Monetary Policy in a Fiscal Theory Regime *

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Abstract

This paper considers the role for monetary policy in a regime in which the “Fiscal Theory of the Price Level” (FTPL) applies and in which the government issues long-term debt. Cochrane (2001) shows that the fiscal authority can use a commitment to maintain a fixed maturity structure of government debt to fully determine the impact and dynamic effects of present-value-surplus shocks on inflation in a cashless economy. We show that in a monetary economy, in which Cochrane’s fiscal commitment to maturity structure is replaced with a monetary commitment to a state-contingent interest rate rule, the monetary authority becomes responsible for the timing of inflation. We derive interest rate rules, which minimize the variance of inflation around a target, and show the interest rate will respond positively to shocks raising inflation and to shocks raising output above trend, as with a Taylor Rule.

Key Words: Fiscal Theory of the Price Level, Monetary Policy, Taylor Rule, Inflation, Long-term Government Debt

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1 Introduction

This paper considers the role of monetary policy in a regime in which the “Fiscal Theory of the Price Level” (FTPL) applies. Leeper (1991), Woodford (1995, 1997, 1998a, 1988b, 2001), Sims (1994, 1997), and Cochrane (1999, 2000, 2001) have espoused the view that the price level is determined by the level of nominal government debt together with the government’s ability to generate net revenue to repay that debt. A decrease in the expected present value of government surpluses raises the demand for goods in current and future periods. If an equilibrium exists, then excess demand is eliminated by a reduction in the real value of an agent’s wealth, where agent net wealth includes nominal government debt. In the standard model with one-period government bonds, monetary policy, in the form of an open-market operation, has no effect on the price level, given the present-value of government surpluses. Woodford (1998a) and Cochrane (1998, 2000) move to a cashless economy and demonstrate that the FTPL determines the price level even in the absence of money. This has left many with the impression that monetary policy is irrelevant for the determination of the price level under the FTPL.\footnote{This is not quite true, as monetary policy in the form of the determination of the nominal interest rate determines expected inflation, as is obvious from the Euler equation derived in an intertemporal model with money.}

When government debt has a single-period maturity and the FTPL applies, a reduction in the present value of expected future surpluses must raise the current price level. However, Cochrane (2001) shows that when the government has long-term debt, a reduction in the
present-value of future surpluses must be met by a price level increase on some date, but not necessarily on the current date. Long-term debt gives the government the ability to trade off current inflation for future inflation. The government’s debt policy, in the form of the expected pattern of state-contingent debt sales and repurchases, determines the impact of a fiscal shock on current and future inflation. Cochrane assumes that the government commits to a maturity structure for the debt, and solves for the maturity structure which minimizes the variance of inflation. The time pattern of expected present-value surpluses, together with a commitment to maintain a fixed maturity structure for the debt, fully determine the time path for inflation. The analysis is conducted in a cashless economy, so monetary policy has no explicit role. However, there is an implicit role for monetary policy because an open market operation, whereby debt of different maturities is exchanged, does impact the time path of inflation.

This paper brings the FTPL with long-term government debt back into a monetary economy with the purpose of examining the roles of the fiscal and monetary authorities in the determination of the time path of inflation. From Cochrane’s work, we know that long-term government bonds allow the government to respond to a negative fiscal shock with some combination of current and future inflation. Cochrane lets the fiscal authority make this choice through its commitment to a maturity structure for debt. We replace Cochrane’s fiscal authority commitment with a commitment by the monetary authority to state-contingent short-term interest rate rules, and examine the roles of fiscal and monetary

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2 In our framework, the monetary authority is given the role of managing the the maturity structure of the government debt at short horizons as it conducts open market operations in money and one-period debt to control the nominal short-term interest rate. The fiscal authority is assumed to determine the portion of its
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policy in the determination of inflation under the FTPL. Fiscal policy is the choice of present-value surpluses, and monetary policy is the choice of state-contingent interest rate rules, a division of responsibility reflecting institutions in many countries. Additionally, we derive the optimal state-contingent interest rate rules under an inflation target and compare them to those actually observed.

The paper is organized as follows. Section 2 contains a presentation of the Fiscal Theory of the Price Level, modified as in Cochrane (2001) and Woodford (1998b), to contain long-term government bonds. Section 3 describes monetary policy in regimes without and with long-term bonds. Section 4 provides conclusions.

2 Fiscal Theory of the Price Level

Following the FTPL, the fiscal authority is assumed to choose the present value of government surpluses and to finance any budget deficits with government debt. Fiscal shocks take the form of changes in the present-value of government surpluses. It is important to recognize that shocks to the present value of government surpluses could be positively or negatively correlated with shocks to the current surplus. Indeed, Cochrane (2001) argues that negative innovations to the current surplus are generally accompanied by positive innovations to the present-value surplus, allowing the fiscal authority to increase real debt in times of deficits.

We conduct the analysis in a flexible-price, money-in-the-utility-function model with debt which has a longer horizon than a single period.

3 Davig Leeper and Chung (2003) model an interesting alternative way to give monetary policy a role without long-term government bonds. They assume that the fiscal regime which can switch stochastically between Ricardian and non-Ricardian behavior. Monetary policy is assumed to switch between strong and weak Taylor Rules.
technology shocks and shocks to the present value of government surpluses. The choice of a simple model serves two purposes. First, the flexible price model simplifies the presentation by confining the monetary authority’s concerns to inflation since monetary policy has no real effects in the model.\footnote{Other authors also choose to deal with monetary policy issues, including the liquidity trap and price-level indeterminacy, using flexible price models. Examples include: Alstadheim and Henderson (2002), Benhabib, Schmitt-Grohe and Uribe (2001a, 2001b), Schmitt-Grohe and Uribe (2000).} Second, many of the results on the conduct of monetary policy under an objective function yielding inflation-smoothing are similar to those derived in sticky-price models. Using a flexible price FTPL model allows comparison of results, due to the FTPL assumption, with those due to the sticky-price assumption. We acknowledge that a full understanding of monetary policy is likely to require including nominal rigidities, but we think that insights are clearer at this point if issues due to the FTPL are separated from issues due to price stickiness.

2.1 Long-term Government Bonds

The maturity structure of government debt is important, as in Cochrane (2001). We assume a long-term debt structure which yields both declining debt obligations over time and an analytically tractable model. Specifically, we assume that each unit of long-term bonds \((B^N_t)\) obligates the government to pay one dollar every period for \(N\) periods. Additionally, the government sustains only \(N\) period bonds by buying up all bonds issued in the previous period (now \(N - 1\) period bonds) at the market price and replacing them with new debt, including \(N\)-period bonds. This is equivalent to a an arrangement where \(B^N_t\) dollars of discount bonds mature each of the next \(N\) periods. While constant in face value terms,
such a debt structure implies that the market value of outstanding debt declines with its maturity.\textsuperscript{5} The fiscal authority chooses $N$ together with the long-run fraction of $N$-period bonds in total debt.

The debt of the consolidated monetary and fiscal authorities includes one-period nominal bonds $(B^S_t)$, nominal money $(M_t)$, and $N$-period bonds. Letting $\rho^N_t$ be the time $t$ price of a bond paying coupons for $N$ periods, the consolidated government flow budget constraint can be written as:

$$D_t = \rho^N_t B^N_t + B^S_t + M_t = D_{t,t-1} + P_t y_t (g_t - \tau_t),$$  \hspace{1cm} (1)

where

$$D_{t,t-1} = (1 + i_{t-1}) B^S_{t-1} + (1 + \rho^{N-1}_t) B^N_{t-1} + M_{t-1},$$

such that $D_{t,t-1}$ is the period $t$ nominal value of debt outstanding in period $t - 1$. It includes interest and nominal capital gains and losses on long-term debt outstanding in period $t - 1$. Additionally, $i_{t-1}$ is the contractual nominal interest rate on one-period bonds issued in period $t - 1$, $P_t$ is the price level, $y_t$ is real income, $g_t$ is real government spending as a fraction of real income, and $\tau_t$ is the government’s tax rate on nominal income. The time $t$ price of a long-term bond issued in period $t - 1$ with $N$ periods to maturity, becomes $\rho^{N-1}_t$ since there are only $N - 1$ periods to maturity from period $t$. Equation (1) requires that the government issue debt, consisting of long-term bonds, short-term bonds, and money, to finance its current liabilities. Liabilities include interest and principle on one-period bonds, $N$-period bonds, issued one period ago valued at their current price together with their

\textsuperscript{5} The declining maturity structure is consistent with US data.
coupon, money, and the excess of spending over taxes.

To solve for the price of long-term bonds in terms of interest rates, it is necessary to specify and solve the consumer’s optimization problem. Assume that the economy is populated by a representative agent who receives an exogenous endowment of real income \((y_t)\) each period. Expected utility is given by:

\[
E_t U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \eta \ln (y_s c_s) + (1 - \eta) \ln (y_s m_s) \right), \quad 0 < \eta < 1,
\]

where \(c_t\) is real consumption as a fraction of real income and \(m_t\) denotes the real value of nominal money as a fraction of real income. The agent faces a budget constraint given by:

\[
\rho_t^N B_t^N + B_t^S + M_t = D_{t,t-1} + P_t y_t (1 - \tau_t - c_t).
\]

Letting small letters denote values as a fraction of nominal income (for example \(m_t = \frac{M_t}{P_t y_t}\)), and defining

\[
d_{t,t-1} = \frac{D_{t,t-1}}{P_{t-1} y_{t-1}} = \left(1 + \rho_t^{N-1}\right) b_{t-1}^N + (1 + i_{t-1}) b_{t-1}^S + m_{t-1},
\]

\[
1 + \pi_t \equiv \frac{P_t}{P_{t-1}} \quad \text{and} \quad 1 + \gamma_t \equiv \frac{y_t}{y_{t-1}},
\]

the agent’s budget constraint can be written as:

\[
\rho_t^N b_t^N + b_t^S + m_t = \frac{d_{t,t-1}}{(1 + \pi_t) (1 + \gamma_t)} + (1 - \tau_t - c_t).
\]

The agent maximizes expected utility, given by equation (2), with respect to choices for the time paths of \(b^S, b^N, m_t,\) and \(c_t,\) subject to the budget constraint given by equation (5), an initial value for \(d,\) and a “no Ponzi game” constraint requiring

\[
\lim_{T \to \infty} \left( \sum_{t=0}^{T} \frac{d_{t+1} y_{t+1}}{(1 + \pi_{t+1})(1 + \gamma_{t+1})} \right) \Pi_{t=0}^{T} \left( \frac{(1+i_{t+1})}{(1 + \pi_{t+1})(1 + \gamma_{t+1})} \right) = 0
\]
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0. The first order conditions can be expressed as:

\[
E_t \left[ \beta \frac{c_t (1 + i_t)}{c_{t+1} (1 + \pi_{t+1}) (1 + \gamma_{t+1})} - 1 \right] = 0. \tag{6}
\]

\[
m_t = \left( \frac{1 - \eta}{\eta} \right) \left( \frac{1 + i_t}{i_t} \right) c_t \tag{7}
\]

\[
E_t \left[ \frac{1 + \rho_{t+1}^{N-1} - (1 + \pi_t) \rho_t^N}{c_{t+1} (1 + \pi_{t+1}) (1 + \gamma_{t+1})} \right] = 0 \tag{8}
\]

Equation (6) is a standard Euler equation. The term \((1 + \gamma_{t+1})\) appears because consumption is defined as a fraction of nominal income. Equation (7) is a money demand equation, and equation (8) is an asset-pricing equation for long-term bonds.

Using equation (6), the asset pricing equation for long-term bonds, given by (8), can be written as:

\[
E_t \left( 1 + \rho_{t+1}^{N-1} \right) = \rho_t^N (1 + i_t) (1 + \varphi_t) ; \quad \varphi_t = -\beta \text{cov}_t \left[ \frac{1 + \rho_{t+1}^{N-1}}{\rho_t^N} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \right]. \tag{9}
\]

Therefore, the expected value of an \(N\)-period bond (coupon plus resale price), one period after it is issued, equals its previous price, augmented by the short-term interest rate and the risk premium. The latter is defined by the negative of the discounted conditional covariance of the rate of increase of the bond price and the inverse of nominal consumption growth, where \(C_t = P_t y_t c_t\).

Solving forward, the price of the long-term bond can be expressed as:

\[
\rho_t^N = E_t \sum_{h=0}^{N-1} \Pi_{j=0}^h \left( \frac{1}{1 + i_{t+j}} \right) \left( \frac{1}{1 + \varphi_{t+j}} \right). \tag{10}
\]

Equation (10) demonstrates that the price of long-term bonds depends on current and expected future interest rates. Note that interest rates have a smaller effect on the market
value of outstanding debt \( \left( \rho_t^N \right) \) the farther they are into the future. This is because the government has a debt structure whereby its obligations are, in present-value terms, decreasing over time. Additionally, since government debt has finite maturity, interest rates in the very distant future have no effect on the real value of outstanding debt.

### 2.2 FTPL with Long-Term Government Debt

We derive the FTPL by considering the behavior of the representative agent in an economy in which the government issues both one-period and N-period nominal bonds. To solve for consumption, it is necessary to compute that agent’s intertemporal budget constraint. Using equation (5), this becomes:

\[
\frac{d_{t,t-1}}{(1 + \pi_t) (1 + \gamma_t)} = \sum_{j=0}^{\infty} \Omega(t, j) \left[ \tau_{t+j} + c_{t+j} - 1 + \frac{i_{t+j}}{1 + i_{t+j}} m_{t+j} - \varphi_{t+j} \rho_{t+j} b_{t+j}^N + \frac{E'_{t+j+1} \rho_{t+j+1} b_{t+j}^N}{1 + i_{t+j}} \right],
\]

where

\[
\Omega(t, j) = \prod_{h=1}^{j} \left( \frac{(1 + \pi_{t+h}) (1 + \gamma_{t+h})}{1 + i_{t+h-1}} \right), \quad \Omega(t, 0) = 1.
\]

The notation \( E'_t x_{t+i} \) is generally used to denote revisions in expectations of the variable \( x_{t+i} \), such that \( E'_t x_{t+i} \equiv E_t x_{t+i} - E_{t-1} x_{t+i} \). When \( i = 0 \), revisions in expectations become forecast errors since \( E_t x_t = x_t \). Therefore, \( E_t \rho_t^{N-1} \equiv \rho_t^{N-1} - E_{t-1} \rho_t^{N-1} \) denotes the forecast error in the price of long-term bonds. The forecast error represents capital gains and losses on long-term debt due to an unexpected change in the price of long-term bonds.

The expected present value of consumption is determined by taking the expected present-
value of the intertemporal budget constraint and solving, yielding:

\[ E_t \sum_{j=0}^{\infty} \Omega(t, j) c_{t+j} = \frac{d_{t,t-1}}{(1 + \pi_t)(1 + \gamma_t)} + E_t \sum_{j=0}^{\infty} \Omega(t, j) \left[ -\tau_{t+j} + 1 - \frac{i_{t+j}}{1 + i_{t+j}} m_{t+j} + \varphi_{t+j} \rho_{t+j} b_{t+j}^N \right]. \]

(11)

Note that \( E_t \left( \frac{E_t\rho_{t+j} b_{t+j}^N}{1+i_{t+j}} \right) = 0 \) since revisions in expectations in future periods are not currently anticipated. Equation (11), together with the Euler equation (6), determine current consumption. Therefore, current consumption is based on a forecast of the present value of taxes, expenditures on money, and risk-premia on long-term bonds.

We can derive the FTPL from the agent’s intertemporal budget constraint. Define the government’s flow primary surplus as ordinary taxes net of spending together with any revenue or costs from issuing debt in a form other than one-period bonds (seigniorage net of the risk premium on long-term bonds), but to exclude interest payments on debt. As a fraction of nominal income, the primary surplus can be expressed as:

\[ s_t = \tau_t - g_t + \frac{i_t}{1 + i_t} m_t - \varphi_t \rho_t^N b_t^N. \]

(12)

Substituting equation (12) on the right-hand side of the expression for the expected present-value of consumption yields:

\[ E_t \sum_{j=0}^{\infty} \Omega(t, j) c_{t+j} = \frac{d_{t,t-1}}{(1 + \pi_t)(1 + \gamma_t)} + E_t \sum_{j=0}^{\infty} \Omega(t, j) \left[ 1 - s_{t+j} + g_{t+j} \right]. \]

Imposing goods market equilibrium,

\[ c_t + g_t = 1, \]

yields:

\[ \frac{d_{t,t-1}}{(1 + \pi_t)(1 + \gamma_t)} = E_t \sum_{j=0}^{\infty} \Omega(t, j) s_{t+j} \equiv E_t \Psi_t. \]

(13)
The expected present-value of future primary surpluses \( E_t \Psi_t \) must equal the real value of outstanding debt, all computed as a fraction of income. Note that this equation is identical to intertemporal government budget balance.\(^6\) Additionally, recall that \( E_t \Psi_t \) need not be positively correlated with \( s_t \).

Following the FTPL, we assume that fiscal policy is the choice of \( E_t \Psi_t \). That is, we assume that taxes net of spending can be adjusted to offset any seigniorage or long-term bond premium costs and any equilibrium changes in the real interest rate. This serves to simplify the analysis at the cost of ignoring the effect of monetary policy on the present value of surpluses.\(^7\) With the components of \( d_{t,t-1} \), given by \( b^S_{t-1}, b^N_{t-1} \), and \( m_{t-1} \) predetermined, \( \gamma_t \) determined by the real side of the model, and \( E_t \Psi_t \) determined by the fiscal policy choice, some combination of \( \rho_t^{N-1} \) and \( \pi_t \) must adjust to assure that equation (13) holds. This is the equation for the “Fiscal Theory of the Price Level,” amended to include long-term bonds. Woodford (1998b) and Cochrane (2001) derive similar equations. It differs from the simple FTPL by including long-term bond prices, as a determinate of the current real value of government debt.

Equation (10) together with equation (13) can be used to understand the set of choices facing the monetary authority. From equation (10), the monetary authority’s choice of current and future interest rates determines the current price of long-term bonds. Once \( \rho_t^{N-1} \) is determined, equation (13) determines current inflation \( \pi_t = \frac{P_t}{P_{t-1}} - 1 \). For \( b^N_{t-1} \neq 0 \),

\(^6\) The equation explicitly requires expected intertemporal balance. However, given the price level that emerges, actual intertemporal balance must result. For a discussion of this in the context of the fiscal theory, see Daniel (2003).

\(^7\) These effects are likely to be small as they include only changes in seigniorage and risk premia induced by interest rate changes.
the monetary authority can choose to let innovations in the price of long-term bonds absorb shocks as an alternative to letting innovations in inflation absorb them.

Equation (13) is useful to distinguish two classes of shocks, which can cause inflation. The first class includes innovations to variables in the equation, other than inflation or the price of long-term bonds, that is, innovations in either $E_t \Psi_t$ or in $\gamma_t$. With an innovation in one of these variables, then some combination of $\rho_t^{N-1}$ and $\pi_t$ must adjust, and the combination reflects the trade-off between current and future inflation. The second class includes innovations in either $\rho_t^{N-1}$ or $\pi_t$, due to an innovation in a variable outside this equation. In the absence of long-term bonds, an innovation in current inflation is not possible without an innovation in either $E_t \Psi_t$ or $\gamma_t$. With long-term bonds, an innovation directly affecting either the price of long-term bonds or current inflation is possible as long as both $\rho_t^{N-1}$ and $\pi_t$ adjust in the same direction, keeping the real value of outstanding debt constant. The FTPL is useful in determining the inflationary effects of shocks other than fiscal shocks, a point often overlooked.

3 Monetary Policy

Now, consider the role for monetary policy in a regime in which $E_t \Psi_t$ in equation (13) is exogenous, that is, in a regime in which the FTPL applies. The monetary authority is assumed to observe current shocks. Also, we assume that the short-term nominal interest rate is the instrument of monetary policy, and that the monetary authority can fully commit to a state-contingent set of rules for current and future interest rates $N - 1$ periods into the
future.\footnote{We recognize that the full commitment assumption is strong, and that it would be interesting to relax this assumption following recent work on monetary policy. However, it is necessary to understand the full commitment solution before considering how to implement the full commitment solution in the absence of commitment. We leave the solution without commitment to future research.}

The mechanism through which the monetary authority chooses the time path of the nominal interest rate is open-market operations in money and one-period nominal government bonds. From equation (7), $m$ must take on the value necessary to achieve the desired value for the nominal interest rate. This requires that there be a large enough quantity of short-term debt for $m$ to rise to levels desired at low interest rates, placing an upper bound on the fiscal authority’s choice of long-term debt as a fraction of total debt, where the fraction depends on the total quantity of debt. That is, if the monetary authority is to have the freedom of managing the nominal interest rate, there must be a large enough quantity of short-term government debt. We this assumption.\footnote{There are issues here which are beyond the scope of this paper. We consider the zero lower bound on the nominal interest rate as non-binding and do not impose it. We do not consider redefining monetary policy such that the authority is allowed to transact in long-term government debt or in private debt.}

An equilibrium for this economy requires satisfaction of the first-order conditions, expressed as (6), (7), and (10), together with the equation for the FTPL (13), which imposes goods market equilibrium and agent intertemporal budget balance.

To obtain analytical results, we work with a linearized approximation of the model about an equilibrium with a constant growth rate. The Euler equation (6), becomes:

\begin{equation}
\hat{i}_t = E_t (\hat{\pi}_{t+1} + \hat{\gamma}_{t+1}),
\end{equation}

where a hatted variable denotes a proportionate deviation of the variable from its constant-growth equilibrium value. When the variable is a rate of change, the assumption of a
constant-growth equilibrium, implies that the rate of change will have a steady-state equilibrium value. When the variable is a rate of return, the hatted variable denotes the proportionate deviation of the gross rate of return from its steady-state value \((\hat{i}_t = \frac{i_t - i_{1+}}{1+i})\). We have also assumed that government spending as a fraction of income \((g)\) is constant, making \(c\) constant.

A linear approximation for equation (13) is given by:

\[
\hat{D}_{t,t-1} = \pi_t + \gamma_t + \psi_t, \tag{15}
\]

where \(\psi_t = E_t \Psi_t\). Note that deviations in the present-value of surpluses and growth \((\hat{\gamma}_t + \hat{\psi}_t)\) create deviations in the real value of government debt \((\hat{D}_{t,t-1} - \pi_t)\), a restatement of the FTPL.

Linearizing equation (10) about a certainty equivalence steady state, holding \(\phi_t\) constant at its certainty value of zero, yields:

\[
\hat{\rho}^N_t = -\sum_{j=0}^{N-1} R(j, N) E_t (\hat{i}_{t+j}), \tag{16}
\]

where

\[
R(j, N) = \left(\frac{1}{1+i}\right)^j \left[\frac{(1+i)^N - (1+i)^j}{(1+i)^N - 1}\right],
\]

unsupscripted variables denote steady state values, and \(\rho^N = \frac{1}{i} \left(1 - \left(\frac{1}{1+i}\right)^N\right)\). Expected future deviations of interest rates from their long-term equilibrium levels determine the deviation of the price of the long-term bond about its long-run level.

Equations (14), (15), and (16) can be used to solve for current values and expected future paths for the nominal interest rate, inflation, and the price of long-term bonds. To
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illustrate the roles of monetary and fiscal policy in determining inflation, it is useful to break the solution into two parts, one for expected current values and one for revisions in expectations. Consider, first, the solution for expected values of current variables. Solving equation (14) for expected inflation yields:

\[ E_t \hat{\pi}_{t+1} = \hat{i}_t - E_t \hat{\gamma}_{t+1}. \]  \( (17) \)

Therefore, the monetary authority’s choice of the nominal interest rate, together with expected growth, determines expected inflation over the coming period. This occurs in any money-in-the-utility-function model with a fixed ratio of consumption to output, whether the FTPL applies or not. Fiscal policy, defined as the expected present-value of future surpluses, has no impact on expected inflation.\(^{11}\) Monetary policy’s choice of the time path of future interest rates also determines expected future interest rates and, therefore, the price of long term bonds from equation (16). Taking expectations of equation (15), the expected value of nominal government debt is determined by expected inflation, present-value surpluses, and expected growth.

To understand the role of fiscal policy in determining inflation, consider unexpected shocks. Taking deviations about previous period expectations for equations (14), (15), and (16), yields:

\[ E_t' \hat{i}_t = E_t' \left( \hat{\pi}_{t+1} + \hat{\gamma}_{t+1} \right) \]  \( (18) \)

\[ E_t' \hat{D}_{t,t-1} = \lambda E_t' \hat{\rho}_{t}^{N-1} = E_t' \left( \hat{\pi}_t + \hat{\gamma}_t + \hat{\psi}_t \right), \]  \( (19) \)

\(^{11}\)We can break this by assuming that government spending as a fraction of output varies, breaking the constancy of the consumption-output ratio. This is a different type of fiscal policy, and we do not consider its role in this paper.
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\[
E_t^\prime \rho_t^N = - \sum_{j=0}^{N-1} \left( \frac{1}{1+i} \right)^j \left( \frac{(1+i)^N - (1+i)^j}{(1+i)^N - 1} \right) E_t^\prime (\hat{\delta}_{t+j}), \tag{20}
\]

where \( \lambda = \frac{\rho^{N-1}B^N}{D} \), the steady state value of long-term debt, net of coupon payment, as a fraction of total debt.\(^{12}\) Note that capital gains and losses on long-term debt are due to revisions in expectations about future interest rates.

Using equation (16) to solve for \( \hat{\rho}_t^{N-1} \), the time t price of a bond yielding coupon payments for \( N-1 \) periods,\(^{13}\) computing expectations revisions from equation (20), substituting into the equation for the FTPL, given by equation (19), and solving for revisions in expectations about current inflation, yields an expression for the time t revision in inflationary expectations as:

\[
E_t^\prime \tilde{\pi}_t = - \left\{ E_t^\prime \left( \hat{\psi}_t + \hat{\gamma}_t \right) + \lambda \sum_{j=0}^{N-2} R (j, N-1) E_t^\prime (\hat{\delta}_{t+j}) \right\}. \tag{21}
\]

Equation (21) is the fiscal constraint faced by the monetary authority in its decisions about the time path of the nominal interest rate. Note that in the presence of long-term bonds \( \lambda > 0 \), the monetary authority can offset the effect of a current inflationary shock (a fall in \( E_t^\prime \left( \hat{\psi}_t + \hat{\gamma}_t \right) \)) on inflation by raising current and/or future interest rates.\(^{14}\) In the absence of a shock to \( E_t^\prime \left( \hat{\psi}_t + \hat{\gamma}_t \right) \), current inflation and the price of long-term bonds must move in opposite directions.

Using equation (18), to substitute for interest rates in equation (21), reveals that the

\(^{12}\)The monetary authority is assumed to conduct open-market operations in money and short-term government bonds, leaving \( \lambda \) unaffected. As noted before, the value of \( \lambda \) cannot be too large.

\(^{13}\)Substitute \( N - 1 \) for \( N \).

\(^{14}\)Note that it is straightforward to consider perpetuities by letting \( N \to \infty \). In this case \( R (j, \infty) = \left( \frac{1}{1+i} \right)^j \).
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monetary policy choice is about the time path of expected inflation.

\[ E_t^e \hat{\pi}_t = - \left\{ E_t^e (\hat{\psi}_t + \hat{\gamma}_t) + \lambda \sum_{j=0}^{N-2} R(j, N-1) E_t^e (\hat{\pi}_{t+j+1} + \hat{\gamma}_{t+j+1}) \right\}. \] (22)

For example, an increase in demand, created by a reduction in expected future fiscal surpluses \((E_t^e \hat{\psi}_t \text{ falls})\), requires an increase in either current inflation or in expected future inflation. The monetary authority can choose the time path of expected inflation through its choice for the nominal interest rate path, subject to the constraint, given by equation (22).

To summarize, monetary policy’s choice of the interest rate path together with expected future growth determine expected inflation. Fiscal innovations, together with the response of monetary policy (innovations in the expected path of interest rates), determine both revisions in future inflationary expectations and deviations of current inflation from its expected level. Given a fiscal innovation reducing present-value surpluses, the monetary authority must accept an increase in inflation on at least one date. The time path for that increase in inflation is constrained by equation (22), which incorporates a trade-off between current and expected future inflation.

Another way to think about inflation and the fiscal theory is to view inflation as determined by excess demand. Under the assumptions of the FTPL, government debt is viewed by agents as net wealth. When the monetary authority increases the interest rate, the price of long-term bonds falls, reducing wealth, thereby reducing demand and inflation. This provides a role for the nominal interest rate, independent of the real rate, in determining demand and inflation. Since government debt is viewed as net wealth by agents, the nominal interest rate directly affects demand through its effect on the real value of agents’ wealth.
3.1 Monetary Policy without Long-term Bonds

As a benchmark, consider the role of monetary policy in the absence of long-term bonds, as in standard presentations of the FTPL. We assume that fiscal policy determines the path of surpluses, in accordance with the FTPL, while monetary policy determines the path of interest rates. The monetary authority’s choice of the nominal interest rate determines expected inflation as in equation (17). Equation (22) with $\lambda = 0$ has only one endogenous variable, revisions in expectations about current inflation. A decrease in expected future surpluses must create an upward revision in inflationary expectations. Actual inflation is given by its prior expectation, determined by monetary policy, plus the current revision in expectations, determined by fiscal policy. The monetary authority has no tools which it can use to offset the innovation in inflation.

3.2 Monetary Policy with Long-term Bonds

With long-term bonds, inflation becomes more interesting. Now, the monetary authority can use unexpected changes in interest rates, conditional on innovations in fundamentals, to offset inflationary shocks, at the cost of future inflation.

Determination of the optimal path for inflation requires a loss function. In the flexible price, money-in-the-utility function model used here, output is always at full employment, and the optimal rate of inflation sets the nominal interest rate to zero, implying a target inflation rate equal to the negative of the real interest rate. However, empirical evidence on actual policy choice suggests that the target inflation rate is positive. We follow Cochrane (2001) in not explicitly modeling the justification for the positive target, and assume that the
monetary authority seeks to minimize the expected present-value of squared deviations of inflation from a positive target, where that target is the long-run inflation rate about which we have linearized. This objective yields inflation-smoothing as a goal, which Cochrane (2001) notes is necessary to match data.\textsuperscript{15} Additionally, the positive inflation target makes the assumption, of the non-binding zero lower bound on nominal interest rates,\textsuperscript{16} more reasonable.

The loss function can be expressed as:

\begin{equation}
L_t = \frac{1}{2} \left[ \sum_{s=0}^{\infty} \beta^s \hat{\pi}_{t+s}^2 \right].
\end{equation}

The monetary authority derives its optimal rule by minimizing the period $t-1$ expectation of loss at period $t$. This requires that the monetary authority choose its response to disturbances before it observes the disturbances, equivalently that it commit to a rule.

To express the loss function, as dependent on variables under the control of the monetary authority, note that:

\[ \hat{\pi}_{t+s} = E_{t-1} \hat{\pi}_{t+s} + E'_{t} \hat{\pi}_{t+s} + \ldots + E'_{t+s} \hat{\pi}_{t+s} + \ldots. \]

Expectations revisions must have zero means and auto-covariances, implying that the expected loss function can be expressed as:\textsuperscript{17}

\begin{equation}
E_{t-1} L_t = \frac{1}{2} E_{t-1} \left[ \sum_{s=0}^{\infty} \beta^s \left( (E_{t-1} \hat{\pi}_{t+s})^2 + \sum_{h=0}^{s} (E'_{t+h} \hat{\pi}_{t+s})^2 \right) \right].
\end{equation}

\textsuperscript{15} Indeed, avoiding this lower bound can be used as an argument for a positive target. At the end of the paper, we explicitly consider this zero lower bound.

\textsuperscript{16} Indeed, avoiding this lower bound can be used as an argument for a positive target. At the end of the paper, we explicitly consider this zero lower bound.

\textsuperscript{17} The ability to commit prevents the monetary authority from using revisions in inflationary expectations to offset expected deviations of inflation from target.
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Since the monetary authority cannot affect the first term with its choices in period $t$, focus on the second term of the loss function. The first component of this term contains the time $t$ revisions in inflationary expectations and is obtained by setting $h = 0$ to yield:

$$
\frac{1}{2} E_{t-1} \left[ \left( E'_t \tilde{\pi}_t \right)^2 + \beta \left( E'_t \tilde{\pi}_{t+1} \right)^2 + \beta^2 \left( E'_t \tilde{\pi}_{t+2} \right)^2 + \ldots \right]
$$

Define $v_t$ as the time $t$ revision in expectations about current and future shocks to fundamentals according to:

$$
v_t \equiv E_t \left( \hat{\psi}_t + \hat{\gamma}_t \right) + \lambda \sum_{j=0}^{N-2} R(j, N-1) E'_t \tilde{\gamma}_{t+j+1}, \quad (25)
$$

This allows the time $t$ revision in inflationary expectations to be expressed, using the fiscal constraint given in equation (22), as:

$$
E'_t \tilde{\pi}_t = - \left[ v_t + \lambda \sum_{j=0}^{N-2} R(j, N-1) E'_t \tilde{\gamma}_{t+j+1} \right]. \quad (26)
$$

Equation (26) is the constraint on the time path of inflation, imposed by the FTPL. Monetary policy chooses state-contingent revisions in the time path of inflationary expectations, subject to this constraint.

From equation (18), the monetary authority directly affects inflationary expectations with its choice of interest rates. In specifying the monetary rule, we assume that it can directly choose time $t$ revisions in future inflationary expectations, conditional the time $t$ disturbance. Assume that the monetary rule is given by:

$$
E'_t \tilde{\pi}_{t+j+1} = \alpha_{j+1} v_t. \quad (27)
$$
Substituting equation (27) into (26) yields an expression for the expectations revisions for current inflation:

\[ E_t^\pi = - \left[ 1 + \lambda \sum_{j=0}^{N-2} R(j, N-1) \alpha_{j+1} \right] v_t. \]  

(28)

Assuming that the monetary authority will continue to follow the rule given by (27), such that contemporaneous revisions in inflationary expectations are given by equation (26) for all \( t \), the second term of the loss function, given by equation (24), can be expressed as:

\[ \frac{1}{2} \left\{ \left[ 1 + \lambda \sum_{j=0}^{N-2} R(j, N-1) \alpha_{j+1} \right]^2 + \sum_{j=0}^{N-2} \beta^{j+1} \alpha_{j+1}^2 \right\} \left( \sigma_v^2 \right) \left( \frac{1}{1 - \beta} \right), \]  

(29)

where \( \sigma_v^2 \) denotes variance. Note that \( \alpha_{j+1} = 0 \) for \( j > N-2 \) since inflation and interest rate changes beyond the horizon of the long-term bond have no role in offsetting the effect of inflationary shocks on current inflation. Minimizing with respect to \( \alpha_{j+1} \) for \( 0 \leq j \leq N-2 \), yields:

\[ \lambda R(j, N-1) \left[ 1 + \lambda \sum_{j=0}^{N-2} R(j, N-1) \alpha_{j+1} \right] + \beta^{j+1} \alpha_{j+1} = 0, \quad 0 \leq j \leq N-2. \]  

(30)

In general, equation (30) can be used to show that:

\[ \alpha_{j+1} = \alpha_1 \frac{R(j, N-1)}{\beta^j}, \quad 0 \leq j \leq N-2. \]  

(31)

Substituting equation (31) into the first order condition (30) yields a solution for \( \alpha_1 \) as:

\[ \alpha_1 = \frac{-\lambda}{\beta + \lambda^2 \sum_{h=0}^{N-2} \beta^{-h} [R(j, N-1)]^2} \]  

(32)

Finally, substituting \( \alpha_1 \) and \( \alpha_{j+1} \) into equations (27) and (28) yields solutions for optimal
revisions in expectations of inflation according to:\(^18\)

\[
E_t^t \hat{\pi}_t = \left\{ \frac{-\beta}{\beta + \lambda^2 \sum_{h=0}^{N-2} \beta^{-h} [R(h, N - 1)]^2} \right\} v_t
\]

\[
E_t^t \hat{\pi}_{t+j+1} = \left\{ \frac{-\lambda \beta^{-j} R(j, N - 1)}{\beta + \lambda^2 \sum_{h=0}^{N-2} \beta^{-h} [R(h, N - 1)]^2} \right\} v_t = \frac{\lambda R(j, N - 1) E_t^t \hat{\pi}_t}{\beta^{j+1}} \quad 0 \leq j \leq N - 2.
\]

Equation (33) implies that under a policy of inflation smoothing, the effect of a shock on current inflation is smaller in an economy with long-term bonds (\(\lambda > 0\)).\(^19\) The monetary authority uses interest rate policy to smooth the inflationary impact of the shock over time. Additionally, equation (34) implies that the effect of the shock on future inflation is declining over time since \(R(j, N - 1) \beta^{-(j+1)}\) is declining in \(j\).\(^20\) This is because nominal interest rate changes in the near future have larger effects on the market value of government debt than interest rate changes in the distant future due to the declining debt structure. It follows that front-loading inflation will satisfy the fiscal constraint with a smaller increase in total inflation.

Note that fiscal policy choice of \(\lambda\) and \(N\) also affects the time path of inflation. If the fiscal authority shares the monetary authority’s objective of minimizing the squared deviation of inflation from target, then it will choose \(\lambda\) and \(N\) to minimize this loss, given

\(^{18}\)With perpetuities, these expressions simplify to: \(E_t^t \hat{\pi}_t = - \left\{ \frac{\beta(1+j)^2-1}{(\beta+\lambda^2)(1+j)^2-1} \right\} v_t \) and \(E_t^t \hat{\pi}_{t+j+1} = \lambda^j \left[ \frac{1}{(1+j)^2} \right] E_t^t \hat{\pi}_t.\)

\(^{19}\)Woodford (1998b) derives this result imposing a Taylor rule with a weak response to inflation, instead of solving for optimal monetary policy under a specific loss function. The results are similar because the Taylor rule with a weak response to inflation is similar to the optimal interest rate rule, as shown below.

\(^{20}\)Note that \( R(j, N - 1) \beta^{-j} = [(1 + \pi)(1 + \gamma)]^j \left[ \frac{(1 + j)^2}{(1+j)^2-1} \right]. \) For positive \((1 + \pi)(1 + \gamma) > 0\) and \((1 + i) > 0\), as assumed, the term is declining in \(j.\)
optimal monetary policy. Substituting equations (32) and (31) for optimal monetary policy into equation (29) yields:

\[-\beta \frac{\beta + \lambda^2 \sum_{h=0}^{N-2} \beta^{-h} [R(h, N-1)]^2}{\beta + \lambda^2 \sum_{h=0}^{N-2} \beta^{-h} [R(h, N-1)]^2} \]
as the expression to be minimized. Comparing this with the expression for the current revision in inflationary expectations from equation (33) reveals that the fiscal authority chooses characteristics of the maturity structure, specifically \( \lambda \) and \( N \), to minimize the impact of shocks on current inflation. The fiscal authority therefore chooses \( N \to \infty \), selecting perpetuities, and chooses \( \lambda \) as large as possible, subject to the constraint that there be sufficient short-term debt for the monetary authority to freely choose the interest rate.\(^{21}\) Therefore, fiscal policy determines the effect of shocks on current inflation, while the monetary authority’s role is to chose the optimal path of deviations of inflation from target over time.

### 3.2.1 Optimal Interest Rate Rule

When the interest rate is the monetary policy instrument, optimal monetary policy sets current and future interest rates to deliver the desired revision in inflationary expectations expressed in equations (33) and (34). Using these equations, together with equation (14), it is possible to write the set of optimal interest rate rules at time \( t \) as:

\[
E_t \hat{\pi}_{t+j} = E_{t-1} \hat{\pi}_{t+j} + \frac{\lambda R(j, N-1)}{\beta^{j+1}} E_t' \hat{\pi}_t + E_t' \hat{\gamma}_{t+j+1} \quad 0 \leq j \leq N - 2.
\]

\(^{21}\)The fact that governments do not generally choose to issue their long-term debt as perpetuities suggests that their loss function includes other arguments.
Consider the optimal response of interest rates to a negative innovation in $\nu_t$, created by a revision in expectations about the present value of future government budget surpluses such that $E_t^{t^*}\hat{\psi}_t < 0$. The FTPL requires that inflation increase on at least one date. We therefore designate the shock as inflationary. For $\lambda = 0$, current inflation must rise to fully offset the shock, and there is no effect on current or future interest rates. This is the implication of the FTPL without long-term bonds. However, when $\lambda > 0$, it is optimal to spread the inflation over time. From equation (33), current inflation will rise less than the shock, and from equation (34), future inflation will also rise, with the increases in inflation decreasing in time. Equation (35) shows that the interest rate policy, which achieves this result, is an increase in current and future interest rates with the magnitude of the increase declining into the future. Therefore, both long and short interest rates rise in response to an inflationary fiscal shock. From the perspective of equation (13), the reduction in the real value of debt, required by the negative fiscal shock, is obtained through an increase in current inflation and a reduction in the price of long-term bonds.

Note that this result potentially explains why short-term interest rates and long-term rates move together. When the monetary authority raises short-term rates in response to an inflationary shock, it is smoothing the inflationary impact of the shock, implying that it will also raise future short-term rates. This implies a current increase in both long and short rates. And it explains interest rate smoothness, whereby the monetary authority does not dramatically change nominal interest rates over time.  

22 It is interesting to compare this to a result in the regime switching model by Davig, Leeper, and Chung (2003), in which they find that an interest rate response to inflation implies a persistent increase in inflation and in interest rates. This paper demonstrates that this is optimal policy in response to an inflationary
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Now, consider the optimal response of monetary policy to a negative revision in expectations about future growth \( \left( E_t \gamma_{t+1} < 0 \right) \), which creates a negative innovation in \( \nu_t \). This is also an inflationary shock, and the optimal time path of expectations revisions is again given by equations (33) and (34). The monetary authority must manipulate current and future interest rates to achieve these revisions in inflationary expectations, given the shock. If the monetary authority were to leave expected future interest rates unchanged, then from equation (14), expected inflation in period \( t + 1 \) would rise, and there would be no effect on current inflation, the price of long-term bonds, or on inflation beyond period \( t+1 \). This is not optimal under the loss function assumed here. The monetary authority wants to smooth the future inflation, implying that it will manipulate interest rates to reduce inflation in period \( t + 1 \), relative to what it would be with no change in interest rates, accepting an increase in inflation on other dates. This requires a reduction in the current interest rate and an increase in future interest rates with the increase declining over time. From the perspective of equation (13), the interest rate changes raise the price of long-term bonds, allowing current inflation to rise. For this type of shock, interest rate smoothing does not occur between the first two periods. Smooth interest rates across initial periods would require current revisions in expectations about future growth to be autocorrelated.

The rule for the current interest rate, equation (35) with \( j = 0 \), has some resemblance to a Taylor Rule. In response to an inflationary shock, specifically a negative innovation in \( \nu_t \), the monetary authority raises the nominal interest rate by \( \lambda/\beta \), which is less than shock. It achieves the objective of smoothing inflation over time.
one, as long as the fraction of government debt held as long-term debt is less than the discount factor. Therefore, the Taylor Principle of responding to inflation with a coefficient greater than one does not emerge.\textsuperscript{23} And in response to a shock which would decrease the output gap (defined as deviation of output from a pre-determined trend), faster growth, the monetary authority also raises the nominal interest rate.\textsuperscript{24}

A distinction between optimal interest rate policy under the fiscal theory and the Taylor rule is that the former specifies a rule for the revision of expectations about current and future interest rates in response to current disturbances. The Taylor rule specifies the actual deviation of the current interest rate from its steady state as a function of current deviations of output and inflationary expectations. Under optimal policy with the fiscal theory, earlier shocks might have increased inflationary expectations for period $t$ relative to their constant-growth equilibrium. However, in the absence of period $t$ shocks, the monetary authority, acting with commitment, will not respond with a change in the interest rate from its prior expectation, even though inflation could be above target. In this framework, the monetary authority plans policy each period for $N - 1$ periods, instead of planning only for the current period.

### 3.2.2 Change in Target Inflation Rate

Now, consider the ability of the monetary authority to reduce inflation to a new lower target rate in the absence of any shocks in the economy. In a fiscal theory world without long-term

\textsuperscript{23}Indeed, as Woodford (1998b) shows, such a policy in a fiscal theory regime implies that absence of a stable equilibrium.

\textsuperscript{24}This is true even though a decrease in the output gap reduces $E_{t}^{'}\pi_{t}$. To prove this, write $E_{t}^{'}\pi_{t}$ as a function of fundamental shocks, including $E_{t}^{'}\gamma_{t+1}$ and differentiate.
government bonds, a permanent reduction in the nominal interest rate will reduce the rate of
growth of nominal government debt, permanently reducing the rate of inflation in the future
with no consequences for current inflation.

The policy becomes more complicated in a fiscal theory regime in which there are long-
term government bonds. Assuming no real shocks in the economy, using equation (22), the
fiscal constraint can be expressed as:

$$E_t^0 \hat{\pi}_t = -\lambda \sum_{j=0}^{N-2} R(j, N-1) E_t^0 \hat{\pi}_{t+j+1},$$

where inflation deviations are about the original target. In the absence of demand or supply
shocks, upward revisions in expectations about current inflation must be offset by weighted
downward revisions in expectations about future inflation. It is not possible for the monetary
authority to reduce the inflation rate in some periods without raising it in others. A reduction
in current inflation requires a reduction in the price of long-term bonds, created by an increase
in future interest rates and future inflation, to satisfy the fiscal constraint. Alternatively, a
reduction in future inflation requires a reduction in future interest rates, raising the price of
long-term bonds, requiring an increase in current inflation to satisfy the fiscal constraint.

A successful reduction in current and future inflation must be accompanied by some
additional change. One possibility is fiscal reform whereby the present-discounted value of
future surpluses rises. Another would be an increase in the rate of technological progress,
that is, fortuitous supply shocks. Perhaps it is not a coincidence that many countries in
the world succeeded in reducing inflation over a decade characterized by an acceleration in
growth.
Equivalently, monetary policy alone cannot increase both current and future inflation. This is particularly interesting when a country begins to experience deflation and encounters the zero lower bound on the nominal interest rate. At the zero lower bound, a decrease in the current interest rate is not feasible. And an increase in the interest rate will raise future inflation, but only at the cost of even larger current deflation. Monetary policy affects the timing of inflation, but it cannot move current and future inflation in the same direction without fundamental change.

4 Conclusion

This paper considers the roles of monetary policy and fiscal policy in determining inflation in a regime in which the “Fiscal Theory of the Price Level” applies and in which the government issues long-term nominal debt. Monetary policy determines expected inflation through its choice of the nominal interest, as is usual in monetary models. When the government issues long-term bonds, monetary and fiscal policy together determine deviations of current inflation from target and revisions in expectations about future inflation. In the presence of a shock, such as a reduction in the expected present-value of future surpluses, the monetary authority cannot eliminate the implied inflation, but it can choose when to accept it. Using a flexible-price, money-in-the-utility-function model, we find that when the monetary authority wants to smooth inflation around a target, it will react both to inflationary shocks and to shocks raising expected future output above trend by raising the nominal interest rate. This policy has some resemblance to a Taylor Rule, which has been suggested as optimal policy in
the traditional sticky-price paradigm, except that the interest rate is reacting to exogenous shocks instead of to endogenous inflation and output gaps. Additionally, the optimal policy specifies a reaction of the path of future interest rates and expected inflation to exogenous shocks.

This paper advances over Cochrane’s (2001) determination of optimal inflation policy under the FTPL by explicitly introducing monetary policy. Although fiscal policy can be specified such that it fully determines the time path of inflation in a cashless economy, it is also possible to specify monetary and fiscal policy, more reflective of current institutions, in which monetary policy retains a substantive role in managing inflationary shocks. The monetary authority is assumed to manage short-term debt through open-market operations to maximize an objective function, which includes an inflation target. Under this assumption, the role of monetary policy in a fiscal theory regime is to determine the timing of the inflationary response to both fiscal and other inflationary shocks. State-contingent interest rate rules, which minimize the variance of inflation around a target, have some resemblance to a Taylor Rule.
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