(1) (a) We roll a die 80 times and get a sum of 268. Do we have evidence at the 5% level that the die is unfair (we will call it unfair if the theoretical mean isn’t 3.5)? Do we have evidence at the 5% level that it’s weighted in favor of lower numbers? (Assume the standard deviation for the die is 1.708 in any case.)

\[ \mu = \text{average of all possible rolls} \]
\[ H_0: \mu = 3.5 \]
\[ H_a: \mu \neq 3.5 \]

\[ \bar{x} = \frac{268}{80} = 3.35 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as far from the center):

Left tail is:
\[ \bar{x} \leq 3.35 \]
\[ Z \leq \frac{3.35 - 3.5}{\frac{1.708}{\sqrt{80}}} \]
\[ Z \leq -0.79 \]

\[ p - \text{value} = 0.2148 \times 2 = 0.4296 \text{ (both tails)} \]
\[ 0.4296 \not< 0.05 \text{ no evidence of } H_a; \text{ do not reject } H_0 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as small):

\[ \bar{x} \leq 3.35 \]
\[ Z \leq \frac{3.35 - 3.5}{\frac{1.708}{\sqrt{80}}} \]
\[ Z \leq -0.79 \]

\[ p - \text{value} = 0.2148 \text{ (left tail)} \]
\[ 0.2148 \not< 0.05 \text{ no evidence of } H_a; \text{ do not reject } H_0 \]

(b) Same question, when the die sums to 2680 after 800 rolls.

\[ \mu = \text{average of all possible rolls} \]
\[ H_0: \mu = 3.5 \]
\[ H_a: \mu < 3.5 \]

\[ \bar{x} = \frac{2680}{800} = 3.35 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as far from the center):

Left tail is:
\[ \bar{x} \leq 3.35 \]
\[ Z \leq \frac{3.35 - 3.5}{\frac{1.708}{\sqrt{800}}} \]
\[ Z \leq -2.48 \]

\[ p - \text{value} = 0.0066 \times 2 = 0.0132 \text{ (both tails)} \]
\[ 0.0132 < 0.05 \text{ evidence of } H_a; \text{ reject } H_0 \]
\[ \bar{x} = \frac{2680}{800} = 3.35 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as small):

\[ \bar{x} \leq 3.35 \quad \quad \frac{Z}{1.708} \leq \frac{3.35 - 3.5}{\sqrt{800}} \leq -2.48 \]

\[ p - value = 0.0066 \text{ (left tail)} \]
\[ 0.0066 < 0.05 \quad \text{evidence of } H_a; \text{ reject } H_0 \]

(2) A famous astronomer claims that the average number of continents on "type M" planets is 5.2 We inspect such 70 planets and find a mean of 4.8 continents. Assume the standard deviation for number of continents is known to be 1.2 Do we have evidence at the 1\% level that he’s overestimating?

\[ \mu = \text{average #continents on all such planets} \]
\[ H_0: \mu = 5.2 \]
\[ H_a: \mu < 5.2 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as small):

\[ \bar{x} \leq 4.8 \quad \quad \frac{Z}{1.2} \leq \frac{4.8 - 5.2}{\sqrt{70}} \leq -2.79 \]

\[ p - value = 0.0026 \text{ (left tail)} \]
\[ 0.0026 < 0.01 \quad \text{evidence of } H_a; \text{ reject } H_0 \]

Do we have evidence at the 0.5\% level that he’s wrong?

\[ \mu = \text{average #continents on all such planets} \]
\[ H_0: \mu = 5.2 \]
\[ H_a: \mu \neq 5.2 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as far from the center):

\[ \bar{x} \leq 4.8 \quad \quad \frac{Z}{1.2} \leq \frac{4.8 - 5.2}{\sqrt{70}} \leq -2.79 \]

\[ p - value = 0.0026 \times 2 = 0.0052 \text{ (both tails)} \]
\[ 0.0052 < 0.005 \quad \text{no evidence of } H_a; \text{ do not reject } H_0 \]

(3) Picking 120 pregnant women at random, we find the mean pregnancy length is 264 days. Assuming the standard deviation for pregnancy length is 16 days, do we have evidence at the 10\% level that the mean length of all pregnancies is under 266 days?

\[ \mu = \text{average length of all pregnancies} \]
\[ H_0: \mu = 266 \]
\[ H_a: \mu < 266 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as small):

\[ \bar{x} \leq 264 \quad \quad \frac{Z}{16} \leq \frac{264 - 266}{\sqrt{120}} \leq -1.37 \]

\[ p - value = 0.0853 \text{ (left tail)} \]
\[ 0.0853 < 0.10 \quad \text{evidence of } H_a; \text{ reject } H_0 \]
That it’s not 266 days?

\[ \mu = \text{average length of all pregnancies} \]

\[ H_0: \mu = 266 \]
\[ H_a: \mu \neq 266 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as far from the center):

Left tail is:

\[
\bar{x} \leq 264 \quad Z \leq \frac{264 - 266}{\frac{16}{\sqrt{120}}} \quad Z \leq -1.37
\]

\[ p - \text{value} = 0.0853 \times 2 = 0.1706 \text{ (both tails)} \]

\[ 0.1706 \ll 0.10 \quad \text{no evidence of } H_a; \ \text{do not reject } H_0 \]

(4) Whenever someone calls up to sell something, I ask them their I.Q. After 85 responses, I get an average I.Q of 95. Do I have evidence at the 1% level that the mean I.Q. of all telephone salespeople is below 100, the official average for everyone? Assume the standard deviation for salespeople is the same as the overall one, which is 16.

\[ \mu = \text{average I.Q of all telephone salespeople} \]
\[ H_0: \mu = 100 \]
\[ H_a: \mu < 100 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as small):

\[
\bar{x} \leq 95 \quad Z \leq \frac{95 - 100}{\frac{16}{\sqrt{85}}} \quad Z \leq -2.88
\]

\[ p - \text{value} = 0.0020 \text{ (left tail)} \]

\[ 0.0020 < 0.01 \quad \text{evidence of } H_a; \ \text{reject } H_0 \]

(5) Someone claims the accident rate at a hazardous intersection is 3 accidents per week. Using the data in (12.5) above, do I have evidence that it’s higher?

\[ \mu = \text{average I.Q of all telephone salespeople} \]
\[ H_0: \mu = 3 \]
\[ H_a: \mu > 3 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as large):

\[
\bar{x} \geq 3.1 \quad Z \geq \frac{3.1 - 3}{\frac{1.4}{\sqrt{85}}} \quad Z \geq 0.52
\]

\[ p - \text{value} = 0.3015 \text{ (right tail)} \]

\[ 0.30150 \ll 0.05 \quad \text{no evidence of } H_a; \ \text{don’t reject } H_0 \]

A level is not specified, but 5% is the most common. Something that happens 30% of the time through chance is not “suspiciously unlikely” by any reasonable standard.

(6) An engineer has designed a new type of light bulb. The mean lifetime for the old type of light bulb is 1200 hours. A test of 400 bulbs of the new type produces a mean of 1206 hours. Assume that the standard deviation for any type of light bulb is 80 hours. Do we have evidence that the new type is bulb is better (lasts longer) than the old type (a) at the 5% level

\[ \mu = \text{average lifetime of all new type bulbs} \]
\[ H_0: \mu = 1200 \]
$H_a: \mu > 1200$

Assuming $H_0$, find the probability of a sample at least as extreme as ours (as large):

$$\bar{x} \geq 1206 \quad Z \geq \frac{1206 - 1200}{\frac{80}{\sqrt{400}}} \quad Z \geq 1.50$$

$p$ - value = 0.0668 (right tail)

0.0668 < 0.05 no evidence of $H_a$; don't reject $H_0$

(b) at the 10% level?

0.0668 < 0.10 evidence of $H_a$; reject $H_0$

(7) Statistics can help decide the authorship of literary works. Sonnets by a certain Elizabethan poet are known to contain an average of 6.9 new words (words not used in the poet's other works). The standard deviation of the number of new words is $\sigma = 2.7$. Now a manuscript with 5 new sonnets is discovered, and scholars are debating whether it is the poet's work. The new sonnets contain an average of 11.2 words not used in the poet's known works. We expect poems by another author to contain more new words. Do we have evidence that sonnets are not his work?

$$\mu = \text{average #new words in all hypothetical sonnets by our unknown poet}$$

$H_0: \mu = 6.9$

$H_a: \mu > 6.9$

Assuming $H_0$, find the probability of a sample at least as extreme as ours (as large):

$$\bar{x} \geq 11.2 \quad Z \geq \frac{11.2 - 6.9}{\frac{2.7}{\sqrt{5}}} \quad Z \geq 3.56$$

$p$ - value $\approx 0$ (right tail) (less than 0.0002 anyway)

$p$ - value $< 0.0002 < 0.05$ evidence of $H_a$; reject $H_0$

Again, a level is not specified, but something "off the chart unlikely" is suspiciously unlikely at any reasonable level.

(8) The mean yield of corn in the USA is about 120 bushels per acre. A survey of 40 farmers this year gives a sample mean yield of 123.8 bushels per acre. Assuming that the farmers surveyed are a SRS from the population of all commercial corn growers and that the standard deviation for this population is $\sigma = 10$ bushels per acre, do we have evidence at the 5% level that the national mean this year is not 120 bushels per acre?

$$\mu = \text{average corn yield for all farmers this year}$$

$H_0: \mu = 120$

$H_a: \mu \neq 120$

Assuming $H_0$, find the probability of a sample at least as extreme as ours (as far from the center):

Right tail is:

$$\bar{x} \geq 123.8 \quad Z \geq \frac{123.8 - 120}{\frac{10}{\sqrt{40}}} \quad Z \geq 2.40$$

$p$ - value $= 0.0082 * 2 = 0.0164$ (both tails)

0.0164 < 0.05 evidence of $H_a$; reject $H_0$

At the 1% level?

0.0164 < 0.05 no evidence of $H_a$; don't reject $H_0$

(9) In the past, the mean score of seniors at South High on the ACT has been 20. This year a special prep course is offered, and all 53 seniors planning to take the ACT enroll in the course. The mean of their scores is 22.1. Assuming that
ACT scores vary normally with standard deviation 6, do we have good evidence that the mean of all students who'd take
the course is greater than 20 (i.e., that the course does some good)?

\[ \mu = \text{average score of all hypothetical students who could take this course} \]
\[ H_0: \mu = 20 \]
\[ H_a: \mu > 20 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as large):

\[ \bar{x} \geq 22.1 \quad Z \geq \frac{22.1 - 20}{\sqrt{6}} \quad Z \geq 2.55 \]

\[ p - \text{value} = 0.0054 \text{ (right tail)} \]
\[ 0.0054 < 0.05 \text{ evidence of } H_a; \text{ reject } H_0 \]

Again, a level is not specified, but 0.0054 is suspiciously unlikely at any reasonable level, and in particular at the most
commonly used 5% level.

(10) A computer has a random number generator designed to produce random numbers that are uniformly distributed
on the interval 0 to 1. If this is true, the numbers generated come from a population with mean 0.5 (this is obvious) and
standard deviation 0.2887 (we haven't proved this). A command to generate 100 random numbers gives outcomes with
mean 0.4365. Assume that the standard deviation really is as advertised. Do we have evidence at the (a) 5% level (b) 1%
level that the mean \( \mu \) of this random number generator (i.e., of all numbers it could possibly produce) is not 0.5?

\[ \mu = \text{average of all random numbers that could be generated} \]
\[ H_0: \mu = 0.5 \]
\[ H_a: \mu \neq 0.5 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as far from the center):

Left tail is:

\[ \bar{x} \leq 0.4365 \quad Z \leq \frac{0.4365 - 0.5}{\sqrt{0.2887}\sqrt{100}} \quad Z \leq -2.20 \]

\[ p - \text{value} = 0.0139 \times 2 = 0.0278 \text{ (both tails)} \]
\[ 0.0278 < 0.05 \text{ evidence of } H_a; \text{ reject } H_0 \]
\[ 0.0278 < 0.01 \text{ no evidence of } H_a; \text{ don't reject } H_0 \]

(11) The SSHA is a psychological test that measures the motivation, attitude toward school, and study habits of students.
Scores range from 0 to 200. The mean score for U.S. college students is about 115, and the standard deviation is about 30.
A teacher who suspects that older students have better attitudes toward school gives the SSHA to 20 students who are at
least 30 years old. Their mean score is 135.2. Assuming that the standard deviation is unchanged for older students, do we
have evidence that the mean \( \mu \) of all older students (students over 30) is more than 115?

\[ \mu = \text{average score of all older students} \]
\[ H_0: \mu = 115 \]
\[ H_a: \mu > 115 \]

Assuming \( H_0 \), find the probability of a sample at least as extreme as ours (as large):

\[ \bar{x} \geq 135.2 \quad Z \geq \frac{135.2 - 115}{\sqrt{30}} \quad Z \geq 3.01 \]

\[ p - \text{value} = 0.0013 \text{ (right tail)} \]
\[ 0.0013 < 0.05 \text{ evidence of } H_a; \text{ reject } H_0 \]

Once again, a level is not specified, but 0.0013 is suspiciously unlikely at any reasonable level, and in particular at the
most commonly used 5% level.