Section 11 answer key

1) The U.S.R.S. (Union of Starfleet Red Shirts) claims that 40% of the Red Shirts who beam down with Captain Kirk end up dying. If 35 of the last 93 Red Shirts who beamed down with Kirk met an unfortunate end, (a) do we have evidence at the 5% level that it’s really less than they claim?

\[ p = \text{proportion who would die in such circumstances} \]
\[ H_0: p = 0.4 \]
\[ H_a: p < 0.4 \]

\[ \hat{p} = \frac{35}{93} = 0.3763 \]

\[ \hat{p} \leq 0.3763 \]
\[ Z \leq \frac{0.3763 - 0.4}{0.4 \times (1 - 0.4)} \]
\[ Z \leq -0.47 \]

\[ p - \text{value} = 0.3192 \text{ (left tail)} \]
\[ 0.3192 < 0.05 \]
\[ \text{no evidence of } H_a; \text{ do not reject } H_0 \]

b) Do we have evidence at the 5% level that it’s not what they claim?

\[ H_0: p = 0.4 \]
\[ H_a: p \neq 0.4 \]

\[ p - \text{value} = 0.3192 \times 2 = 0.6384 \text{ (both tails)} \]
\[ 0.6384 < 0.05 \]
\[ \text{no evidence of } H_a; \text{ do not reject } H_0 \]

2) The Responsible Cookie Corporation claims that only 4% of their cookies contain mouse hairs. If we inspect 720 cookies and find that 34 of them contain mouse hairs, do we have evidence at the 10% level that the actual percentage is higher? At the 20% level?

\[ p = \text{proportion of cookies with mouse hairs} \]
\[ H_0: p = 0.04 \]
\[ H_a: p > 0.04 \]

\[ \hat{p} = \frac{34}{720} = 0.0472 \]

\[ \hat{p} \geq 0.0472 \]
\[ Z \geq \frac{0.0472 - 0.04}{0.04 \times (1 - 0.04)} \]
\[ Z \geq 0.99 \]

\[ p - \text{value} = 0.1611 \text{ (right tail)} \]
\[ 0.1611 < 0.10 \]
\[ \text{no evidence of } H_a; \text{ do not reject } H_0 \]
\[ 0.1611 < 0.20 \]
\[ \text{evidence of } H_a; \text{ reject } H_0 \]

3) Someone once said, "94% of all statistics are meaningless." If a newspaper contains 340 statistics and 310 of them are meaningless, do we have evidence at the 2% level that the quote is not true? Do we have evidence at the 2% level that the figure in the quote is too high?

(a)

\[ p = \text{proportion of all statistics that are meaningless} \]
\[ H_0: p = 0.94 \]
\[ H_a: p \neq 0.94 \]

\[ \hat{p} = \frac{310}{340} = 0.9118 \]
4) We wish to know whether a suspect coin is fair or not. We flip the coin 400 times and 190 times it lands heads. Do we have evidence at the 2% level that the coin is unfair? Do we have evidence at the 2% level that the coin is weighted against "heads"?

(a) 
\[ p = \text{proportion of all tosses that would land head (prob. of head)} \]
\[ H_0: p = 0.5 \]
\[ H_a: p < 0.5 \]
\[ \hat{p} = \frac{190}{400} = 0.475 \]
\[ \hat{p} \leq 0.475 \quad Z \leq \frac{0.475 - 0.5}{\sqrt{0.5 \times (1 - 0.5) / 400}} \quad Z \leq -1.00 \]
\[ p - \text{value} = 0.1587 \times 2 = 0.3174 \text{ (both tails)} \]
\[ 0.3174 < 0.02 \quad \text{no evidence of } H_a; \quad \text{do not reject } H_0 \]

(b) 
\[ p = \text{proportion of all statistics that are meaningless} \]
\[ H_0: p = 0.94 \]
\[ H_a: p < 0.94 \]
\[ \hat{p} = \frac{310}{340} = 0.9118 \]
\[ \hat{p} \leq 0.9118 \quad Z \leq \frac{0.9118 - 0.94}{\sqrt{0.94 \times (1 - 0.94) / 340}} \quad Z \leq -2.19 \]
\[ p - \text{value} = 0.0143 \times 2 = 0.0286 \text{ (both tails)} \]
\[ 0.0286 < 0.02 \quad \text{no evidence of } H_a; \quad \text{do not reject } H_0 \]
5) Picking 250 orders randomly from a mail-ordering company, we find that 210 arrived on time. The company claims that 90% of its orders are on time. Do we have evidence at the 0.1% level that their claim is too high?

\[ p = \text{proportion of all orders that are on time} \]

\[ H_0: p = 0.9 \]
\[ H_a: p < 0.9 \]

\[ \hat{p} = \frac{210}{250} = 0.84 \]

\[ \hat{p} \leq 0.84 \quad Z \leq \frac{0.84 - 0.9}{\sqrt{0.9 \times (1 - 0.9) / 250}} \quad Z \leq -3.16 \]

\[ p - \text{value} = 0.0008 \text{ (left tail)} \]
\[ 0.0008 < 0.001 \quad \text{evidence of } H_a; \text{ reject } H_0 \]

6) The National Bureau of Mail Order Research claims that 90% of the preceding company’s orders are on time. Do we have evidence at the 0.1% level that the claim is untrue?

\[ p = \text{proportion of all orders that are on time} \]

\[ H_0: p = 0.9 \]
\[ H_a: p \neq 0.9 \]

\[ \hat{p} = \frac{210}{250} = 0.84 \]

\[ \hat{p} \leq 0.84 \quad Z \leq \frac{0.84 - 0.9}{\sqrt{0.9 \times (1 - 0.9) / 250}} \quad Z \leq -3.16 \]

\[ p - \text{value} = 0.0008 \times 2 \text{ (both tails)} \]
\[ 0.0016 < 0.001 \quad \text{no evidence of } H_a; \text{ do not reject } H_0 \]

7) In a rural area, only about 30% of the wells that are drilled find adequate water at a depth of 100 feet or less. A local man claims to be able to find water by "dowsing" – using a forked stick to indicate where the well should be drilled. You check with 80 of his customers are find that 27 have wells less than 100 feet deep. Do we have evidence at the 5% that he really can "dowse"?

\[ p = \text{proportion of all customers who’d find water} \]

Being able to “dowse” would mean probability of finding water \( p \) is higher than it would be for blind chance (0.3).

\[ H_0: p = 0.3 \]
\[ H_a: p > 0.3 \]

\[ \hat{p} = \frac{27}{80} = 0.3375 \]

\[ \hat{p} \geq 0.3375 \quad Z \geq \frac{0.3375 - 0.3}{\sqrt{0.3 \times (1 - 0.3) / 80}} \quad Z \geq 0.73 \]

\[ p - \text{value} = 0.2327 \text{ (right tail)} \]
\[ 0.2327 < 0.05 \quad \text{no evidence of } H_a; \text{ do not reject } H_0 \]
8) In the 1980’s it was generally believed that autism affected about 5% of the nation’s children. We wish to know whether there’s been an increase in autism. A recent study examined 384 children and found that 46 of them showed signs of some form of autism. Is there evidence at the 5% level that autism has increased? Is there evidence at the 1% level that autism has increased? Is there evidence at the .1% level that autism has increased?

\[ p = \text{proportion of all modern children who are diagnosed as autistic} \]

\[ H_0: p = 0.05 \]
\[ H_a: p > 0.05 \]

\[ \hat{p} = \frac{27}{384} = 0.1198 \]

\[ \hat{p} \geq 0.1198 \]
\[ Z \geq -\frac{0.1198 - 0.05}{\sqrt{0.05 \times (1 - 0.05) / 384}} \]
\[ Z \geq 6.28 \]

\[ p - \text{value} \approx 0 \text{ (right tail)} \]
\[ 0 < 0.05 \text{ evidence of } H_a; \text{ reject } H_0 \]
\[ 0 < 0.01 \text{ evidence of } H_a; \text{ reject } H_0 \]
\[ 0 < 0.001 \text{ evidence of } H_a; \text{ reject } H_0 \]

Specifically, something which is definitely less than 0.0002 (end of chart)*2 = 0.0004, is definitely less than (for instance) 0.001 and definitely implausible by any remotely reasonable standard. In fact (using a computer rather than normal chart) this probability (two-tailed) turns out to be approximately 0.0000000000339, so the null hypothesis is definitely unbelievable. But this only implies that it is highly unlikely that the proportion of all children who would be diagnosed (for whatever reason) as autistic today, has not risen.

9) A widely-accepted report indicates that about 3% of all births produce twins. Data from a large hospital shows that only 7 sets of twins were born to 469 teenage girls. Does that suggest that mothers under age 20 may be less likely to have twins: Is there evidence at the 10%, 5%, 1% levels?

\[ p = \text{proportion of all teenage mothers who have twins} \]

\[ H_0: p = 0.03 \]
\[ H_a: p < 0.03 \]

\[ \hat{p} = \frac{7}{469} = 0.0149 \]

\[ \hat{p} \leq 0.0149 \]
\[ Z \leq -\frac{0.0149 - 0.03}{\sqrt{0.03 \times (1 - 0.03) / 469}} \]
\[ Z \leq -1.91 \]

\[ p - \text{value} = 0.0281 \text{ (left tail)} \]
\[ 0.0281 < 0.10 \text{ evidence of } H_a; \text{ reject } H_0 \]
\[ 0.0281 < 0.05 \text{ evidence of } H_a; \text{ reject } H_0 \]
\[ 0.0281 < 0.01 \text{ no evidence of } H_a; \text{ do not reject } H_0 \]