THE DIRECT SOLAR RADIATION ANALYTICAL MODEL AND THE SOLAR PLANAR MODULES’ TILT SELECTION TO MAXIMIZE THE ANNUAL INSOLATION OF THEIR SURFACES

Sergey Kivalov  
P.O. Box #0355,  
Brooklyn NY, 11218.  
United States.  
snk2105@gmail.com

Vladimir Evdokimov,  
Dmitry Strebkov,  
Eduard Tver’yanovich  
All-Russian Research Institute for Electrification of Agriculture (VIESH)  
1-Veshniakovski proezd 2,  
Moscow 109456, Russia.  
viesh@dol.ru

ABSTRACT.

This paper is devoted to the development of the direct solar radiation analytical model and its application to determine the tilt of planar modules at an installation phase for maximization of the annual insolation of their surfaces. For evaluation, the Solar Photovoltaic (PV) modules have been chosen.

The model is based on an improved analytical formula for air mass numbers for the sea level. The results of comparison of the ISO Standard Atmosphere with the improved air mass formula and with Kasten’s approximation formula are provided.

The cumulative simulation results of the tilt selections to maximize the annual insolations of the PV modules installed at the places with the different latitudes and with the different transparencies of atmosphere during the year are presented. The comparisons of our simulation results with Christensen and Barker’s tilt estimations shows that their tilt estimation values are mostly located in the estimation ranges predicted by our model for the different transparencies of atmosphere.

The paper also estimates the effect of the diffuse solar radiation on the tilt deviations. The estimation shows that to maximize the amount of the diffuse energy received by the planar module, its tilt also has to be deviated toward zenith.

1. INTRODUCTION

1.1. The tilt optimization

The issue of optimization the tilt of planar collectors and PV modules was primarily addressed by Twidell and Weir [1] and Rauschenbach [2]. They noted that sometimes it is convenient to install solar collectors with the latitude angles, but in general, the tilts have to be counted from the conditions of the sites of installations and from their energy consumption needs during the year or during the specified time interval. Stine and Geyer [3] emphasized that the maximal annual irradiance level occur when the module tilt slightly less than the latitude of the site of installation. Basing on the NREL solar radiation database, Christensen and Barker [4] estimated “Effects of Tilt and Azimuth on Annual Incident Solar Radiation for United States Locations,” and suggested the optimal angles for the positioning of solar collectors.

1.2. The solar radiation models

One of the first references on the modeling the solar radiation level on the slopped surface is Duffie and Beckman’s book “Solar Engineering of Thermal Processes” [5]. Duffie and Beckman developed an isotropic model to estimate the irradiance level ($\mathcal{H}_T$) on the surfaces with the different tilts ($\phi$). The model equation is:

$$
\mathcal{H}_T = \mathcal{H}_hR_h + \mathcal{H}_d\frac{1+\cos\phi}{2}+(\mathcal{H}_h+\mathcal{H}_d)\frac{1-\cos\phi}{2}\rho. \tag{1}
$$
Where: $H_b$ - direct and $H_d$ – diffuse components of the incident solar radiation on the horizontal plane; $R_b$ – correction coefficient for the direct component of solar radiation; $\rho$ - albedo of landscape. The correction coefficient ($R_b$) is the ratio of cosines of two radiation deviation angles. For the south oriented surfaces:

$$
R_b = \frac{\cos \theta_s}{\cos \theta_t} = \frac{\cos(\varphi - s) \cdot \cos \delta \cdot \cos \omega + \sin(\varphi - s) \cdot \sin \delta}{\cos \varphi \cdot \cos \delta \cdot \cos \omega + \sin \varphi \cdot \sin \delta}
$$

(2)

Where: $\theta_s$ - angle from the normal of the slopped surface; $\theta_t$ - zenith angle; $\varphi$ - latitude angle; $\delta$ - angle of solar declination at noon; $\omega$ - hour angle.

However, to use the formula (1) for the estimation of the irradiance level on slopped planes, we have to know measurements of the direct and diffuse components of the incident solar radiation on the horizontal plane.

There is a number of papers which are devoted to the diffuse radiation model development. Liu and Jordan developed an isotropic sky model [6]. Starting from Liu and Jordan’s isotropic model, Klusher developed anisotropic model [7]. This model returns to the isotropic configuration in the absent of direct sunlight. The most complete anisotropic hourly diffuse radiation models for horizontal and slopped surfaces were developed by R. Perez et. al [8, 9]. The models divide sky hemisphere on three zones following the natural anisotropy in atmosphere: “circumsolar brightening due to forward scattering by aerosol, horizon brightening due primarily to multiple Rayleigh scattering” and the rest of the dome. Perez’s model equation for the diffuse irradiance is:

$$
\frac{F_c}{F_{0c}} = \frac{\sin \theta}{\sin \theta_0} \cdot \frac{\sin \alpha}{\sin \alpha_0} \cdot \frac{\cos \gamma}{\cos \gamma_0} \cdot \left(1 - \frac{2 \delta}{\rho_\varphi} \right) \cdot \frac{R_{0h}}{R} \cdot \cos \gamma
$$

(3)

2. THE ANALYTICAL MODEL

2.1. The model description

To analytically estimate amount of the direct solar radiation on slopped planes, we started with the formula for air mass numbers (4) and the representation of the radiation deviation angle ($\theta_s$) from the formula (2) [5]:

$$
\cos \theta_t = \cos(\varphi - s) \cdot \cos \delta \cdot \cos \omega + \sin(\varphi - s) \cdot \sin \delta.
$$

The resulting expression for the irradiance level ($I_s W/m^2$) on the plain slopped with the angle $s$ due to the direct component of solar radiation ($I_\perp$) is:

$$
I_s = I_\perp \cdot \cos \theta_s = I_0 \cdot p^{M_s(\theta_s)} \cdot \cos \theta_s.
$$

(8)

Using the standard relationship between the time and the hour angle ($\omega$) (15° for 1 hour):

$$
T_{\text{hour}} = \frac{12}{\pi} \cdot \omega.
$$

We can derive the expression for the amount of incident solar energy ($E_{\text{day}} Wh/m^2$) (direct component) received by the slopped plain during the day:

$$
E_{\text{day}} = I_0 \cdot p^{M_s(\theta_s)} \cdot \cos \theta_s \cdot 24h.
$$

The simplest formula for the calculation of the relative air mass numbers which neglects the curvature of atmosphere for the different solar altitude angles $\gamma$ is [12]:

$$
M_a = \frac{1}{\sin \gamma} \quad \gamma \geq \frac{\pi}{3}.
$$

(5)

Following to Link’s approach [12] to take in account both the curvature of atmosphere and atmospheric refraction, Kasten developed improved relative air mass formula [13]:

$$
M_a = \frac{1}{\sin \gamma + a(\gamma + b)\gamma}
$$

(7)

2. Outside the Earth atmosphere, the irradiance level on the surface normal to the solar beam is equal $I_0 = 1367 W/m^2$ [10]. To calculate the direct component of solar radiation on the Earth’s surface normal to the solar beam ($I_\perp$), we can use an expression for relative air mass numbers ($M_a$) and the atmosphere transparency factor ($p$) related to the one air mass number (path through atmosphere in the direction to zenith). The correspondent formula is [11]:

$$
I_\perp = I_0 \cdot p^{M_s}
$$

(4)
\[ E_{\text{day}} (\varphi, \delta, s) = I_0 \cdot \frac{12}{\pi} \int_{-\omega_0}^{\omega_0} p(M_s(\varphi_0)) \cdot \cos \vartheta_1 d\vartheta \] (9)

Where: \( \omega_0 \) - the maximal hour angle at which the unit is still able to capture the direct solar radiation. From [5]:

\[ \omega_0 = \min \arccos \left\{ -\tan \delta \cdot \tan \varphi, -\tan \delta \cdot \tan (\varphi - s) \right\} \] (10)

Using the formula for the solar declination angle at noon \( \delta(n) \) as a function of a day number \( n \) in a year [5]:

\[ \delta(n) = 23.45 \cdot \sin \left( 360 \cdot \frac{284 + n}{365} \right) \]

We can derive the final expression of the analytical model to evaluate the amount of incident solar energy (direct component) for the period \( [\delta_1, \delta_2] \), where: \( \delta(n_1) = \delta_1 \) and \( \delta(n_2) = \delta_2 \):

\[ E_{[\delta_1, \delta_2]} (\varphi, s) = I_0 \cdot \frac{12}{\pi} \cdot \sum_{n=n_1}^{n_2} \int p(\delta(n), \omega) M_s(\varphi_0) \cdot \cos \vartheta_1 d\vartheta \] (11)

2.2. Choice of parameters

Before starting the computer simulation for the model (11), we defined two functions located under the integral sign.

\( p(\delta(n), \omega) \) - describes the transparency of atmosphere as a function of the declination and hour angles. In general, this is a complicated sky transparency distribution which could be modeled with using the probability approach described in R. Perez et al. papers [16, 17]. However, in the homogeneous atmospheric conditions, we can assume \( p=\text{const} \) that can be calculated from the formula (4) at the noon:

\[ p = \left( I_{\perp}/I_0 \right)^{M_s} \] (12)

In this case, we can also take the follow constant values for \( p \) [5]: \( p_{\text{average}}=0.8; \ p_{\text{max}}=0.9; \ p_{\text{mix}}=0.6. \)

To describe the air mass numbers as a function of the zenith angle - \( M_s(\vartheta_z) \), we have chosen the previously developed formula [18] which was improved to fit data from the ISO Standard Atmosphere:

\[ M_s(\vartheta_z) = \sqrt{684^2 \cdot \cos^2 \vartheta_z + 37^2} - 684 \cdot \cos \vartheta_z \] (13)

Comparison of the ISO Standard Atmosphere data [14, 15] with the formula (13) and with Kasten’s formula (7) shows that the root mean square error (RMSE) for the formula (13) (RMSE=0.0092) is smaller than the RMSE for Kasten’s formula (7) (RMSE=0.0103).

3. SIMULATION AND VALIDATION

3.1. The simulation results

The computer simulation of the model (11)-(12)-(13) was conducted for the different latitudes and the different shapes of the atmospheric transparency during the year.

The resulting graphs of the math simulation are presented at the Figure 1 \( p_{\text{winter}}=0.7 \) and at the Figure 2 \( p_{\text{winter}}=0.6, p_{\text{summer}}=0.8 \). All values on the vertical axes are interpreted as fractions of the amount of incident solar energy collected by the PV module installed with the tilt deviated from the latitude toward the zenith to the amount of incident solar energy collected by the similar PV module installed with the tilt equals to the latitude. The different lines represent the fraction values for the different latitudes \( \pm 40^\circ, \pm 45^\circ, \pm 50^\circ, \pm 55^\circ, \pm 60^\circ \) of the PV modules installation sites.

Fig. 1: The relative amount of energy collected by the PV modules installed with the tilt deviated toward zenith from the latitude for \( p_{\text{winter}}=p_{\text{summer}}=0.7 \)
Fig. 2: The relative amount of energy collected by the PV modules installed with the tilt deviated toward zenith from the latitude for $\varphi_{\text{winter}}=0.6$, $\varphi_{\text{summer}}=0.8$

The cumulative simulation results of maximization of the incident energy annually collected by the PV modules installed at the places with the different latitudes are presented in the Table 1. $(\varphi - s)$ - is a deviation angle of the PV module installation toward zenith from the latitude. $\Delta E$ - is an additional amount of energy collected by the installed module.

<table>
<thead>
<tr>
<th>TABLE 1: CUMULATIVE MAXIMIZATION RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>latitude</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>40°</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>45°</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>50°</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>55°</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>60°</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

3.2. Justification of modeling assumptions

The model (11)-(12)-(13) only estimates the amount of energy delivered by the direct component of the solar radiation. To estimate the effect of the diffuse solar radiation, we can use Perez’s anisotropic models [8, 9]. From Duffie and Beckman’s work, we can emphasize that the ratio of the average daily amount of the diffuse radiation ($E_{d}^{\text{day}}$) to the amount of the total daily radiation ($E_{b}^{\text{day}}+E_{d}^{\text{day}}$) on the horizontal surface is only the function of the transparency of atmosphere ($p$). For $p=0.8$, $E_{d}^{\text{day}} \approx 0.18 \cdot (E_{b}^{\text{day}}+E_{d}^{\text{day}})$, and for $p=0.6$, $E_{d}^{\text{day}} \approx 0.36 \cdot (E_{b}^{\text{day}}+E_{d}^{\text{day}})$. And, the amount of total solar radiation in “winter” for Madison, WI is about 3 times less than the same amount in “summer.” So, in the absolute values $E_{d}^{\text{winter}} \approx \frac{2}{3} \cdot E_{d}^{\text{summer}}$.

From Perez’s anisotropic model, we can estimate the sky clearness ($\varepsilon$) and anisotropy coefficients $F_1$ and $F_2$: for $p=0.8; \varepsilon \approx 12, F_1=0.2, F_2=0.2$; for $p=0.6; \varepsilon \approx 5.5, F_1=0.4, F_2=0.1$. Paying attention to the model dependency on the zenith and incident radiation angles, we can estimate that the circumsolar region gives about 30% of all diffuse radiation in “summer” time and about 40-45% in “winter” time. And, we can also estimate that the dome region gives about 50% of diffuse radiation during the year. Because we don’t not count “horizon brightening” at “summer” and because its “winter” coefficient is really small, we can neglect the effect of “horizon brightening” on our model estimations.

Finally, combining all these estimations together, we can conclude that in the absolute values, both the circumsolar and the dome regions bring more energy in “summer” than in “winter”. This means that to maximize the amount of the diffuse energy received by the PV module, we still have to deviate its tilt toward zenith from the latitude value. This deviation is in the agreement with our original model concept.

3.3. The model validation

To compare the tilt deviation results of our direct radiation model with the real data, we have chosen Christensen and Barker’s model results [4] based on the NREL solar radiation database as a major reference. The results of comparison between these models are presented in the Table 2 for some different sites in the USA.

<table>
<thead>
<tr>
<th>TABLE 2: TILT DEVIATION COMPARISON</th>
</tr>
</thead>
<tbody>
<tr>
<td>latitude</td>
</tr>
<tr>
<td>40°45'</td>
</tr>
<tr>
<td>39°07'</td>
</tr>
<tr>
<td>40°10'</td>
</tr>
</tbody>
</table>
TABLE 2 (CONTINUE):

<table>
<thead>
<tr>
<th>latitude</th>
<th>city / place</th>
<th>Our results</th>
<th>[4]result</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°39'</td>
<td>Eagle L. Calif.</td>
<td>7.6° / 10°-12°</td>
<td>≈ 10°</td>
</tr>
<tr>
<td>45°22'</td>
<td>Lincoln, Maine</td>
<td>8.8° / 12.5°-15°</td>
<td>≈ 9°</td>
</tr>
<tr>
<td>45°28'</td>
<td>Aberdeen, S. Dak.</td>
<td>8.8° / 12.5°-15°</td>
<td>≈ 7°</td>
</tr>
<tr>
<td>45°34'</td>
<td>Cambridge, Minn.</td>
<td>8.8° / 12.5°-15°</td>
<td>≈ 9°</td>
</tr>
<tr>
<td>45°32'</td>
<td>Portland, OR</td>
<td>8.8° / 12.5°-15°</td>
<td>≈ 12°</td>
</tr>
<tr>
<td>51°32'</td>
<td>Amchitka L.</td>
<td>10.2° / 15°-17°</td>
<td>≈ 16°</td>
</tr>
<tr>
<td>60°24'</td>
<td>St. Matthew I.</td>
<td>15° / 20°</td>
<td>≈ 16°</td>
</tr>
<tr>
<td>60°48'</td>
<td>Bethel, Alaska</td>
<td>15° / 20°</td>
<td>≈ 16°</td>
</tr>
</tbody>
</table>

The comparison shows that the energy maximization tilt predicted by [4] is mostly located in the tilt range predicted by our model for the different transparencies of atmosphere. The real values are never bigger than the upper bound values predicted by our model, but in some cases they can be a little smaller than the lower bound values of our model. This can be explained by the effect of the diffuse radiation data on the model [4] calculations.

4. CONCLUSION

We can conclude that the deviation of the installation tilt of the planar collectors and the PV modules toward zenith maximizes the annual amount of incident solar energy on their surfaces. For the specific locations, the difference in the transparency of atmosphere during the year plays an important role and should be considered for the selection of the optimal installation tilt for the modules.

We can also confirm that there is an average agreement between the maximization results of our direct radiation model and Christensen and Barker’s results based on the NREL solar radiation database. However, the exact matching doesn’t occur. This might happen because our model doesn’t count both the effect of the diffuse radiation and the effect of the real transparency of atmosphere in the specific locations. It would be useful to conduct the extended research involving the combined solar radiation model which will count effects of both the direct and the diffuse radiation on the tilt changing.

In the regions with latitudes 50° - 60°, it is possible to increase the annual insolation of the planar PV modules on 4-6% by decreasing their tilts on 15°-20° toward zenith from the latitude angle of the site of installation.

5. NOMENCLATURE

$I_0$ - extra terrestrial irradiation level, W/m²
$I_\perp$ - irradiation of the surface normal to the solar beam, W/m²
$I_s$ - irradiation of the slopped surface, W/m²
$M_\alpha$ - relative air mass numbers, -
$p$ - atmosphere transparency factor, -
$\varphi$ - latitude angle, °
$\delta$ - angle of solar declination at noon, °
$\omega$ - solar hour angle, °
$\gamma$ - solar altitude angles, °
$s$ - tilt of the slopped surface, °
$\theta_s$ - beam radiation angle from the normal of the slopped surface, °
$\theta_z$ - zenith angle from the normal of the slopped surface, °
$E_{day}$ - amount of incident solar energy (direct component) received by a slopped plain during the day, Wh/m²
$E_{[\delta, \delta_z]}$ - amount of incident solar energy (direct component) for the period $[\delta, \delta_z]$, Wh/m²
$\Delta E$ - an additional amount of energy collected by the installed module, %

6. ACKNOWLEDGEMENTS

I would like to thanks Prof. Richard Perez for his attention to this work and advices, very useful consultations about his diffuse radiation model, and for the provided references.

7. REFERENCES

(10) WMO (1982)
(14) ISO Standard Atmosphere (1972)