Notes on Seigniorage and The Fiscal Theory of the Price Level

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Abstract

This note compares the implications of Sargent and Wallace’s (1981) “Unpleasant Monetarist Arithmetic” and the “Fiscal Theory of the Price Level” for the restrictions on monetary policy and inflation.
1 Introduction

We present a simple version of Sargent and Wallace’s (1981) “Unpleasant Monetarist Arithmetic” (UMA) and show that its implications for the price level and for monetary policy and inflation are identical to those in the “Fiscal Theory of the Price Level” (FTPL) when the later is modified to have real bonds instead of nominal bonds. Both models assume that the time path of government deficits, defined as spending less ordinary taxes, is exogenous. The UMA model requires that seigniorage revenues, appropriately defined, assure intertemporal government budget balance. Therefore, UMA requires responsiveness of monetary policy to fiscal policy to assure satisfaction of the government’s intertemporal budget constraint. Similarly, it is well-known that under the FTPL, monetary policy also faces restrictions for an equilibrium to exist. We show that when we modify the FTPL model to make government bonds are real, that the restrictions faced by the monetary authority under the FTPL are identical to those under the UMA, and the models are equivalent. Money growth and inflation finance the increase in the deficit, as in UMA. And the price level is determined to equate real government debt with the present-value of future primary surpluses, as in the FTPL.

When we allow nominal bonds, as required by the FTPL, the restrictions placed on the monetary authority, following a reduction in the present-value of primary surpluses, are less severe due to the capital loss on nominal bonds created by the price level increase. The
price level is determined by the FTPL. Nonetheless, money growth and inflation finance the residual increase in present-value deficits net of bond debt, as in UMA. The restrictions on monetary policy due to UMA mechanisms disappear from the FTPL only in the limit when nominal money vanishes. In the limit of a non-monetary economy, the model is pure FTPL since a reduction in the present-value of government spending must be met with a reduction in the real value of bonds.

To simplify the presentation, assume that output and government spending are constant, fixing equilibrium consumption and forward-looking real interest rates. Additionally, assume that utility is logarithmic in consumption and real money balances, fixing expenditures on money \( \left( \frac{i_t}{1+i_t}m_t \right) \), where \( i_t \) is the nominal interest rate and \( m_t \) is the level of real balances. The economy has perfect foresight except in the initial period.

2 UMA

The theory in “Unpleasant Monetarist Arithmetic” is based on the government’s intertemporal budget constraint. Consider an economy, in which the government issues money and one-period debt. The government’s flow budget constraint is given by:

\[
M_t + B_t = (1 + i_{t-1})(M_{t-1} + B_{t-1}) + P_t(g_t - \tau_t) - i_{t-1}M_{t-1},
\]

where \( P_t \) represents the price level, \( M_t \) and \( B_t \) represent the dollar value of end-of-the-period money balances and bonds respectively, \( i_{t-1} \) represents the nominal interest rate agreed upon by borrowers and lenders at the end of the period \( t - 1 \), \( g_t \) is real government spending, and \( \tau_t \) is real lump-sum taxes. Sargent and Wallace (1981) assume that all government bonds
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are real. Letting small letters denote real values, the flow budget constraint in real terms can be expressed as:

\[ b_t = (1 + r_{t-1}) b_{t-1} - s_t - \frac{\mu_t}{1 + \mu_t} m_t \]

where \( s_t = \tau_t - g_t \), that is, the primary surplus net of seigniorage revenues, \( \mu_t = \frac{M_t - M_{t-1}}{M_{t-1}} \), \( \pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \) so that \( \frac{\mu_t}{1 + \mu_t} m_t = \frac{M_t - M_{t-1}}{P_t} \). Taking the present-discounted value of the flow budget constraint yields:

\[ (1 + r_{t-1}) b_{t-1} = \sum_{t=0}^{\infty} \left( s_t + \frac{\mu_t}{1 + \mu_t} m_t \right) R_{0,t} + \lim_{T \to \infty} b_T R_{0,T} \]  

(2)

where

\[ R_{0,t} = \Pi_0^{t-1} \left( \frac{1}{1 + r_j} \right) \]

\[ 1 + r_j = (1 + i_j) \frac{P_j}{P_{j+1}} = \frac{1 + i_j}{1 + \pi_{j+1}}. \]

The UMA can be illustrated using equation (2) and imposing \( \lim_{T \to \infty} b_T R_{0,T} = 0 \). Consider the effect of a previously unexpected reduction in the present-value of future surpluses caused by a tax cut today. To avoid outright default, the monetary authority must finance the budget deficit with an increase in the present value of seigniorage, defined as \( \frac{\mu_t}{1 + \mu_t} m_t \). Note that the assumptions fixing \( \frac{\mu_t}{1 + \mu_t} m_t \) imply that \( \frac{\mu_t}{1 + \mu_t} m_t \) is increasing in \( \mu_t \). Hence, “Unpleasant Monetarist Arithmetic” shows that inflation is due to fiscal policy because the fiscal authority forces the monetary authority to finance its deficits with money.

UMA goes further, however, by showing that the increase in money growth can create inflation today even when the money growth does not occur until the future. The expression
for seigniorage in equation (2) can be written as:

$$\sum_{t=0}^{\infty} \left( \frac{\mu_t}{1+\mu_t} m_t \right) R_{0,t} = \sum_{t=0}^{\infty} \left( \frac{i_t}{1+i_t} m_t \right) R_{0,t} - \frac{M_{-1}}{P_0} + \lim_{T \to \infty} m_T R_{0,T}. \quad (3)$$

If we impose $\lim_{T \to \infty} m_T R_{0,T} = 0$ in equilibrium,\(^1\) then equation (3) shows that the increase in expected future money growth must cause an increase in the current price level ($P_0$), creating current inflation, irrespective of when the increase in future money growth occurs. In the next section, we show that this is the same price level increase required by the FTPL were that theory to be applied to a model with real bonds.

3 FTPL

3.1 The Theory

The fiscal theory of the price level (FTPL) is also based on the government’s intertemporal budget constraint, but, in the FTPL, government bonds are assumed to be nominal instead of real. The budget constraint, given by equation (1), can be expressed in real terms as:

$$m_t + b_t = (1 + i_{t-1}) \left( \frac{P_{t-1}}{P_t} \right) (m_{t-1} + b_{t-1}) - s_t - i_{t-1} \left( \frac{P_{t-1}}{P_t} \right) m_{t-1}, \quad (4)$$

The present-value of initial outstanding debt can be expressed as:

$$\frac{(1 + i_{-1}) B_{-1} + M_{-1}}{P_0} = \sum_{t=0}^{\infty} \left( s_t + \frac{i_t}{1+i_t} m_t \right) R_{0,t} + \lim_{T \to \infty} (m_T + b_T) R_{0,T}. \quad (5)$$

The FTPL can be illustrated using equation (5), imposing $\lim_{T \to \infty} (m_T + b_T) R_{0,T-1} = 0$, and ruling out outright default. Under the FTPL, the intertemporal budget constraint is viewed as an equilibrium condition, determining equilibrium price, not as a budget constraint.\(^1\) This is equivalent to ruling out equilibria in which the limit condition fails.
Again, consider the effect of a previously unexpected reduction in the present-value of future surpluses, created by a tax cut. Equation (5) shows that this is possible, under the simplifying assumptions of the model, only if $P_0$ rises, reducing the real value of the government’s outstanding nominal debt. Hence, a negative shock to the present value of future primary surpluses must cause an increase in the current price level for any monetary policy consistent with existence of equilibrium.

3.2 Real Bonds

Now apply the FTPL to an economy in which government bonds are real instead of nominal. The intertemporal government budget constraint with real bonds can be expressed as:

$$
(1 + r_{-1}) b_{-1} + \frac{M_{-1}}{P_0} = \sum_{t=0}^{\infty} \left( s_t + \frac{i_t}{1+i_t} m_t \right) R_{0,t} + \lim_{T \to \infty} (m_T + b_T) R_{0,T}.
$$

(6)

Using equations (2) and (3), in equation (6) shows that the price level increase in the FTPL, modified to have real bonds, is identical to the price level increase in UMA. Additionally, equation (3) shows that the present-value increase in seigniorage, due to an increase in money growth, necessary for a non-explosive equilibrium to exist, is identical to that under UMA.

UMA focuses on the effect of the surplus reduction on money growth, while FTPL focuses on its effect on the price level. When all government bonds are real, equation (3) shows that these are different ways of expressing the same effect.

3.3 Nominal Bonds

Now, return to the FTPL with nominal bonds. We have shown that if we replace the nominal bonds in the FTPL with real bonds, that the two models become identical. However, the
FTPL explicitly requires nominal bonds, as in equation (5). With nominal bonds, a price level increase reduces the real value of money and bonds, requiring a smaller increase in the price level for any given reduction in present-value surpluses. However, equation (3) makes clear that, as long as there is nominal money in the economy, there are implications for seigniorage. The reduction in the real value of money must be accompanied by an increase in seigniorage revenue, defined as the present value of $\frac{\mu}{1+\mu} m_t$. Since the increase in $P_0$ is smaller with nominal bonds, then the increase in present-value seigniorage revenue can be smaller. However, as long as there is money in the economy, the UMA mechanisms, requiring that an increase in the money supply follow a reduction in present-value surpluses, operate in the FTPL. Note, additionally, that the FTPL states that the increase in $P_0$ is independent of how the monetary authority responds, provided that it responds so money and bonds each are on a non-explosive path. A monetary authority, operating in an economy with nominal bonds faces restrictions on policy very similar to those in UMA. The UMA mechanisms disappear only in the limit as money vanishes.