CSI 604 – Spring 2016
Greedy Algorithms for Interval Scheduling

I. Finding a Largest Compatible Subset of Intervals:

In this problem, we are given \( n \) requests (intervals). For each request \( i, 1 \leq i \leq n \), the start time and finish time are denoted by \( s(i) \) and \( f(i) \) respectively. Two requests are compatible if they don’t overlap. The greedy algorithms below finds a maximum set of compatible requests.

Outline of the Greedy Algorithm:

1. Let \( R \) denote the set of requests (intervals). Set \( A = \emptyset \). (At the end, \( A \) contains the solution.)
2. while \( R \neq \emptyset \) do
   a. Choose a request \( i \in R \) with the smallest finish time.
   b. Add request \( i \) to \( A \).
   c. Delete from \( R \) all requests that conflict with \( i \).
3. Output \( A \).

An \( O(n \log n) \) implementation of the above Greedy Algorithm:

1. Sort the requests in increasing order of their finish times. Rename the requests if necessary so that the sorted order of requests is \( \langle 1, 2, \ldots, n \rangle \).
2. Let \( A = \{1\} \). (\( A \) will contain the solution at the end of the algorithm.)
3. Let \( j = 1 \). (Variable \( j \) represents the index of the last request chosen in \( A \) so that \( f(j) \) will be the maximum finish time among all the requests in \( A \).)
4. for \( i = 2 \) to \( n \) do
   if \( s(i) \geq f(j) \) then
   a. Add \( i \) to \( A \). (Request \( i \) is compatible with all the requests in \( A \).)
   b. Let \( j = i \). (Now, \( f(i) \) is the maximum finish time among the requests in \( A \).)
5. Output \( A \).

(over)
II. Partitioning All Intervals into a Minimum Number of Compatible Subsets:

Let $d$ denote the depth of the given collection of intervals. Recall that the number of compatible sets into which the collection can be partitioned is at least $d$. The following greedy algorithm produces a partition with exactly $d$ subsets.

The algorithm assigns a label from the set $\{1, 2, \ldots, d\}$ to each interval such that for each label $\ell$, $1 \leq \ell \leq d$, all the intervals assigned the label $\ell$ are compatible.

Outline of the Greedy Algorithm for Scheduling All Intervals:

1. Sort the requests in increasing order of their start times. Rename the requests if necessary so that the sorted order of requests is $\langle 1, 2, \ldots, n \rangle$. (The requests will be assigned labels in that order.)

2. for $j = 2$ to $n$ do

   for each interval $r$ that precedes $j$ and is incompatible with $j$ do

      Exclude the label of $r$ from consideration for $j$.

   if any of the labels in $\{1, 2, \ldots, d\}$ has not been excluded

      then Assign any non-excluded label to $j$.

   else Leave $j$ unlabeled. /* This will never happen. */

3. For each request, output its label.

Exercise: Develop an $O(n \log n + dn)$ implementation of the above algorithm.