Section 15: Comparing Means (population stdev’s known)
1) IQ’s are normally distributed with mean 100 and standard deviation 16. Two large groups of people are studied, population X and population Y. 100 people picked from population X have a mean IQ of 105, and 108 people picked from population Y have a mean IQ of 110. (a) Do we have evidence at the 10%, 5%, 1% levels that these populations have different average IQ’s?

\[ \mu_1 = \text{mean IQ of population X} = ? \]
\[ \mu_2 = \text{mean IQ of population Y} = ? \]
\[ \sigma_1 = \text{standard deviation of IQ’s of population X} = 16 \]
\[ \sigma_2 = \text{standard deviation of IQ’s of population Y} = 16 \]
\[ x_1 = 105 \quad x_2 = 110 \quad n_1 = 100 \quad n_2 = 108 \]
\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 \neq \mu_2 \]

\[ SD = \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} = \sqrt{\frac{16^2}{100} + \frac{16^2}{108}} = 2.2205 \]

\[ z = \frac{x_1 - x_2}{SD} = \frac{105 - 110}{2.2204} \quad z = -2.25 \]

The single tail is 0.0122

\[ p - \text{value} = 0.0122 \times 2 = 0.0244 (\text{both tails}) \]

0.0244 < 0.10; evidence of \( H_a \); reject \( H_0 \)

0.0244 < 0.05; evidence of \( H_a \); reject \( H_0 \)

0.0244 < 0.01; no evidence of \( H_a \); do not reject \( H_0 \)

(b) Do we have evidence at the 10%, 5%, 1% levels that these population Y has a higher average IQ’s?

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 < \mu_2 \]

The single tail is 0.0122

0.0122 < 0.10; evidence of \( H_a \); reject \( H_0 \)

0.0122 < 0.05; evidence of \( H_a \); reject \( H_0 \)

0.0122 < 0.01; no evidence of \( H_a \); do not reject \( H_0 \)

(b) Do we have evidence at the 10%, 5%, 1% levels that these population Y has a higher average IQ’s?

2) 280 men are found to have an average pulse rate of 73 (beats per minute), and 240 women are found to have an average pulse rate of 72 (beats per minute). It is known that the standard deviation of men's pulse rates is 5.4, and the standard deviation of women's pulse rates is 7.7.
(a) Do we have evidence at the 5%, 1%, 0.1% levels that men have higher pulse rates?

\[ \mu_1 = \text{mean pulse of all men} \]
\[ \mu_2 = \text{mean pulse of all women} \]

\[ \sigma_1 = \text{standard deviation of men’s pulse rates} = 5.4 \]
\[ \sigma_2 = \text{standard deviation of women’s pulse rates} = 7.7 \]

\[ \bar{x}_1 = 73 \quad \bar{x}_2 = 72 \quad n_1 = 280 \quad n_2 = 240 \]

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 > \mu_2 \]

\[ SD = \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} = \sqrt{\frac{5.4^2}{280} + \frac{7.7^2}{240}} = 0.5926 \]

\[ z = \frac{\bar{x}_1 - \bar{x}_2}{SD} = \frac{73 - 72}{0.5926} \quad z = 1.69 \]

The single tail is 0.0455

0.0455 < 0.05; evidence of \( H_a \); reject \( H_0 \)

0.0455 < 0.01; no evidence of \( H_a \); do not reject \( H_0 \)

0.0455 < 0.001; no evidence of \( H_a \); do not reject \( H_0 \)

(b) Do we have evidence at the 5%, 1%, 0.1% levels that the genders have different pulse rates?

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 \neq \mu_2 \]

\[ p - \text{value} = 0.0455 \times 2 = 0.0910 \text{(both tails)} \]

0.0910 < 0.05; no evidence of \( H_a \); do not reject \( H_0 \)

0.0910 < 0.01; no evidence of \( H_a \); do not reject \( H_0 \)

0.0910 < 0.001; no evidence of \( H_a \); do not reject \( H_0 \)

3) In 1996, 7537 people who took a famous exam got an average score of 150. In 2000, 15,109 people got an average score of 147. (a) Assuming from past experience that the standard deviation of the scores is 1.22, do we have evidence that scores have gone down?

\[ \mu_1 = \text{mean score in 1996} = ? \]
\[ \mu_2 = \text{mean score in 2000} = ? \]

\[ \sigma_1 = \text{standard deviation scores in 1996} = 1.22 \]
\[ \sigma_2 = \text{standard deviation of scores in 2000} = 1.22 \]
\[
\begin{align*}
\bar{x}_1 &= 150 \quad \bar{x}_2 = 147 \quad n_1 = 7537 \quad n_2 = 15,109 \\
H_0: \mu_1 &= \mu_2 \\
H_a: \mu_1 &> \mu_2
\end{align*}
\]

\[
SD = \sqrt{\left(\frac{\sigma_1}{n_1}\right)^2 + \left(\frac{\sigma_2}{n_2}\right)^2} = \sqrt{\frac{1.22^2}{7537} + \frac{1.22^2}{15,109}} = 0.0172
\]

\[
z = \frac{\bar{x}_1 - \bar{x}_2}{SD} = \frac{150 - 147}{0.0172} = 174.37
\]

The single tail is seriously close to 0 indeed

at any serious level: evidence of \(H_a\); reject \(H_0\)

(b) Do we have evidence that scores have changed?

\[
\begin{align*}
H_0: \mu_1 &= \mu_2 \\
H_a: \mu_1 &\neq \mu_2
\end{align*}
\]

2 * basically nothing is still basically nothing

at any serious level: evidence of \(H_a\); reject \(H_0\)

4) Advertisements on tv featuring sexual material, watched by 106 people resulted in an average of 2.72 products remembered the next day, and advertisements on tv featuring neutral material, watched by 103 people resulted in an average of 4.65 products remembered the next day. Suppose it’s known that the standard deviation of remembered ads featuring sexual material is 1.85 products and that the standard deviation of ads featuring neutral material is 1.62 products.

Do we have evidence that the type of advertisement makes a difference in whether the product is remembered?

\[
\begin{align*}
\mu_1 &= \text{mean # of products remembered (sexual)} \Rightarrow \\
\mu_2 &= \text{mean # of products remembered (neutral)} \Rightarrow \\
\sigma_1 &= \text{standard deviation of # of products remembered (sexual) = 1.85} \\
\sigma_2 &= \text{standard deviation of # of products remembered (neutral) = 1.62}
\end{align*}
\]

\[
\begin{align*}
\bar{x}_1 &= 2.72 \quad \bar{x}_2 = 4.65 \quad n_1 = 106 \quad n_2 = 103 \\
H_0: \mu_1 &= \mu_2 \\
H_a: \mu_1 &\neq \mu_2
\end{align*}
\]

\[
SD = \sqrt{\left(\frac{\sigma_1}{n_1}\right)^2 + \left(\frac{\sigma_2}{n_2}\right)^2} = \sqrt{\frac{1.85^2}{106} + \frac{1.62^2}{103}} = 0.2403
\]

\[
z = \frac{\bar{x}_1 - \bar{x}_2}{SD} = \frac{2.72 - 4.65}{0.2403} = -8.03
\]

The single tail is very close to 0

at any reasonable level: evidence of \(H_a\); reject \(H_0\)
Do we have evidence that “neutral” ads work better?

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 < \mu_2 \]

2 * approximately 0 is still approximately 0

at any reasonable level: evidence of \( H_a \); reject \( H_0 \)

5) We wish to know whether the population of ancient Egypt changed considerably in body structure during its existence. 126 skulls found from around 4000 B.C. had an average breadth of 70mm, and 129 skulls found from 200 B.C. had an average breadth of 84mm. Suppose it is known that the standard deviation of skull breadth in 4000 B.C. was 7.3mm and was 6.7mm in 200 B.C. Assume that the standard deviations are those for their respective populations. Do we have evidence that skull breadth changed?

\[ \mu_1 = \text{mean skull size in 4000 B.C.} = ? \]
\[ \mu_2 = \text{mean skull size in 200 B.C.} = ? \]
\[ \sigma_1 = \text{standard deviation of skull size in 4000 B.C.} = 7.3 \]
\[ \sigma_2 = \text{standard deviation of skull size in 200 B.C.} = 6.7 \]
\[ \bar{x}_1 = 70 \quad \bar{x}_2 = 84 \quad n_1 = 126 \quad n_2 = 129 \]

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 \neq \mu_2 \]

\[ SD = \sqrt{ \frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2} } = \sqrt{ \frac{7.3^2}{126} + \frac{6.7^2}{129} } = 0.8780 \]

\[ z = \frac{\bar{x}_1 - \bar{x}_2}{SD} = \frac{70 - 84}{0.8780} \quad z = -15.94 \]

The single tail is very close to 0

2 * approximately 0 is still approximately 0

at any reasonable level: evidence of \( H_a \); reject \( H_0 \)

Do we have evidence that it increased?

\[ H_0: \mu_1 = \mu_2 \]
\[ H_a: \mu_1 < \mu_2 \]

The single tail is very close to 0

at any reasonable level: evidence of \( H_a \); reject \( H_0 \)

6) A bank has two plans for a Christmas Club account. Plan A offers a higher interest rate, and plan B offers a free toaster. 250 customers of plan A saved an average of $1787 (standard deviation $125), and 200 customers of plan B saved an average of $1818 (standard deviation $140). Assume that the standard deviations are those for their respective populations. Do we have evidence at the 5%, 1% levels that these means are different?
\( \mu_1 = \text{mean savings of all plan A customers} = \) ?

\( \mu_2 = \text{mean savings of all plan b customers} = ? \)

\( \sigma_1 = \text{standard deviation of all plan A customers} = 125 \)

\( \sigma_2 = \text{standard deviation of all plan B customers} = 140 \)

\( \bar{x}_1 = 1787 \quad \bar{x}_2 = 1818 \quad n_1 = 250 \quad n_2 = 200 \)

\[ SD = \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} = \sqrt{\frac{125^2}{250} + \frac{140^2}{200}} = 12.6689 \]

\[ z = \frac{\bar{x}_1 - \bar{x}_2}{SD} = \frac{1787 - 1818}{12.6689} \quad z = -2.45 \]

The single tail is 0.0071

\( 2 \times 0.0071 = 0.0142 \) (p - value)

0.0142 < 0.05 ; evidence of \( H_a \); reject \( H_0 \)

0.0142 < 0.01 ; no evidence of \( H_a \); do not reject \( H_0 \)

That plan B’s is higher?

\[ \begin{align*}
H_0: \mu_1 & = \mu_2 \\
H_a: \mu_1 & < \mu_2 
\end{align*} \]

The single tail is 0.0071

\( 0.0071 < 0.05 \); evidence of \( H_a \); reject \( H_0 \)

\( 0.0071 < 0.01 \); evidence of \( H_a \); reject \( H_0 \)

7) 80 Big Macs had an average calorie count of 593, and 75 Whoppers had an average calorie count of 625. Suppose it is known that the standard deviations of Big Mac and Whopper calorie count are 12 and 15 respectively. Do we have evidence that Whoppers have, on average, more calories?

\( \mu_1 = \text{mean calories of all Big Macs} = ? \)

\( \mu_2 = \text{mean calories of all Whoppers} = ? \)

\( \sigma_1 = \text{standard deviation of calories of all Big Macs} = 12 \)

\( \sigma_2 = \text{standard deviation of calories of all Whoppers} = 15 \)

\( \bar{x}_1 = 593 \quad \bar{x}_2 = 625 \quad n_1 = 80 \quad n_2 = 75 \)

\[ \begin{align*}
H_0: \mu_1 & = \mu_2 \\
H_a: \mu_1 & < \mu_2 
\end{align*} \]
\[SD = \sqrt{\frac{(\sigma_1)^2}{n_1} + \frac{(\sigma_2)^2}{n_2}} = \sqrt{\frac{12}{80} + \frac{15^2}{75}} = 2.1909\]

\[z = \frac{\bar{x}_1 - \bar{x}_2}{SD} = \frac{593 - 625}{2.1909} = -14.61\]

The single tail is very close to 0 at any reasonable level: evidence of \(H_a\); reject \(H_0\)