_i(\epsilon; R) \quad \forall\epsilon \in [0, \bar{v}_b + \bar{v}_s - R]\}, \quad \text{where}$$

$$G_i(\epsilon; R) = -\Psi_j'(\bar{v}_j - \epsilon)\Psi_i(R - \bar{v}_j + \epsilon) + \Psi_j(\bar{v}_j - \epsilon)\Psi_i'(R - \bar{v}_j + \epsilon) \tag{10}$$

(11)

$$\Gamma_i(R, \epsilon_i^*) = \frac{\lambda_b + \lambda_s}{2r}\left[\int_{v_i=R-\bar{v}_j+\epsilon_i^*}^{\bar{v}_i} \Psi_j(R - v_i)dF_i + \int_{v_j=\bar{v}_j-\epsilon_i^*}^{\bar{v}_j} \Psi_i(R - v_j)dF_j\right] + \frac{\lambda_i\Psi_j(\bar{v}_j - \epsilon_i^*)\Psi_i'(R - \bar{v}_j + \epsilon_i^*)}{r} + \frac{\lambda_j\Psi_j'(\bar{v}_j - \epsilon_i^*)\Psi_i(R - \bar{v}_j + \epsilon_i^*)}{r}$$

Furthermore, for each $R \in [0, \bar{v}_b + \bar{v}_s]$, $\quad \widehat{\Gamma}_i(R) = \{\Gamma_i(R, \epsilon_i^*) : \epsilon_i^* \in \Phi_i(R)\}$.

Proof of Claim A.3. It is easy to show that equations 1 and 3 can be respectively rewritten as follows:

$$\frac{2rV_i}{\lambda_i} = \int_{v_i=R_i}^{\bar{v}_i} \Psi_j(R-v_i)dF_i + \int_{v_j=R_j}^{\bar{v}_j} \Psi_i(R-v_j)dF_j + 2\Psi_j(R_j)\Psi_i'(R_i) \tag{12}$$

$$\frac{2rV_j}{\lambda_j} = \int_{v_i=R_i}^{\bar{v}_i} \Psi_j(R-v_i)dF_i + \int_{v_j=R_j}^{\bar{v}_j} \Psi_i(R-v_j)dF_j + 2\Psi_i(R_i)\Psi_j'(R_j), \tag{13}$$

where $R_b = V_b + p_i^*$, $R_s = V_s - p_i^*$, $R = R_b + R_s$ and, for each $x = b, s$ and for any $z \in \Re$, $\Psi_x(z)$ is define in (6). It follows immediately from the arguments in the proof of Proposition 2 above that in any ME, the equilibrium posted price p_i^* is as follows:

$$p_i^* = \begin{cases} V_s - \bar{v}_s + \epsilon_b^* & \text{if } i = b \\ \bar{v}_b - V_b - \epsilon_s^* & \text{if } i = s, \end{cases}$$

for some $\epsilon_i^* \in \Phi_i(R)$, where $\Phi_i(R)$ is defined in Claim A.3. It thus follows that a ME can be characterized by a triple (V_b, V_s, ϵ_i^*) which satisfies equations 12 and 13 *with* $R_j = \bar{v}_j - \epsilon_i^*$ and $R_i = R - \bar{v}_j + \epsilon_i^*$, and such that $\epsilon_i^* \in \Phi_i(R)$. Substituting for R_i and R_j in (12) and (13) using these latter expressions, it follows (by adding (12) and (13)) that $R = \Gamma_i(R, \epsilon_i^*)$, where $\Gamma_i(R, \epsilon_i^*)$ is defined in Claim A.3. Hence, (V_b, V_s, p_i^*) defines a ME *only if* $R(= V_b + V_s)$ is a fixed point of $\widehat{\Gamma}_i$, where the correspondence $\widehat{\Gamma}_i$ is defined in Claim A.3. This, then, establishes Claim A.3.

Proof of Proposition 4. Fix an arbitrary ME. In order to emphasize the dependence of the sum of the equilibrium payoffs to a pair of agents of the opposite types on the discount rate, we write it as $R(r)$. Now consider two arbitrary values of the discount rate, $r_H > r_L$. We need to show that $R(r_L) > R(r_H)$. We argue by contradiction. Thus, suppose, to the contrary, that $R(r_L) \leq R(r_H)$. It follows from equation (7) — since for any R, $\xi(R; r)$ is strictly decreasing in r — that therefore $r_H V_j(r_H) < r_L V_j(r_L)$. But this implies that $V_j(r_H) < V_j(r_L)$, which, in turn, implies that $V_i(r_H) < V_i(r_L)$. Hence, it follows that $R(r_H) < R(r_L)$, which contradicts our supposition.

We now show that $R \to \bar{v}_b + \bar{v}_s$ as $r \to 0$. In the limit as $r \to 0$, the LHS of (7) converges to zero, and hence, the RHS must also converge to zero. This implies that in the limit as $r \to 0$,

$$\iint_{v_b+v_s \geq R} [v_b + v_s - R]dF_b dF_s$$

must converge to zero. Hence, R must converge to $\bar{v}_b + \bar{v}_s$.

We now show that $R \to 0$ as $r \to \infty$. After dividing equation 7 by r, it follows that in the limit as $r \to \infty$, it must be the case that $V_j/k + V_i \to 0$. Hence, $R \to 0$.

Proof of Lemma 2

Proof of Part (a)

Here we establish the existence of a ME by allowing agents of type i to randomize over their choice of the posted price. This existence result only requires that both F_b and F_s are continuous, which allows us to use the *Theorem of the Maximum*, from which it follows that $\Phi_i(.)$ is compact valued and upper hemi-continuous (uhc), where Φ_i is defined in Claim A.3 above.

A type i agent randomizes over two prices — that is, over two values of ϵ_i^*. The value of ϵ_i^* is realized after he encounters an agent of the opposite type but before the match-specific values are realized. A randomization is a triple $(\epsilon_{i1}^*, \epsilon_{i2}^*, \pi)$ where p is the probability with which ϵ_{i1}^* is selected.

From the definition and derivation of $G_i(\epsilon; R)$ — which is provided in the proof of Proposition 1 — it should be clear that the amended problem for the type i agent is now to pick $(\epsilon_{i1}, \epsilon_{i2}, \pi)$ from $[0, \bar{v}_i + \bar{v}_j - R]^2 \times [0, 1]$ to maximize $\pi G_i(\epsilon_{i1}; R) + (1-\pi) G_i(\epsilon_{i2}; R)$. Hence, the optimal pair ϵ_{i1}^* and ϵ_{i2}^* can be chosen arbitrarily from $\Phi_i(R)$. Given this choice, π^* is arbitrary.

Amending (12) and (13) to allow for randomizations, and then adding them implies that in equilibrium $R \in \Lambda(R)$, where

$$\Lambda(R) = \{\gamma : \gamma = \pi \min\{\widehat{\Gamma}_i(R)\} + (1 - \pi) \max\{\widehat{\Gamma}_i(R)\} \quad \text{for some} \quad \pi \in [0, 1]\}.$$

Because $\Gamma_i(R, .)$ is continuous, $\widehat{\Gamma}_i(.)$ is compact valued and uhc. Hence, $\Lambda : [0, \bar{v}_i + \bar{v}_j] \to [0, \bar{v}_i + \bar{v}_j]$ is compact valued, convex valued and uhc. An equilibrium is fully characterized as a fixed point of Λ. This is because any $R^* \in \Gamma(R^*)$ identifies a randomization $\{\epsilon_{i1}^*, \epsilon_{i2}^*, \pi^*\}$ such that $\Gamma_i(R^*, \epsilon_{i1}^*) = \min\{\widehat{\Gamma}_i(R^*)\}$ and $\Gamma_i(R^*, \epsilon_{i2}^*) = \max\{\widehat{\Gamma}_i(R^*)\}$ and $\pi = [R^* - \min\{\widehat{\Gamma}(R^*)\}]/[\max\{\widehat{\Gamma}(R^*)\} - \min\{\widehat{\Gamma}(R^*)\}]$, which is consistent with type i agents' optimal behaviour at $R = R^*$. Given this randomization, the threshold value for the sum of the match specific preference shocks, $v_i + v_j$, at which trade occurs is exactly R^*. Existence of a ME is now a consequence of *Kakutani's Fixed Point Theorem.*

Proof of Part (b)

In order to establish this part of the proposition, we shall make use of the characterization of the set of ME given in Claim A.3 above.

It is straightforward to show that when F_i is degenerate, the expression for $G_i(\epsilon; R)$ defined in Claim A.3 reduces to:

$$G_i(\epsilon; R) = -(v_i^* + \bar{v}_j - \epsilon - R)\Psi_j'(\bar{v}_j - \epsilon) - \Psi_j(\bar{v}_j - \epsilon).$$

Hence, if F_j is differentiable then the derivative of G_i with respect to ϵ,

$$G_i'(\epsilon; R) = (v_i^* + \bar{v}_j - \epsilon - R)\Psi_j''(\bar{v}_j - \epsilon) + 2\Psi_j'(\bar{v}_j - \epsilon). \tag{14}$$

It follows from Claim A.1 above that for any $\epsilon_i^* \in \Phi_i(R)$, it must be the case that $0 < \epsilon_i^* < \bar{v}_b + \bar{v}_s - R$. Hence, it follows that $\epsilon_i^* \in \Phi_i(R)$ *only if* $G_i'(\epsilon_i^*; R) = 0$. It thus follows immediately from (14) that $\epsilon_i^* \in \Phi_i(R)$ *only if* ϵ_i^* is a solution to the following equation in ϵ (where $\epsilon \in [0, v_i^* + \bar{v}_j - R]$):

$$v_i^* + \bar{v}_j - \epsilon - R = -\frac{2\Psi_j'(\bar{v}_j - \epsilon)}{\Psi_j''(\bar{v}_j - \epsilon)}. \tag{15}$$

Notice that the left-hand side of (15) is strictly decreasing in ϵ over the closed interval $[0, v_i^* + \bar{v}_j - R]$ — taking a strictly positive value at $\epsilon = 0$, and taking a value of zero when $\epsilon = v_i^* + \bar{v}_j - R$. Over the same closed interval, log-concavity of $1 - F_j$ implies that the right-hand side is increasing from zero to a strictly positive number. Hence, there exists a unique solution in ϵ to (15). It is thus follows that this unique solution constitutes the unique element of $\Phi_i(R)$. Hence, $\widehat{\Gamma}_i$ is a function, and thus, R is a fixed point of $\widehat{\Gamma}_i$ if and only if $R = \widehat{\Gamma}_i(R, \epsilon_i^*(R))$. That is, using (11) — after simplifying it using the assumption that F_i is degenerate —

if and only if R satisfies

$$(16) \quad 2rR = (\lambda_i + \lambda_j)\Psi_j(R - v_i^*) + (\lambda_j - \lambda_i)[(v_i^* + \bar{v}_j - \epsilon_i^*(R) - R)\Psi_j'(\bar{v}_j - \epsilon_i^*(R)) + \Psi_j(\bar{v}_j - \epsilon_i^*(R))].$$

We now establish that there exists a unique solution in R to the above equation. Hence, it follows from Claim A.3 that there exists a unique ME. The set of feasible values of R is the closed interval $[0, v_i^* + \bar{v}_j]$. The left-hand side of (16) increases over this interval — taking the value of zero when $R = 0$, and taking the value of $2r(v_i^* + \bar{v}_j)$ when $R = v_i^* + \bar{v}_j$. Differentiating the right-hand side of (16) with respect to R yields

$$(\lambda_i + \lambda_j)\Psi_j'(R - v_i^*) - (\lambda_j - \lambda_i)\Bigg[(v_i^* + \bar{v}_j - \epsilon_i^*(R) - R)\epsilon_i^{*\prime}(R)\Psi_j''(\bar{v}_j - \epsilon_i^*(R)) + (2\epsilon_i^{*\prime}(R) + 1)\Psi_j'(\bar{v}_j - \epsilon_i^*(R))\Bigg].$$

Then, using (15) this derivative reduces to

$$(\lambda_i + \lambda_j)\Psi_j'(R - v_i^*) - (\lambda_j - \lambda_i)\Psi_j'(\bar{v}_j - \epsilon_i^*(R)),$$

which is less than or equal to zero — since $0 \le \epsilon_i^*(R) \le v_i^* + \bar{v}_j - R$. Finally, note that at $R = v_i^* + \bar{v}_j$, the right-hand side of (16) is zero — since $\epsilon_i^*(v_i^* + \bar{v}_j) = 0$. Hence, since both sides of (16) are continuous in R, there exists a unique solution in R to (16).

Proof of Lemma 3. Under the restriction that $\lambda_i = \lambda_j$, equation 16 reduces to $\hat{r}R = \Psi_j(R - v_i^*)$. Hence, we obtain that

$$0 < \frac{\partial R}{\partial v_i^*} < 1 \quad \text{and} \quad \frac{\partial R}{\partial \hat{r}} < 0.$$

The lemma now follows immediately from this results.

Proof of Proposition 6. Fix $x = \hat{r}, v_i^*$. It follows from (4) that

$$\frac{\partial \tau}{\partial x} = \frac{1}{\gamma^2}\Big[-\gamma F_j'(R_j)\frac{\partial R_j}{\partial x} + [1 - F_j(R_j)]F_j'(R - v_i^*)\frac{\partial (R - v_i^*)}{\partial x}\Big].$$

Equation 15 can be written as:

$$v_i^* + R_j - R + \frac{2\Psi_j'(R_j)}{\Psi_j''(R_j)} = 0.$$

Given the comparative-static results on R derived above in the proof of Lemma 3, and since (by the assumption that $1 - F_j$ is log-concave) $\Psi_j'(R_j)/\Psi_j''(R_j)$ is strictly increasing in R_j, it is straightforward to show that therefore R_j is strictly decreasing in x. Hence, it follows that the derivative of τ with respect to x is in general ambiguous.

Now suppose that $v_i^* + \underline{v}_j > 0$. Once again, fix $x = \hat{r}, v_i^*$. It is straightforward to show from the equation that determines R — namely, $\hat{r}R = \Psi_j(R - v_i^*)$ — that there exists a value of x — call it $\bar{x}$ — such that for any $x > \bar{x}$, $R - v_i^* \le \underline{v}_j$. Thus, for any $x > \bar{x}$, the equilibrium probability of trade $\gamma = 1$. Now note that since R_j is strictly decreasing in x (as shown above), the equilibrium probability of trade at the posted price $1 - F_j(R_j)$ is strictly increasing in x. The appropriate results in Proposition 6 (when $v_i^* + \underline{v}_j > 0$) concerning the relationship between τ and x (for values of $x > \bar{x}$) now follow immediately.

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