1. **Two-period life cycle model.** An individual’s lifetime utility function is given by $U = \log(c_1) + \beta \log(c_2)$. The individual has initial real wealth equal to $20,000$, current real income equal to $90,000$, and future real income equal to $110,000$. The real interest rate $r$ is 10%, and the discount rate $\beta$ is $1/1.1$.

a. **Find the individual’s present value of lifetime resources ($PVLR$).**

$$PVLR = 20,000 + 90,000 + \frac{110,000}{1.1} = 210,000.$$

b. **How much will the individual save and consume in each period?**

We use two conditions to find the optimal level of consumption:

- $PVLC = PVLR$
- $u'(c_1) = (1+r) \beta u'(c_2)$

In our problem, these two conditions are:

- $c_1 + \frac{c_2}{1.1} = 210,000$
- $\frac{1}{c_1} = 1.1 \frac{1}{1.1 c_2} \Rightarrow c_1 = c_2$

Using the fact that consumption is the same in both periods, we can use the first condition to solve for optimal consumption in the first and second periods.

$$c_1^* + \frac{c_2^*}{1.1} = 210,000$$

$$c_1^* \left( 1 + \frac{1}{1.1} \right) = 210,000$$

$$c_1^* = c_2^* = 110,000$$

We can solve for saving by subtracting consumption from income in each period:

$$s_1^* = y_1 - c_1^* = 90,000 - 110,000 = -20,000$$

$$s_2^* = y_2 - c_2^* = 110,000 - 110,000 = 0$$

c. **Depict the optimal consumption and saving decisions on a graph with second-period variables (consumption and income) on the y-axis.** Make sure to label the intercepts and slope.
d. How will saving and consumption change if current income rises by $11,000?
If current income rises by $11,000, the present value of lifetime resources will rise by $11,000.

\[
\Delta c_i^* = \Delta c_2^* = $115,762 - $110,000 = $5,762
\]

\[
\Delta s_i^* = (\Delta y_i - \Delta c_i^*) = $11,000 - $5,762 = $5,238
\]

\[
\Delta s_2^* = (\Delta y_2 - \Delta c_2^*) = $0 - $5,762 = -$5,762
\]

e. How will saving and consumption change if future income rises by $11,000?
The increase in future income increases the PVLR by $11,000/1.1.

\[
\Delta c_i^* = \Delta c_2^* = $115,238 - $110,000 = $5,238
\]

\[
\Delta s_i^* = (\Delta y_i - \Delta c_i^*) = $0 - $5,238 = -$5,238
\]

\[
\Delta s_2^* = (\Delta y_2 - \Delta c_2^*) = $11,000 - $5,238 = -$5,762
\]

f. How will saving and consumption change if wealth rises by $11,000?
An increase in wealth increases the \( PVLR \) by the same amount as part (c), but there is no change in income, so saving falls in both periods by the drop in consumption.

\[
\begin{align*}
\frac{c_i^* + c_i^*}{1.1} &= 210,000 + 11,000 \\
c_i^* (1 + \frac{1}{1.1}) &= 221,000 \\
\Delta c_i^* = \Delta c_2^* &= 115,762 - 110,000 = 5,762 \\
\Delta s_i^* &= (\Delta y_1 - \Delta c_i^*) = 0 - 5,762 = -5,762 \\
\Delta s_2^* &= (\Delta y_2 - \Delta c_2^*) = 0 - 5,762 = -5,762
\end{align*}
\]

2. Assume that the capital share is 1/4, and that next year there will be 150 million workers and TFP will be 1,500. The effective tax rate on capital in the U.S. is about 25%. Suppose capital depreciates at a rate of 20% per year and the price of capital is $1 in 1992 dollars. Banks charge an average interest rate of 10% and inflation is expected to be 3%.

\[\begin{align*}
\text{a. What is the desired level of the capital stock?} \\
MPK' &= \alpha AK^{\alpha-1} N^{1-\alpha} \\
&= (1/4)l,500 K^{-3/4} (150 \times 10^6)^{3/4} \\
&= \frac{1}{4} l,500 \left( \frac{K}{150 \times 10^6} \right)^{-3/4} \\
K^* &= 150 \times 10^6 \left( \frac{1 \times (0.2 + 0.07)}{1 - 0.25} \times \frac{4}{1,500} \right)^{-4/3} \approx 1,584 \text{ billion}
\end{align*}\]

\[\begin{align*}
\text{b. If TFP rises to 2,000, what is the change to the desired level of capital?} \\
K^* &= 150 \times 10^6 \left( \frac{1 \times (0.2 + 0.07)}{1 - 0.25} \times \frac{4}{2,000} \right)^{-4/3} \approx 2,324 \text{ billion} \\
\Delta K^* &= 2,324 \text{ billion} - 1,584 \text{ billion} = 740 \text{ billion}
\end{align*}\]

\[\begin{align*}
\text{c. If the tax rate on capital is removed, what is the change to the desired level of capital?} \\
K^* &= 150 \times 10^6 \left( \frac{1 \times (0.2 + 0.07) \times 4}{1,500} \right)^{-4/3} \approx 2,324 \text{ billion} \\
\Delta K^* &= 2,324 \text{ billion} - 1,584 \text{ billion} = 740 \text{ billion}
\end{align*}\]
d. Explain why the user cost of capital does not include the total price of a new unit of capital.
The user cost of capital does not include the total price since the capital can be resold. We are interested in the flow or annual cost of capital.

3. Use the saving-investment diagram to analyze the effects of the following on national saving, investment, and the real interest rate. Explain your reasoning.

a. Consumers become more impatient.
This changes the slope of the indifference curve map such that consumers want more current consumption and less future consumption, reducing current saving. National saving shifts left, and the interest rate rises.

b. The government reduces current taxes and current spending by equal amounts.
National saving is the sum of private and public saving. There is no effect on government saving. Households raise savings because they want to spread the tax cut over current and future consumption. Alternatively, viewing national savings as \( S = Y - C - G \), we find that the fall in \( G \) raises saving while the increase in \( C \) reduces saving. However, since \( C \) rises by less than the tax cut, equivalently the increase in government spending, the rise in \( C \) is less than the fall in \( G \) so that national saving rises. The equilibrium interest rate falls.

c. An improvement in computer chip design increases the future MPK of computers and increases expected future income.
Investment demand shifts to the right immediately, and national saving shifts to the left as consumption rises in expectation of a higher PVLR. Eventually, saving will rise as the higher future income is realized (national saving is output less consumption and government spending). The interest rate immediately rises and falls as incomes rise.

d. The stock market posts record gains.
There are two effects. The first is an increase in consumer wealth. Consumers will spend some of the increase in wealth, raising consumption. With income constant and consumption up, saving falls. Second, the record gains raise Tobin’s q, inducing firms to acquire more capital, that is, to invest more. This shifts investment to the right. With higher investment demand and lower demand for saving, the interest rate rises.
4. Abel & Bernanke Numerical Problem # 6

\[ GDP = Y = \$1,000,000 = \text{total production of coconuts} \]
\[ GNP = \$1,025,000 = \text{production of coconuts} + \text{net factor income from abroad} \]
\[ NFP = \$25,000 \]
\[ I = \$0 \]
\[ S = Y + NFP - C - G = \$1,000,000 + \$25,000 - \$1,025,000 = \$0 \]
\[ CA = S - I = \$0 \]
\[ NX = CA - NFP = \$0 - \$25,000 = -\$25,000 \]
\[ KFA = -CA = \$0 \]

The $500,000 in foreign bonds provides income of $500,000 \times 0.05 = \$25,000 per year. Since people consume exactly $1,025,000, they must be using the $25,000 of foreign interest receipts to buy imported consumption goods. So net exports are –$25,000 and the current account balance is zero. Since the current account balance is zero, the capital and financial account balance is also zero.

5. Abel & Bernanke Analytical Problem # 3

In Figure 5.3, before the capital controls are imposed, the home country has a current account deficit of the amount \( CA \), while the foreign country has a matching current account surplus. The effect of the capital controls is to make saving equal investment in each country. In the home country, the real interest rate rises, investment declines, saving increases, and the current account balance increases to zero. The world real interest rate (the interest rate in the foreign country) declines.

Figure 5.3
6. Abel & Bernanke Analytical Problem # 5

(a) The home country’s saving curve shifts to the right, from $S^1$ to $S^2$ in Figure 5.5. The real world interest rate falls, so that the current account surplus in the home country equals the current account deficit in the foreign country. From Figure 5.5, $S$ rises, $I$ rises, $CA$ rises, $r^w$ falls.

![Figure 5.5](image-url)
(b) The foreign country’s saving curve shifts to the right, from $S^1_{For}$ to $S^2_{For}$ in Figure 5.6. The real world interest rate must fall, so the current account surplus in the foreign country equals the current account deficit in the home country. As shown in the figure, $S$ falls, $I$ rises, $CA$ falls, $r^w$ falls.

![Figure 5.6](image)

(c) The foreign country’s saving curve shifts to the left, from $S^1_{For}$ to $S^2_{For}$ in Figure 5.7. The real world interest rate must rise, so the current account deficit in the foreign country equals the current account surplus in the home country. As shown in the figure, $S$ rises, $I$ falls, $CA$ rises, $r^w$ rises.

![Figure 5.7](image)

(d) If Ricardian equivalence holds, there is no effect. If Ricardian equivalence does not hold, then the result is the same as in part (b), as the foreign country’s saving curve shifts to the right.
The sum of exports and imports (instead of their difference) is often used to measure the share of trade in GDP. Note a strong increase over time as countries have reduced barriers to trade. Trade was only about 10% of GDP in 1960 and had risen to over 25% in 2005.