Implications of Productive Government Spending for Fiscal Policy

Betty C. Daniel Si Gao
Department of Economics,
University at Albany, SUNY*
March 2015

Abstract

The standard assumption in macroeconomics that government spending is unproductive can have substantive implications for tax and spending policy. Productive government spending introduces a positive feedback between the tax rate, the productive capacity of the economy, and tax revenue. We allow marginal tax revenue to be optimally allocated between productive subsidies to human capital and utility-enhancing government consumption and calculate Laffer Curves for the US. Productive government spending yields higher revenue-maximizing tax rates, steeper slopes at low tax rates and higher peaks. The differences are particularly pronounced for the labor-tax Laffer curve. The use of tax revenue is an important determinant of the actual revenue that a tax rate increase generates.

JEL Classification: E62,H20,H52

Keywords: Fiscal policy; Laffer curve; Welfare; Human capital

*The authors thank John Jones, two anonymous referees, and seminar participants at UAlbany for helpful comments on an earlier draft.
1 Introduction

A major role of government is to provide public goods, some of which enhance the productivity of the economy. Examples include the Eisenhower interstate highway system in the US, the extensive rail system in Europe, public education, government-funded research, among other projects. Yet, a standard simplifying assumption in macroeconomics is that government spending is unproductive. An even more extreme but common assumption is that government spending is entirely purposeless with purchases thrown into the ocean or tax revenue redistributed back to the same representative agent who paid it. These standard assumptions eliminate any positive direct effects of government spending on the economy. Those direct effects, however, must be the purpose of the spending and the reason for which the spending is undertaken.

This paper focuses on the importance of including the purpose of government spending when trying to understand the effects of an increase in distortionary tax rates on output, total tax revenue, and welfare. We define productive government spending as spending which raises output per worker. When a distortionary tax increase finances productive government spending, it can increase the productivity of the economy. This increased productivity offsets some of the distortion from the increased tax rate, mitigating the output and welfare loss from the tax rate increase. When the spending is not productive, these effects are absent. Baxter and King (1993) wrote an early paper in which they demonstrated substantial differences in fiscal multipliers under distortionary taxation for the cases of productive and non-productive government spending. We follow their lead and compare Laffer curves and welfare with and without productive government spending. With productive spending, revenue-maximizing tax rates are higher. Additionally, the slope of the Laffer curve at low tax rates and its peak are higher. The net welfare impact of a distortionary tax increase also depends on the type of spending financed.

We model productive government spending as subsidies to education, essentially subsidies to private investment in human capital. Following literature initiated by Lucas (1988)’s endogenous growth model, we provide a role for government subsidies to education by assuming that human capital has an externality in production, inducing the private market to provide too little of it. However, in the presence of distortionary labor
taxation, the government would optimally choose to provide subsidies even in the absence of the externality in order to offset some of the distortion created by the labor tax.

We provide a calibrated model to show that when the government allocates marginal tax revenue optimally between utility-enhancing spending on a public good and subsidies to investment in productive human capital, compared with allocating all marginal revenue to the public consumption good, the difference in the shape of the Laffer curve is economically significant. Revenue maximizing tax rates rise from 0.65 to 0.70 for the labor tax rate and from 0.67 to 0.73 for the capital tax. Additionally, Laffer curve peaks are higher and the slope at low tax rates is steeper. The peak of the labor-tax Laffer curve with optimal allocation provides an additional 71% in revenues compared with an additional 49% with full allocation to government consumption. The capital tax Laffer curve is much flatter and corresponding numbers are 12% and 8%. These numbers imply that optimal allocation of tax revenues yields additional revenue at the peak between 45% and 50% higher than possible with full allocation to the public consumption good. Our focus on a single type of productive government spending ignores other types of productive spending. This implies that our results yield a lower bound on the government’s ability to raise tax revenue with an increase in the tax rate when some marginal tax revenue is allocated toward productive use.

Our assumption that marginal tax revenues are allocated toward welfare-enhancing uses allows us to meaningfully compute optimal labor and capital tax rates, conditional on exogenous government transfers. We find that the optimal capital tax should be zero, as in the optimal tax literature, and that the optimal labor tax is higher than its current rate.

These issues are particularly relevant in current budgetary environments, where countries are facing difficult choices over spending cuts and tax increases needed to achieve long-run fiscal sustainability. Education expenditures have been widely targeted for cuts. Our analysis demonstrates that cuts to productive government spending are considerably less effective in achieving fiscal sustainability than cuts to utility-enhancing spending since the former will reduce the long-run productive capacity of the economy. This does not mean that all spending cuts should be to utility-enhancing spending because choices should be guided by welfare, not by maximizing tax revenues. However, in comparing
costs and benefits of alternative spending cuts, their differing effects on marginal tax revenues should be included in both the budget-balancing and welfare calculations.

Our paper is related to recent papers by Trabandt and Uhlig (2006, 2011, 2012) (TU), which compute steady-state Laffer curves by calibrating the steady state of an exogenous neoclassical growth model with capital and labor as inputs. We calibrate the Laffer curve to the steady state of a growth model, but make significant significant departures.

The first departure is that we ask a different question. TU asks how far various economies are from the peaks of their Laffer curves. Their answer is that some countries are close. Their paper does not have productive government spending. We ask how the addition of productive government spending alters the shape of the Laffer curve. We want information about how the addition of productive government spending changes the slope of the Laffer curve at tax rates below the peak, equivalently information on the effectiveness of a tax rate increase in raising tax revenue. And we want information about the position of the peak which implies a revenue maximizing tax rate.

We compute Laffer curves in two variants of our model, where the variants differ only in whether or not government spending can be productive. We find that productive government spending increases the slope of the Laffer curve for tax rates below the peak, and increases the revenue-maximizing tax rate. The steeper slopes imply that our ability to raise tax revenues with an increase in tax rates is greater than we might have thought from a Laffer curve which omits productive government spending. Additionally, the higher revenue-maximizing tax rates imply that productive government spending could change the answer to the TU question, implying that countries with productive government spending are further from their peaks than they would be in its absence.

The second departure from TU arises over the need to add productive government spending to our model. Given the large fraction of government spending on education, and work initiated by Lucas (1988) on the productivity of human capital, we model productive government spending as subsidies to education expenditures. This requires that we build a model of human capital accumulation in which expenditures on education serve as a factor of production in human capital. Our model of human capital accumulation differs from that in TU due to the need to allow education expenditures to be a factor of production.
The third departure we make is to change how the government uses the marginal tax revenue due to an increase in the tax rate. In TU, all marginal tax revenue is redistributed as lump-sum transfers. Prescott (2002) has shown that allocation of marginal tax revenue to redistribution, compared with purposeless spending, eliminates the wealth effect of any tax increase, thereby sharpening the labor supply reduction in response to a tax increase. The implication for the Laffer curve is that lump-sum redistribution reduces the slope of the Laffer curve at tax rates below the peak and moves its peak left. And it implies that the welfare effects of any tax increase are negative.

In contrast, we require that marginal tax revenue be allocated optimally between two uses: (1) utility-enhancing government spending, and (2) productive expenditures on education. This use of marginal tax revenue allows the welfare effects of a tax increase the potential to be positive, but the assumption serves another important role. To compute the Laffer curve, we must take a stand on how much of the marginal tax revenue is allocated to productive government spending relative to an alternative use. Given our assumption that both types of government spending potentially increase welfare, we can allow the allocation of marginal tax revenue between the two uses to be optimal.

In summary, our paper changes the TU model sufficiently to allow us to consider how adding productive expenditure on education changes the shape of the Laffer curve, compared to a benchmark in which all spending is for utility-enhancing government spending. Our paper contributes to the literature demonstrating the importance of the purpose of government spending for the effect of an increase in the tax rate on steady-state output and tax revenues. The shape of the Laffer curve is not robust to alternative uses of marginal tax revenue.

The paper is organized as follows. We present the model in section 2. Section 3 discusses the calibration and parameterizations. Section 4 presents the results. Section 5 contains conclusions.
2 The Model

2.1 General Assumptions

In this section we specify and solve for the balanced growth path in an exogenous growth model and compute Laffer curves along the balanced growth path, following Trabandt and Uhlig (2011). We do not consider the transition to the balanced growth path or the implied full welfare effects of a policy change. Therefore, our policy implications are long-run implications only.

2.1.1 Firm

Following both Mankiw, Romer, and Weil (1992) and Trabandt and Uhlig (2011), we use an exogenous growth model, and following Mankiw, Romer, and Weil (1992), we add human capital to the production function.\(^1\) The representative firm produces output using a constant-returns-to-scale production function with inputs of physical capital \((K_t)\), human capital \((H_t)\), and labor \((n_{w,t}L_t)\), where \(n_{w,t}\) is hours per worker and \(L_t\) is the number of workers. Technology, denoted by \(z_t\), grows at the exogenous rate \(\xi\), such that \(z_t = \xi t\). We augment the constant-returns-to-scale Mankiw-Romer-Weil production function with an externality in the aggregate level of per-worker human capital \((h_{a,t})\).

Since all representative firms behave identically, aggregate output is given by

\[
Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} (n_{w,t}L_t)^{1-\theta_k-\theta_h} h_{a,t}^\phi, \tag{1}
\]

where we assume that \(\theta_k + \theta_h + \phi < 1\) to ensure a balanced-growth equilibrium.\(^2\)

\(^1\)Trabandt and Uhlig (2011) also consider an endogenous growth model and an exogenous growth model with human capital as a factor of production.

\(^2\)Both the addition of human capital and the externality in human capital add non-standard features to the model. Our specification is equivalent to specifying output as a function of effective labor, defined as a geometric index of human capital and hours by workers. Letting \(Q_t^{1-\theta_k}\) denote effective labor,

\[
Q_t^{1-\theta_k} = H_t^{\theta_h} (n_{w,t}L_t)^{1-\theta_k-\theta_h},
\]

yields an expression for output as

\[
Y_t = z_t K_t^{\theta_k} Q_t^{1-\theta_k} h_{a,t}^\phi.
\]

A Cobb-Douglas specification in capital and effective labor \((H_t n_{w,t}L_t)\) would yield an endogenous growth model since the returns to reproducible factors would equal unity. Considering the externality, note that even if education externalities are local, they would still be present in the aggregate. We add a sensitivity analysis at the end, omitting the externality.
Letting small letters denote per-worker values, the per-worker value of aggregate output can be expressed as
\[ y_t = z_t h_t^\theta_k h_t^\theta_h (n_{w,t})^{1-\theta_k-\theta_h} h_{w,t}^\phi, \]  
where \( h_t = h_{a,t} \) in equilibrium.

The representative firm maximizes profit by solving the following problem
\[
\max_{k_t, n_{w,t}, h_t} y_t - d_t k_t - w_t n_{w,t} - w_{h,t} h_t,
\]
where \( d_t \) is dividends, \( w_t \) is the wage paid per hour of work, and \( w_{h,t} \) is the wage paid to the value of human capital per worker. In choosing human capital, the firm ignores the effect of his choice for human capital on aggregate human capital, implying that the firm chooses too little human capital.

At the optimum, returns to capital (\( d_t \)) and wages to both hours devoted to work (\( w_t \)) and to human capital (\( w_{h,t} \)) can be expressed, respectively, as
\[
d_t = \theta_k \frac{y_t}{k_t}, \quad w_t = (1 - \theta_k - \theta_h) \frac{y_t}{n_{w,t}}, \quad w_{h,t} = \theta_h \frac{y_t}{h_t}.
\]
The externality in human capital serves to augment payments to all factors in the same way that technology does, such that firm profits, after payment to all factors, are zero,
\[
\Pi_t = y_t - d_t k_t - w_t n_{w,t} - w_{h,t} h_t = 0.
\]

### 2.1.2 Household

The representative infinitely-lived household maximizes the expected discounted stream of utility. The labor endowment is normalized to unity such that leisure equals one minus hours worked (\( n_{w,t} \)) minus hours spent on education (\( n_{h,t} \)). Utility is separable in consumption (\( c_t \)), leisure (\( 1 - n_{w,t} - n_{h,t} \)), and utility-enhancing government consumption (\( g_{c,t} \)). For the baseline model, we assume log utility with the objective function expressed
We also do a sensitivity analysis with a smaller Frisch labor supply elasticity than that implied by the Cobb-Douglas specification.

Letting \( \tau \) denote tax rates, the optimization occurs subject to the household’s budget constraint

\[
ct + (1 - s_t)et + it + b_{t+1} = (1 - \tau_{n,t})(wn_{w,t} + wh_{h,t}) + [(1 - \tau_{k,t})(dt - \delta) + \delta] k_t + TR_t + R_{b,t} b_t + mt. \tag{4}
\]

The household uses his after-tax earnings from labor hours \([(1 - \tau_{n,t})(wn_{w,t})]\) and human capital \([(1 - \tau_{n,t})(wh_{h,t})]\), as well as his after-tax earnings on capital \([(1 - \tau_{k,t})(dt - \delta)k_t + \delta k_t]\), plus the sum of government transfer payments \((TR_t)\), gross interest on the risk-free bond \((R_{b,t} b_t)\), and earnings from net foreign assets \((mt)\) to purchase consumption \((ct)\), subsidized education expenses \([(1 - s_t)et]\), investment \((it)\), and risk-free bonds \((b_{t+1})\).

Capital taxes are imposed on dividends net-of-depreciation \((\delta)\) as in Prescott (2002, 2004) and Trabandt and Uhlig (2011). Firm profits are omitted since they are zero.

We follow Trabandt and Uhlig (2011) in introducing \(mt\) to represent earnings from net foreign assets. In a long-run equilibrium the current account is balanced so that net imports equal earnings from net foreign assets. This additional variable is necessary in calibrating the model, since in an open economy, net imports drive a wedge between expenditure by domestic residents and production.

The accumulation equation for physical capital is standard and is given by

\[
k_{t+1} = (1 - \delta) k_t + it. \tag{5}
\]

with capital depreciating at rate \(\delta\).

We specify the human capital accumulation equation with three inputs in the production of human capital, including human capital itself \((ht)\), expenditures on education...
(ε_t), and hours spent on education (n_{h,t}).

\[ h_{t+1} = (1 - \delta_h) h_t + \epsilon_t \omega_e h_t^{\omega_e} (z_t^{1/(1 - \theta_k - \theta_h - \phi)} n_{h,t})^{1 - \omega_e - \omega_h}. \] (6)

To allow for productive government spending on education, we must have education expenditures in the human capital production function. This contrasts with TU who use a learning-by-doing specification. Their human capital production function has inputs of human capital and a linear combination of hours devoted to school and to work. It does not contain education expenditures and so cannot address the issue of how productive government spending on education expenditures could increase human capital. In a sensitivity analysis, we augment their learning-by-doing specification with expenditures on education to consider whether our results are robust to introducing learning-by-doing.

There is no consensus in the literature on the correct specification for the human capital production function in equation (6), with authors including subsets of our three inputs. Lucas (1988) introduces a human-capital-based endogenous growth model in which human capital itself, augmented by hours devoted to human capital accumulation, is the argument in the human capital production. Mankiw, Romer, and Weil (1992) also use a single input but with the assumption that human capital is fully produced by expenditures, measured as education enrollment. Klenow and Rodriguez-Clare (1997), Ben-Porath (1967), Rebelo (1991), Glomm and Ravikumar (1992) and Manuelli, Seshadri, and Shin (2012) include all three inputs.

There is also no consensus on the appropriate returns to scale for the inputs. We assume that the production function for human capital is jointly constant returns to scale in these three factors and consider sensitivity analysis allowing increasing returns. We also assume that a technology growth factor \((z_t^{1/(1 - \theta_k - \theta_h - \phi)})\), based on the same technology growth factor as output, augments hours, allowing a balanced growth path with constant hours, and with expenditures on education and human capital growing at the rate of growth of the economy.\(^3\) Human capital depreciates at rate \(\delta_h\).

The household’s choice variables include \(c_t, i_t, \epsilon_t, b_{t+1}, k_{t+1}, h_{t+1}, n_{w,t},\) and \(n_{h,t}\). The household takes the hourly wage \(w_t\), the human capital wage \(w_{h,t}\), the dividend

\(^3\)We thank John B. Jones for suggesting this specification.
rate \( d_t \), labor and capital tax rates \( \tau_{n,t}, \tau_{k,t} \), the education subsidy rate \( s_t \), government consumption \( g_{c,t} \) and the gross interest rate \( R_{b,t} \) as exogenously given.

Defining \( \lambda_t, \zeta_t, \) and \( \mu_t \) as the Lagrange multipliers on the consumer’s budget constraint (equation 4) and the physical and human capital accumulation equations, (5) and (6), respectively, the first order conditions with respect to \( c_t, i_t \) and \( e_t \), yield definitions of the multipliers according to

\[
\lambda_t = U_c(t),
\]

\[
\lambda_t = \zeta_t,
\]

\[
(1 - s_t)\lambda_t = \mu_t \omega_e e_t^{\omega_e-1} h_t^{\omega_h} \left( \frac{z_{t+1}^{1/(1-\theta_k-\theta_h-\phi)}}{n_{h,t+1}} \right)^{1-\omega_e-\omega_h}.
\]

Equation (7) states that the multiplier on the budget constraint is the marginal utility of consumption. Equation (8) equates the marginal utility of capital with the marginal utility of consumption. The first order conditions with respect to expenditures on physical and human capital, equations (8) and (9), differ due to both the subsidy on education expenditures and to the difference in production functions for human and physical capital. If \( s_t = 0, \omega_e = 1 \) and \( \omega_h = 0 \), the two first order conditions would be identical, and the marginal value of human capital would equal the marginal value of physical capital.

The first order conditions with respect to \( b_{t+1}, k_{t+1} \) and \( h_{t+1} \) yield Euler equations in each of the three assets according to

\[
\lambda_t = \beta E_t \{ \lambda_{t+1} R_{b,t+1} \},
\]

\[
\zeta_t = \beta E_t \{ \lambda_{t+1} [ (1 - \tau_{k,t+1}) (\theta_k \frac{y_{t+1}}{k_{t+1}} - \delta) + \delta] + \zeta_{t+1} (1 - \delta) \},
\]

\[
\mu_t = \beta E_t \{ \lambda_{t+1} [ \theta_h (1 - \tau_{n,t+1}) \frac{y_{t+1}}{h_{t+1}} ] + \mu_{t+1} [ 1 - \delta_h + \omega_h e_t^{\omega_e} h_t^{\omega_h-1} ( \frac{z_{t+1}^{1/(1-\theta_k-\theta_h-\phi)}}{n_{h,t+1}} )^{1-\omega_e-\omega_h} ] \},
\]

where the agent takes \( h_{a,t+1} \) as exogenous in choosing \( h_{t+1} \). The first order condition on bonds is the standard Euler equation. Defining \( R_t \) as

\[
R_t = (1 - \tau_{k,t}) (d_t - \delta) + 1,
\]

we can substitute into equation (11), using equation (8), to write the first order condition
on capital as

\[ \lambda_t = \beta E_t \{ \lambda_{t+1} R_{t+1} \}. \]

It is useful to compare equations (11) and (12). The marginal values for physical capital and human capital have similar recursive expressions. Each equals the sum of the expected present value of marginal utility of income from the asset plus the marginal value of human or physical in the next period. The marginal value for human capital investment contains an additional positive term \[ \mu_{t+1} \omega_e \omega_h^{-1} (\zeta_{t+1}^{1/(\theta_k - \theta_h - \phi)} n_{h,t+1})^{1-\omega_e - \omega_h}. \] Since human capital is a factor of production in human capital, an additional unit adds to the productive capacity of human capital, raising the marginal product of human capital.

The first order conditions with respect to the allocation of hours, \( n_{w,t} \) and \( n_{h,t} \), can be expressed as

\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \lambda_t w_t (1 - \tau_{n,t}), \quad \text{(14)} \]

\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \mu_t (1 - \omega_e - \omega_h) \omega_e \omega_h^{-1} z_t^{1/(\theta_k - \theta_h - \phi)} n_{h,t+1}^{1-\omega_e - \omega_h}. \quad \text{(15)} \]

The marginal cost of hours, which is the marginal cost of giving up leisure for both of the two uses of hours, must equal the marginal benefit. For labor hours, the later is determined by the after-tax wage multiplied by the marginal utility of consumption. For school hours, the marginal benefit is the addition to human capital created by the additional study hours multiplied by the marginal value of human capital.

### 2.1.3 Government

We assume that government purchases three types of goods and services: utility-enhancing government consumption \( g_{c,t} \), education subsidies \( g_{h,t} \), and something exogenous \( g_t \). Both types of endogenous government spending are potentially welfare-improving. Government consumption raises utility. Government subsidies to education raise investment in human capital, thereby raising the productive capacity of the economy. The government budget constraint requires that total expenditures on goods and services plus transfer payments and interest on government debt equal tax revenue plus
new government debt. The budget constraint is given by

\[ g_{c,t} + g_{h,t} + g_t + TR_t + R_{b,t}b_t = T_t + b_{t+1}, \]  

where government spending on education \((g_{h,t})\) is given by

\[ g_{h,t} = e_t s_t, \]

and tax revenues \((T_t)\) are

\[ T_t = \tau_{n,t}(w_t n_{w,t} + w_{h,t} h_t) + \tau_{k,t}(d_t - \delta) k_t. \] 

We assume that exogenous government spending, transfer payments \((TR_t)\), and government debt \((b_t)\) all grow exogenously. Our assumptions on exogeneity are based on the following. We assume that the only reason government raises taxes is to provide benefits to agents who are being taxed. However, a representative-agent model does not capture some of these benefits very well. We set these benefits as exogenous. The primary purpose of government transfers is redistribution. In a representative agent model, it would never be optimal to raise distortionary taxes for the purpose of redistribution, implying that optimal transfer payments are zero. Since endogenous determination of optimal transfer payments in a representative agent model is meaningless, we let these payments be exogenous. Additionally, government defense spending is set to yield a desired level of security in response to the level of threat in the world. It is arguable that defense spending adjusts to changes in threat level, leaving the security level and therefore utility constant. Since this is not a model about optimal reaction to security threats, we set defense spending as exogenous. Finally, we set government debt as exogenous because government debt patterns are far from those predicted by the optimal tax literature.

Government subsidies to education \((s_t)\) and utility-enhancing government consumption \((g_{c,t})\) are chosen jointly optimally for given tax rates \((\tau_{k,t}, \tau_{n,t})\) to balance the government budget. This assumption differs from the standard in which either transfer payments or government spending, which can be worthless or utility-enhancing and separable, adjusts to balance the government budget.
We characterize the government’s optimal allocation problem by implicitly solving the system for the endogenous variables, derived from the agents’ optimization problems, as a function of the policy variables, \((s_t, g_{c,t})\). For any given tax rates, the benevolent government chooses the policy variables to maximize the consumer’s indirect utility, given by

\[
V(s_t, g_{c,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \{ \log c_t(s_t, g_{c,t}) + \alpha_n \log(1 - n_{w,t}(s_t, g_{c,t}) - n_{h,t}(s_t, g_{c,t})) + \alpha_g \log g_{c,t} \},
\]

subject to the constraints in the economy

\[
g_{c,t} + g_{h,t}(s_t, g_{c,t}) + g_t + R_b t + TR_t = b_{t+1}(s_t, g_{c,t}) + \tau_{n,t} (1 - \theta_k) y_t(s_t, g_{c,t}) + \tau_k \theta_k y_t(s_t, g_{c,t}) - \tau_k \delta k_t, \]

\[
k_{t+1}(s_t, g_{c,t}) = (1 - \delta) k_t + i_t(s_t, g_{c,t}), \]

\[
h_{t+1}(s_t, g_{c,t}) = (1 - \delta_h) h_t + e_t(s_t, g_{c,t}) + \omega_h h_t \omega_h n_{h,t}(s_t, g_{c,t})^{1 - \omega_h - \omega_k}, \]

\[
y_t(s_t, g_{c,t}) + m_t = c_t(s_t, g_{c,t}) + i_t(s_t, g_{c,t}) + e_t(s_t, g_{c,t}) + g_t + g_{c,t}, \]

\[
g_{h,t}(s_t, g_{c,t}) = e_t(s_t, g_{c,t}) s_t. \]

### 2.2 Stationary Equilibrium

Along the balanced growth path, \(n_{w,t}, n_{h,t}, R_t, R_b, \tau_{n,t}, \tau_k, \delta\) and \(s_t\) are stationary, and all other variables grow at the constant rate \(\psi = \xi^{1/(1 - \theta_k - \theta_h - \phi)}\). We require that government bonds, the exogenous component of government spending, transfer payments, and income from net foreign assets all grow exogenously at the rate of growth of the economy. In order to obtain stationary solutions, we detrend all the growing variables before solving the system. We denote detrended growing variables with tildes. Non-growing variables are not detrended and retain their original notation.

All variables, which grow at the rate of growth of the economy, are detrended by dividing by \(\psi^t\) to yield \(\tilde{x}_t = x_t/\psi^t\), where the tilde denotes detrended values. Lagrange multipliers grow at a different rate. Multiplying both sides of equation (7) by \(\psi^t\) implies that \(\tilde{\lambda}_t = 1/\tilde{c}_t = \psi^t \lambda_t\). Similarly, equations (8) and (9) imply that detrended multipliers

\(^4\text{We infer concavity from our numerical calculations.}\)
are expressed as $\hat{\zeta}_t = \psi^t \zeta_t$ and $\hat{\mu}_t = \psi^t \mu_t$. Therefore all the Lagrange multipliers grow at rate $\psi^{-1}$ along their balanced growth paths. Equations for the detrended stationary equilibrium are provided in Appendix A.

A competitive equilibrium is a set of plans $\{\tilde{c}_t, \tilde{e}_t, \tilde{b}_t, \tilde{\dot{h}}_{t+1}, \tilde{\dot{h}}_{t+1}, n_{w,t}, n_{h,t}\}$ satisfying the detrended equations of the model, given exogenous processes $\{\tau_{n,t}, \tau_{k,t}, s_t, T\tilde{R}_t, \tilde{b}_t, \tilde{m}_t, \tilde{g}_{h,t}, \tilde{g}_{c,t}, \tilde{g}_t\}$ and the initial condition $\{k_0, b_0, h_0, m_0, g_0, T\tilde{R}_0\}$.

2.3 Steady State

In the steady state, $\tilde{x}_{t+1} = \tilde{x}_t$. We denote steady state values of the variables by dropping time subscripts to yield $\tilde{x} = \tilde{x}_{t+1} = \tilde{x}_t$. The steady state production function is given by

$$\tilde{y} = \tilde{k}^{\theta_k} \tilde{h}^{\theta_h} \nu^{1-\theta_k-\theta_h},$$

(18)

where $\tilde{h}_t = \tilde{h}_{a,t}$ in equilibrium. The steady-state equation for the accumulation of physical capital becomes

$$(\psi - 1 + \delta) \frac{\tilde{k}}{\tilde{y}} = \frac{\tilde{i}}{\tilde{y}},$$

(19)

The steady state Euler equation, with capital as the asset is given by

$$\psi/\beta - 1 = (1 - \tau_k)(\theta_k \frac{\tilde{y}}{\tilde{k}} - \delta).$$

(20)

These two equations link the net-of-tax return to the capital stock and physical investment. Combining the two equations yields the steady state share of investment as a percentage of output

$$\frac{\tilde{i}}{\tilde{y}} = \frac{(\psi - 1 + \delta)(1 - \tau_k)\theta_k}{\psi/\beta - 1 + \delta(1 - \tau_k)}.$$ 

(21)

The investment-to-output ratio is increasing in capital’s share ($\theta_k$) and decreasing in the capital tax rate ($\tau_k$).

Next, we derive the steady state ratios of hours allocated to education relative to work ($n_h/n_w$) and education expenditures relative to output ($\tilde{e}/\tilde{y}$). Letting $\tilde{y}_h$ denote the detrended production of human capital, its long run value is given by the detrended
steady state of equation (6) as

$$\tilde{y}_h = (\psi - 1 + \delta_h)\tilde{h}. \quad (22)$$

The detrended steady state of equation (9) relates the multipliers on the budget constraint and on human capital accumulation according to

$$(1 - s)\tilde{\lambda} = \frac{\omega_e \tilde{y}_h}{\tilde{e}} \frac{\omega_e (\psi - 1 + \delta_h)}{\tilde{e}} \tilde{\mu}. \quad (22a)$$

The left-hand side is the marginal cost of education expenditures $(1 - s)$, multiplied by their marginal value ($\tilde{\lambda}$), while the right-hand side is the marginal benefit of the expenditures, given by the addition to human capital that the expenditures create $(\omega_e \tilde{y}_h/\tilde{e})$ multiplied by their marginal value ($\tilde{\mu}$). We can use this equation to solve for the ratio of the multipliers, the relative marginal values of the production of goods and of human capital as

$$\frac{\tilde{\lambda}}{\tilde{\mu}} = \frac{\omega_e (\psi - 1 + \delta_h)\tilde{h}}{(1 - s)\tilde{e}}.$$

The relative marginal values depend positively on human capital and negatively on the household’s net-of-subsidy expenditures on education.

We can solve for the ratio of education expenditures to output by taking the detrended steady state of equation (12), using the expression for $\tilde{y}_h/\tilde{h}$ in equation (22). The steady state of equation (12) can be expressed as

$$\psi = \beta \left[ \frac{\tilde{\lambda}}{\tilde{\mu}} (1 - \tau_n) \frac{\tilde{y}}{\tilde{h}} + 1 - \delta_h + \omega_h (\psi - 1 + \delta_h) \right].$$

Substituting for $\tilde{\lambda}/\tilde{\mu}$ yields a solution for the value of education expenditures relative to output as

$$\frac{\tilde{e}}{\tilde{y}} = \frac{(1 - \tau_n)}{(1 - s)} \left[ \frac{(\psi - 1 + \delta_h)\omega_e \theta_h}{\psi/\beta - 1 + \delta_h - \omega_h (\psi - 1 + \delta_h)} \right]. \quad (23)$$

Note that the ratio of education expenditures to output in the long run depends on the ratio of values net of labor taxes and net of education subsidies $(1 - \tau_n)$. Higher subsidies and lower taxes raise education expenditures relative to output. If the subsidy rate and the labor tax rate are identical, their effect on education expenditures cancels out.
To solve for the ratio of work hours to school hours, use detrended steady states of equations (14) and (15), to equate the marginal returns to work hours with the marginal returns to school hours

\[ \tilde{\lambda}(1 - \theta_k - \theta_h)(1 - \tau_n) \frac{\tilde{y}}{n_w} = \tilde{\mu}(1 - \omega_e - \omega_h) \frac{\tilde{y}_h}{n_h}. \]  

(24)

Substituting \( \tilde{\lambda}/\tilde{\mu} \) and using the steady state expression for \( \tilde{y}_h/\tilde{h} \), the ratio of hours devoted to school relative to work becomes

\[ \frac{n_h}{n_w} = \frac{(1 - s)}{(1 - \tau_n)} \left[ (1 - \omega_e - \omega_h) \right] \frac{\tilde{e}}{\tilde{y}}. \]  

(25)

Substituting for education expenditures relative to output implies that the ratio of schooling hours to work is independent of subsidies and taxes according to

\[ \frac{n_h}{n_w} = \frac{\theta_h}{1 - \theta_k - \theta_h} \left[ \frac{(\psi - 1 + \delta_h)(1 - \omega_e - \omega_h)}{[\psi/\beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta_h)]} \right]. \]  

(26)

The ratio raises education expenditures relative to output in the exact way that it reduces school hours relative to work hours, for given education expenditures relative to output, such that the terms cancel.

The ratio of capital to output is derived by solving equation (20) to yield

\[ \frac{\tilde{k}}{\tilde{y}} = \frac{(1 - \tau_k)\theta_k}{\psi/\beta - 1 + \delta(1 - \tau_k)}. \]  

(27)

The capital output ratio is a negative function of the capital-tax rate.

The steady state value of human capital relative to output can be written as

\[ (\psi - 1 + \delta_h) \frac{\tilde{h}}{\tilde{y}} = \left( \frac{\tilde{e}}{\tilde{y}} \right) \omega_e \left( \frac{\tilde{h}}{\tilde{y}} \right) \omega_h \left( \frac{n_h}{\tilde{y}} \right)^{1-\omega_e-\omega_h}. \]  

(28)

The consumption-to-GDP ratio can be solved from the equation for the consumer’s
budget constraint in steady state, equation (4), yielding

\[
\bar{c} = \frac{(1 - \tau_n)(1 - \theta_k) + (1 - \tau_k)\theta_k + TR + \bar{m} + \psi \left( \frac{1 - \delta}{\beta} \right) b}{y} - (1 - s) \frac{\bar{e}}{y} - [\psi - 1 + \delta(1 - \tau_k)] \frac{\bar{k}}{y}.
\] (29)

Consumption relative to output depends on tax rates but not on subsidy rates.\(^5\) Therefore, subsidies affect consumption only to the extent that they affect output.

Hours allocated to work and schooling, respectively, in the steady state can be solved by combining the steady state versions of equations (14) and (15)

\[
n_w = \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h + \frac{\alpha_n}{1 - \tau_n} \frac{\bar{c}}{y} + \frac{(1 - s)}{(1 - \tau_n)} \frac{\bar{e}}{\omega_e} - \phi)} \cdot \frac{(\bar{h} + \phi)}{(1 - \theta_k - \theta_h)} n_w \frac{(1 - \theta_k - \theta_h)}{1 - \theta_k - \theta_h - \phi}.
\] (30)

\[
n_h = \frac{(1 - s) \frac{\bar{e}}{(1 - \tau_n) y} - \phi}{(1 - \theta_k - \theta_h + \frac{\alpha_n}{1 - \tau_n} \frac{\bar{c}}{y} + \frac{(1 - s)}{(1 - \tau_n)} \frac{\bar{e}}{\omega_e} - \phi)} \cdot \frac{(\bar{h} + \phi)}{(1 - \theta_k - \theta_h)} n_w \frac{(1 - \theta_k - \theta_h)}{1 - \theta_k - \theta_h - \phi}.
\] (31)

Subsidies do not distort the decision on hours since, from equation (29), they do not affect consumption relative to disposable income \((\frac{\bar{c}}{(1 - \tau_n) y})\), nor from equation (23), the value for \((\frac{1 - s}{(1 - \tau_n) y})\). Tax rates do distort the decision on hours through their effect on consumption relative to disposable income. An increase in the labor tax rate reduces hours spent working and in school.

Given solutions for physical and human capital, each relative to output, and for labor hours in work and in school, detrended output in the steady state can be expressed as

\[
\bar{y} = \frac{\bar{h}}{y} \frac{\theta_h}{(1 - \theta_k - \theta_h - \phi)} \frac{\bar{h}}{y} \left( \frac{\theta_h + \phi}{(1 - \theta_k - \theta_h)} n_w \frac{(1 - \theta_k - \theta_h)}{1 - \theta_k - \theta_h - \phi} \right).
\] (32)

The expression for \(\bar{y}\) can be solved by substituting for \(\bar{h}/\bar{y}\) from equation (28) to yield detrended output as a function of the capital-to-output ratio, the education-to-output ratio, and labor hours in work and in school.\(^6\) The labor tax rate reduces steady-state detrended output by reducing labor hours in work and school, and the education-to-
output ratio. The capital tax rate reduces the capital-output ratio, thereby reducing steady-state detrended output. In contrast, an increase in the subsidy rate raises the education to output ratio, raising steady-state detrended output.

The education subsidy in the steady state can be written as

$$\tilde{g}_h = \tilde{e}s. \quad (33)$$

Substituting into equation (17), detrended long-run tax revenues can be expressed as

$$\tilde{T} = \left[ \tau_n (1 - \theta_k) + \tau_k \theta_k \left( \frac{\psi/\beta - 1}{\psi/\beta - 1 + \delta (1 - \tau_k)} \right) \right] \tilde{y}. \quad (34)$$

To understand the effect of using marginal tax revenue partially for education subsidies, differentiate tax revenues with respect to the labor tax to yield

$$\frac{\partial \tilde{T}}{\partial \tau_n} = (1 - \theta_k) \tilde{y} + \tau_n (1 - \theta_k) \left( \frac{\partial \tilde{y}}{\partial \tau_n} + \frac{\partial \tilde{y}}{\partial s} \frac{\partial \tilde{T}}{\partial \tau_n} \frac{\partial \tilde{T}}{\partial \tau_n} \right).$$

Solving for the effect of the tax rate increase on tax revenue yields

$$\frac{\partial \tilde{T}}{\partial \tau_n} = \frac{(1 - \theta_k) (\tilde{y} + \tau_n \frac{\partial \tilde{y}}{\partial \tau_n})}{1 - \tau_n (1 - \theta_k) \frac{\partial \tilde{y}}{\partial s} \frac{\partial \tilde{T}}{\partial \tau_n}}.$$

The first term in the numerator is positive while the second is negative. The peak of the Laffer curve occurs at the tax rate which equates the two. Since for a given tax rate, output with subsidies is higher than output without and the fall in output due to a labor tax increase is lower, the Laffer curve peak with subsidies is to the right of the peak without. Additionally, effect of the subsidy on the slope of the Laffer curve is larger the larger the share of output affected by the tax, for the labor-tax rate, \((1 - \theta_k)\). The analogous expression for the capital tax is

$$\frac{\partial \tilde{T}}{\partial \tau_k} = \theta_k \left[ \frac{1 + \frac{\tau_k \delta (\psi/\beta - 1)}{[\psi/\beta - 1 + \delta (1 - \tau_k)]^2} \tilde{y} + \tau_k \frac{\partial \tilde{y}}{\partial s}}{1 - \tau_k \theta_k \frac{\partial \tilde{y}}{\partial s} \frac{\partial \tilde{T}}{\partial \tau_k}} \right].$$

The effect of the subsidy on the slope of the capital-tax Laffer curve is smaller since capital has a smaller share.
2.4 Distortionary Taxes, Externalities, and Subsidies

To facilitate understanding of the welfare implications of distortionary taxes, externalities and subsidies, we provide a solution to the planner’s problem in Appendix B. We denote steady-state values for the planner’s detrended optimal values with hats ($\hat{x}$). The planner’s optimal choice for $k/y$, $e/y$, $n_h/n_w$, $c/y$, $n_h$ and $n_w$ can be expressed as

$$\frac{\hat{k}}{\hat{y}} = \frac{\theta_k}{\psi/\beta - 1 + \delta}$$  \hspace{1cm} (35)

$$\frac{\hat{e}}{\hat{y}} = \frac{(\psi - 1 + \delta_h)\omega_e(\theta_h + \phi)}{[\psi/\beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta)]}$$  \hspace{1cm} (36)

$$\frac{n_h}{n_w} = \frac{1 - \omega_e - \omega_h}{\omega_e(1 - \theta_k - \theta_h)}$$  \hspace{1cm} (37)

$$\frac{\hat{c}}{\hat{y}} = \frac{1}{1 + \alpha_c}\left[1 + \tilde{m} - \tilde{g} - \frac{\hat{e}}{\hat{y}}(\psi - 1 + \delta)\frac{\hat{k}}{\hat{y}}\right]$$  \hspace{1cm} (38)

$$n_w = \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h) + \alpha_n} + \frac{1 - \omega_e - \omega_h}{\omega_e}$$  \hspace{1cm} (39)

$$n_h = \frac{1 - \omega_e - \omega_h}{\omega_e} + \frac{1 - \omega_e - \omega_h}{\omega_e}$$  \hspace{1cm} (40)

where hours are understood to be the planner’s choices and are not given hat notation since they are not detrended. We retain the tilde notation for the detrended values of the exogenous variables since these values are unchanged from the decentralized optimum to the planner’s problem.

The equilibrium in the decentralized economy has two types of distortions relative to the central planner equilibrium, those due to distortionary capital and labor tax rates as well as the subsidy rate, and those due to the education externality. Comparison of equations (27) and (35) demonstrates a reduction in the capital-output ratio with the capital tax. A comparison of equations (14) and (15) with equations (39) and (40) reveals that hours in both work and school depend on consumption relative to income net of labor taxes in the decentralized equilibrium and on consumption relative to income in the planning economy. Since consumption relative to income in the planning equilibrium is most likely to be higher than consumption relative to income net of labor taxes in
the decentralized equilibrium,\footnote{We cannot make a full comparison of the values of labor hours in the two different types of equilibrium since our expressions for the consumption-output ratios in the two equilibria do not allow direct comparisons. We rely on numerical simulations to confirm the apparent effects of the labor tax.} distortionary labor taxes tend to reduce labor and school hours. Additionally, equations (23) and (36) show that labor taxes also reduce education expenditures relative to output unless they are offset by education subsidies.

Now, consider the effects of the externality. From equations (23) and (36), the share of expenditures on education is proportional to its optimal value according to

\[
\frac{\hat{e}}{\hat{y}} = \frac{1 - \tau_n}{1 - s} \left[ \frac{\theta_h}{(\theta_h + \phi)} \right] \frac{\hat{e}}{\hat{y}}.
\]

When subsidies and taxes are zero, the externality implies that agents choose too little expenditure on education. For a given tax rate, the subsidy can be chosen to offset this externality exactly. However, distortionary taxation implies that agents choose too little capital and too few hours for work. These choices yield too little output. The subsidy cannot offset these distortionary effects on hours and physical capital, but it can offset some of the distortionary effect on output of too few labor hours and too little capital by increasing human capital. Therefore, it is unlikely that the optimal subsidy in the presence of distortionary taxation is that which exactly offsets the externality.

### 3 Calibration and Parameterization

The model is calibrated to annual data for the US economy. We follow Trabandt and Uhlig (2011) and choose the sample as 1995 to 2007. The data are from Federal Reserve Economic Database, the World Bank Database, the National Income and Product Accounts (NIPA), and the Digest of Education Statistics as described in Appendix C.

For tax rates, we use flat taxes, calculated using the methodology proposed by Mendoza, Razin, and Tesar (1994), instead of marginal tax rates. Calculations by Trabandt and Uhlig (2011) yield average capital income tax and labor income tax rates of $\tau_k = 0.36$, $\tau_n = 0.28$, respectively.

The exogenous balanced growth factor ($\psi$) is set to 1.03, corresponding to the annual real GDP growth rate of 3% per capita over this period. For the interest rate on bonds,
we average the annual real interest rate over 1995-2007 from the world bank database to yield $R_b = 1.048$. $\beta$ is calibrated from the steady state of equation (10) to yield $\beta = \psi / R_b = 0.9828$.

The capital depreciation rate is calibrated from the capital production function in steady state, equation (19), where capital and investment both include private and public values from NIPA. Substituting values into equation (19) yields a value for depreciation as $\delta = 0.04$.

We assume $\delta_h = \delta$, consistent with Mankiw, Romer, and Weil (1992) and Trabandt and Uhlig (2011). The implied value for human capital depreciation is within the range discussed in Stokey and Rebelo (1995).

The parameter on capital in the production function ($\theta_k$) is calibrated from equation (20), which can be rewritten as

$$\theta_k = \frac{\psi / \beta - 1 + \delta (1 - \tau_k) \bar{k}}{1 - \tau_k} \bar{y}$$

(41)

where $\bar{k} / \bar{y}$ is calibrated from NIPA data, averaged over the period. Substituting for values of parameters on the right hand-side yields $\theta_k = 0.33$.

Following Prescott (2002, 2004), McGrattan and Rogerson (2004), and Trabandt and Uhlig (2011), hours worked $n_w$ is calibrated at 0.25, consistent with evidence on hours worked per person aged 15-64 for the US. Schooling hours $n_h$ is calibrated to 0.06, approximately a quarter of $n_w$, following Trabandt and Uhlig (2011).

We split government spending into an exogenous component ($\tilde{g}$), spending on education ($\tilde{g}_h$), and a component which yields utility ($\tilde{g}_c$). We let defense spending be the exogenous component and calibrate $\tilde{g} / \bar{y}$ to be the average of defense spending over the sample, yielding 4.3% of GDP. $\tilde{g}_h / \bar{y}$ is the average public spending on education, or 5.2% of GDP. From equation (33), the subsidy rate ($s$) is government education expenditures relative to total expenditures by educational institutions ($\tilde{g}_h / \tilde{c}$). We measure total spending on education as a fraction of GDP ($\tilde{c} / \bar{y}$) with data on expenditures of educational institutions as a percent of GDP, yielding 7.2%. Together, these data yield a measure

---

Stokey and Rebelo (1995) take the labor force participation rate into account, thereby reducing the average time spent on work per person to 16 to 19 hours per week, 10% to 12% of total time endowment. We consider 12% as an alternative calibration for $n_w$ and find similar changes in the shape of the Laffer curve due to the addition of productive government spending.

---

8Stokey and Rebelo (1995) take the labor force participation rate into account, thereby reducing the average time spent on work per person to 16 to 19 hours per week, 10% to 12% of total time endowment. We consider 12% as an alternative calibration for $n_w$ and find similar changes in the shape of the Laffer curve due to the addition of productive government spending.
of the subsidy rate as 72%. $\tilde{g}_c/\tilde{y}$ corresponds to total government consumption expenditures, net of government education and defense expenditures, as a percent of GDP, yielding 5.7% of GDP.

Government debt relative to GDP ($\tilde{b}$) is calibrated as 38.2% of GDP, using federal debt held by the public. We set $\tilde{TR}/\tilde{y} = 10.7\%$ which corresponds to the ‘implicit’ government transfer payments to GDP ratio in the data.\(^9\)

The ratio of net imports to GDP ($\tilde{m}/\tilde{y}$) is calibrated as the average value of net imports relative to GDP, yielding 3.8%. The investment-to-GDP ratio ($\tilde{i}/\tilde{y}$) is calibrated using aggregate investment relative to GDP, yielding 19.7%. The consumption-to-GDP ratio ($\tilde{c}/\tilde{y}$) is set to match the average ratio of private consumption to GDP in the data.\(^10\)

We can find no literature that provides directly relevant methods for calibrating the parameters $\theta_h$ and $\phi$ in the human-capital production function. Related literature includes Mankiw, Romer, and Weil (1992), who estimate a constant returns-to-scale production function with human capital in a cross-country regression. They find that the coefficient on human capital is 0.28. Their estimate has been criticized for failing to account for country-specific effects. OLS estimates are biased when unobserved effects are correlated with the right-hand-side variables, in this case, the factors of production. Islam (1995) has argued that the extent of the bias could be large. Lucas (1988) proposed a different production technology and estimated the coefficient on labor hours multiplied by human capital as 0.75 and the externality term as 0.417. However, since the model framework is different, it has limited relevance to our calibration.

Gao (2014) constructed a dynamic panel framework to capture the unobserved country-effects. After accounting for the unobserved country effects, the estimated coefficient on human capital is reduced from 0.28 in Mankiw, Romer, and Weil (1992) to 0.23. This finding is in line with other empirical results which find that the coefficient on human

\(^9\)We follow Trabandt and Uhlig (2011) and compute the government transfer-payment-to-income ratio that is consistent with the model using the steady state of government budget constraint (16),

$$ TR/\tilde{y} = \tilde{T}/\tilde{y} - (R_b - \psi)b/\tilde{y} - \tilde{g}_c/\tilde{y} - \tilde{g}_x/\tilde{y} - \tilde{g}_b/\tilde{y}. $$

We subtract government interest payments, government consumption, defense expenses, and public spending on education as a percentage of GDP in the data from the model-predicted tax revenue-to-income ratio in equation (34).

\(^10\)Private spending on education has been deducted from private consumption expenditures. It is calculated as the difference between total education expenditures and public spending on education.
capital is much lower than one-third.\textsuperscript{11} Additionally, since there is an externality in this model, the total coefficient on human capital is $\theta_h + \phi$ from equation (18) corresponds to the empirical estimate of human capital coefficient in Gao (2014). Therefore, we calibrate $\theta_h + \phi = 0.23$ and use this estimate in the benchmark analysis.\textsuperscript{12}

To calibrate parameter values for factor shares in the production function for human capital, contained in equation (6), we use evidence provided by Kendrick (1976). He finds that the technology for producing human capital is intensive in labor. Approximately 50\% of investment in human capital in the US represents the opportunity cost of student time, with the remaining 50\% composed of human capital and physical resources. Hence, we follow this estimation and set the coefficient on school hours $(1 - \omega_e - \omega_h)$ to 0.5.

Given this condition, together with equations (23) (30) and (31), and the restriction $\theta_h + \phi = 0.23$, there are five equations with five unknown parameters $\{\omega_e, \omega_h, \theta_h, \phi, \alpha_n\}$. Solving this system of equations yields the benchmark calibration: $\omega_e = 0.12$, $\omega_h = 0.38$, $\theta_h = 0.20$, $\phi = 0.03$ and $\alpha_n = 1.41$. We conduct sensitivity analysis for alternative values of $\{\omega_e, \omega_h\}$ and $\{\theta_h, \phi\}$.

The final parameter to calibrate is the coefficient on government consumption in the utility function ($\alpha_g$). We do not have a closed-form solution for the steady-state ratio of government consumption to income from the government’s optimal allocation problem. Therefore, conditional on values for other baseline calibrations and the benchmark tax rates, we numerically compute the ratio obtained from the optimal allocation of government expenditure between government consumption and education subsidies for alternative values of $\alpha_g$. We choose the value of $\alpha_g$ for which the ratio most closely matches the value in the data of 5.7\%. This yields a value of $\alpha_g = 0.06$, which we use in the benchmark calibration.

The benchmark calibration for the model is summarized in Tables 1 and 2. This calibration matches the moments in the data perfectly.

\textsuperscript{11}For example, see Islam (1995) and Caselli, Esquivel, and Lefort (1996).
\textsuperscript{12}The estimation is based on a sample of 89 countries excluding major oil-producers over 1970-2010. As a robustness check, we adopted the same methodology and estimated the coefficients over 1995-2007. The coefficients are almost identical. In particular, the estimated coefficient on human capital is 0.24.
Table 1: Part 1 of the Baseline Calibration, Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>1.03</td>
<td>Exogenous growth rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9828</td>
<td>Subjective discount rate</td>
<td>$\beta = \psi / R_b$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>Depreciation rate for capital</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.04</td>
<td>Depreciation rate for human capital</td>
<td>$\delta_h = \delta$</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>0.33</td>
<td>Parameter on $k_t$ in production</td>
<td>Equation (41)</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.20</td>
<td>Parameter on $h_t$ in production</td>
<td>Equations (23) (30) and (31)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.03</td>
<td>Parameter on $h_{a,t}$ in production</td>
<td>Literature</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>0.12</td>
<td>Parameter on $e_t$</td>
<td>Equations (23) (30) and (31)</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>0.38</td>
<td>Parameter on $h_t$</td>
<td>Literature</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>1.41</td>
<td>Parameter on leisure in utility</td>
<td>Equations (23) (30) and (31)</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>0.06</td>
<td>Parameter on $g_{c,t}$ in utility</td>
<td>Data</td>
</tr>
</tbody>
</table>

Table 2: Part 2 of the Baseline Calibration, Policy Rates and Equilibrium Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_n$</td>
<td>0.28</td>
<td>Labor Tax Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.36</td>
<td>Capital Tax Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$n_w$</td>
<td>0.25</td>
<td>Hours Worked</td>
<td>Literature</td>
</tr>
<tr>
<td>$n_h$</td>
<td>0.06</td>
<td>Hours on Education</td>
<td>Literature</td>
</tr>
<tr>
<td>$R_b$</td>
<td>1.048</td>
<td>Real Interest Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{g}/\tilde{y}$</td>
<td>4.3%</td>
<td>Defense Expenses to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{g}_h/\tilde{y}$</td>
<td>5.2%</td>
<td>Public Spending on Education to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{g}_c/\tilde{y}$</td>
<td>5.7%</td>
<td>Government Non-defense Consumption to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{c}/\tilde{y}$</td>
<td>7.2%</td>
<td>Education Expenses to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$s$</td>
<td>0.72</td>
<td>Subsidy Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{b}/\tilde{y}$</td>
<td>38.2%</td>
<td>Debt to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{T}/\tilde{y}$</td>
<td>10.7%</td>
<td>Government Transfers to GDP</td>
<td>Implicit</td>
</tr>
<tr>
<td>$\tilde{m}/\tilde{y}$</td>
<td>3.8%</td>
<td>Net Imports to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{i}/\tilde{y}$</td>
<td>19.7%</td>
<td>Investment to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{c}/\tilde{y}$</td>
<td>66.9%</td>
<td>Private Consumption to GDP</td>
<td>Data</td>
</tr>
</tbody>
</table>

4 Results

4.1 Steady State Laffer Curves

Laffer curves are obtained by varying the steady state labor (capital) tax rates, while holding the capital (labor) tax rate and other parameters fixed. We characterize the Laffer curves for labor taxes and for capital taxes under two different assumptions about
the use of marginal tax revenue. In one case, all marginal tax revenue is allocated to utility-enhancing government spending. This is the case with no productive government spending. In the other case, marginal tax revenue is optimally allocated between utility-enhancing government spending and subsidies to education, which represents productive government spending. We do not have an analytical solution for the optimal allocation. Therefore, we use a grid search to compare alternative allocations numerically and select the one with highest utility for the representative agent.

We demonstrate that the Laffer curves with the optimal allocation have a substantially different shape from the Laffer curve where marginal tax revenue is not used for productive spending. The Laffer curve with productive spending has a steeper slope at tax rates below the peak. Additionally, the peak is higher and occurs at a higher tax rate.

4.1.1 Labor Taxes

Figure 1 compares Laffer curves for labor income tax rates under the two alternative allocations of marginal tax revenue. The dotted curve is the benchmark Laffer curve with the subsidy rate fixed at its benchmark rate of 0.72 and all marginal tax revenue allocated to government consumption. The solid curve is the Laffer curve with marginal tax revenue optimally allocated between productive government spending and consumption. On the vertical axis, tax revenues at the benchmark tax rate of $\tau_n = 0.28$ are set equal to 100.

Figure 1 illustrates that the Laffer curve with marginal tax revenue optimally allocated between productive spending and consumption has a substantial increase in slope for low values of the tax rate and an increase in the peak compared with the benchmark Laffer curve. The peak of the Laffer curve with optimal allocation occurs with tax revenue equal to 171%. Comparing this to initial tax revenues of 100% implies the peak of the Laffer curve with optimal allocation yields an additional 71% in tax revenues, while the peak yields only 49% more tax receipts with marginal revenue allocated toward government consumption. Under optimal allocation, the peak occurs at the labor tax rate equal to 0.70, implying that substantial increases in the tax rate would continue to yield increases in tax revenues; when the subsidy rate is fixed at the benchmark, the peak occurs at a labor tax rate of 0.65. We are further left of the Laffer curve peak when the government optimally allocates some marginal tax revenue to productive expenditures on education.
Figure 2 graphs the optimal subsidy rate and expenditures on human capital as a function of the tax rate. As the tax rate rises, the optimal subsidy rate for education expenditures initially increases rapidly. This implies that expenditures on human capital are increasing rapidly in the tax rate, peaking at an additional 159%. When the subsidy rate is fixed, expenditures on human capital decline in the tax rate since production is falling in the tax rate.

Figure 3 graphs output as a function of the labor tax rate. Under optimal allocation of marginal tax revenue, production increases as the labor tax rate rises from a low rate, eventually peaking and falling. Under full allocation of marginal revenue to government spending, production falls immediately as the tax rate increases, illustrating the importance of the allocation of marginal tax revenue.

Figure 4 illustrates the effect of increases in the labor tax rate on human and physical capital stocks and on hours allocated to schooling and to work. Human and physical capital stocks both have hump shapes with optimal revenue allocation, while both decline in the tax rate with full allocation to government consumption. Since physical capital is complementary to human capital, physical capital mimics the hump shape for human capital. The peak for human capital is about 7% higher than its current value while the peak in physical capital is only about 0.1% higher. Hours allocated toward either non-leisure activity are downward-sloping in the tax rate and are not affected by the allocation of tax revenue. The higher labor tax rate discourages hours allocated to both work and school because the ultimate return on both depends on the after-tax wage.

Finally, Figure 5 graphs the effect of an increasing labor tax on human capital subsidies and on government consumption. As the labor tax rate rises, both the value of
subsidies to human capital and expenditures on utility-enhancing government spending exhibit a hump shape under optimal allocation. Under full allocation to utility-enhancing government consumption, government subsidies fall as the tax rate falls due to the output response to tax rates. Government consumption exhibits a hump shape with a higher peak.

[Figure 5 inserted here]

4.1.2 Capital Taxes

The Laffer curves for capital taxes are graphed in Figure 6. The capital-tax Laffer curve with the optimal allocation of spending has a less pronounced increase in slope at lower tax rates and a smaller increase in peak compared with the labor-tax curve. This is due to the fact that capital has a smaller share in output than the combination of labor hours and human capital. With optimal allocation of marginal tax revenue, the government could raise an additional 12% in tax revenues by increasing capital taxes, while the peak yields only 8% more tax receipts with marginal revenue allocated toward government consumption. Under optimal allocation, the peak occurs at a capital tax rate equal to 0.67, compared with a capital rate of 0.63 at the peak with the subsidy rate fixed at the benchmark.

[Figure 6 inserted here]

Figure 7 illustrates effects of the increase in the capital tax rate on the optimal subsidy rate and on education expenditures. As the capital tax rate increases, the government optimally raises the subsidy rate, similar to the case with labor taxes.

[Figure 7 inserted here]

Figure 8 illustrates the effect of an increase in the capital tax rate on production. There is a slight hump shape with a peak at a tax rate lower than the current rate. Figure 9 reveals that human and physical capital initially move in opposite directions in response to an increase in the capital tax rate. Human capital increases initially, responding to the higher subsidy rate, while physical capital responds negatively to the reduced return.
implied by the higher capital tax rate. The response of hours is independent of the allocation of marginal tax revenue, as for the case with labor taxes.

\[\text{Figure 8 inserted here}\]
\[\text{Figure 9 inserted here}\]

Figure 10 illustrates additional government expenditures on education and on government consumption allowed by the increase in the tax rate. Both are hump-shaped with optimal allocation, while government expenditures on education fall continuously with allocation only to government consumption. The pattern is similar to that for labor taxes.

\[\text{Figure 10 inserted here}\]

Finally, Figure 11 depicts the Laffer curve ‘surface’ along both the capital and labor tax rate dimensions. The curve is steeper along the labor tax direction; after the peak has been reached, the curve falls sharply in almost any direction. The peak of the Laffer curve surface occurs at $\tau_n = 0.69$ and $\tau_k = 0.44$. With a labor tax rate of 0.28 and a capital tax rate of 0.36, the US is well below the peak of the Laffer curve surface. An additional 73% in tax revenues can be achieved at the peak.

\[\text{Figure 11 inserted here}\]

4.2 Sensitivity Analysis

4.2.1 Human Capital Externalities

There is no consensus among economist about either the existence or magnitude of human capital externalities (Moretti (2004b)). Therefore, we conduct sensitivity analysis allowing for different magnitudes for the externality.

Rauch (1993) argues for a larger human capital externality and estimates a log wage equation including average schooling in cities and own schooling, in addition to other terms. He finds that coefficient on average schooling in cities is 0.033 while the coefficient on own education is 0.048.
We cannot directly use his estimates since there are different conditioning variables in his equation and ours. Defining the total labor income as $W_t \equiv w_t n_{w,t} + w_{h,t} h_t$, our wage equation in logarithms can be expressed as

$$\log W_t = \log(1 - \theta_k) + \log z_t + \theta_k \log k_t + \theta_h \log h_t + (1 - \theta_k - \theta_h) \log n_{w,t} + \phi \log h_{a,t}. \quad (42)$$

We use the Rauch (1993) estimates to reflect the relative magnitudes of the effects of average and own education on wages, subject to the constraint that the sum of the coefficients equals 0.23. Rauch’s estimates imply that the relative strength of externalities is $0.033/0.048$, or roughly 0.69, which corresponds to $\phi/\theta_h$ in equation (42). This ratio implies that $\theta_h = 0.14$ and $\phi = 0.09$. In Lucas (1988), the relative strength of externalities is $0.417/0.7$, approximately 0.60. Ciccone and Peri (2002) and Moretti (2004a), also find significant positive spill-overs with similar magnitudes. Therefore, we use the relative strength of the elasticities from Rauch (1993) as an alternative by considering Laffer curves with $\theta_h = 0.14$, $\phi = 0.09$. To focus on the effect of the externality term alone, we retain benchmark values of all other parameter values.

In contrast, Acemoglu and Angrist (1999) find no significant positive human capital spill-overs. Therefore, we also consider Laffer curves with $\theta_h = 0.23$, $\phi = 0$ with all other parameters at benchmark values.

Figures 12 and 13 depict labor-tax and capital-tax Laffer curves with alternative values for the externality parameter $\phi$. The patterns are very similar to those in our original specification. Whether externalities are stronger or weaker compared with our original specification, Laffer curves with optimal allocation exhibit higher peaks that occur at larger tax rates, compared with the case of subsidies fixed at the optimal rate conditional on the current tax rates.\textsuperscript{13} Subsidies to human capital expenditures allow higher tax revenue even in the absence of externalities because they offset some of the effects of distortionary taxation.

\textsuperscript{13}The benchmark subsidy rate is higher with stronger externalities, since the optimal allocation of revenue from existing tax rates allocates more toward education when externalities are higher.
4.2.2 Technology for Human Capital Production

There is no consensus in the literature on the appropriate technology for human capital production. Our calibration assumes constant returns to scale in the three inputs of education expenditure, labor hours, and human capital. As an alternative specification, Gao (2014) estimates an increasing-returns-to-scale human capital production function

\[ h_{t+1} = (1 - \delta_h)h_t + e_t^C(h_t n_{h,t})^{1-\omega}, \]

and concludes that education expenditures explain approximately one-third of human capital production, with the remaining explained by the combination of two other inputs. If we set \( \frac{\omega_i}{1-\omega} = \frac{\omega_h}{\omega_h} \), and impose constant returns to scale, the human capital production function becomes

\[ h_{t+1} = (1 - \delta_h)h_t + e_t^{0.2} h_t^{0.4} n_{h,t}^{0.4}. \]

This finding is similar to Klenow and Rodriguez-Clare (1997), who estimate a constant returns to scale human capital production function based on factor compensation, and find the shares on expenditures, human capital and students’ time are 0.2, 0.3 and 0.5, respectively.

Following the methodology by Gao (2014), we adopt the constant returns to scale technology and reestimate the coefficients using data over 1995-2007. We find that students’ time has a much smaller coefficient than our baseline specification, with \( \omega_e = 0.11 \) and \( \omega_h = 0.61 \). The details are explained in Appendix D.

Therefore, we consider two alternative sets of \( \{\omega_e, \omega_h\} \) based on the above empirical evidence: \( \{\omega_e = 0.20, \omega_h = 0.40\} \) and \( \{\omega_e = 0.11, \omega_h = 0.61\} \). All other parameters remain the same as in the baseline calibration.

We compute Laffer curves for labor and capital taxes with these alternative values for \( \omega_e \) and \( \omega_h \), and graph them in Figures 14 and 15. For comparison, we also graph Laffer curves under the assumption that subsidies are fixed at their benchmark rate, with the benchmark rate set at optimal allocation under the current tax rates. Patterns are similar to our benchmark results. Laffer curves with optimal allocation exhibit higher peaks which occur at larger tax rates than those with subsidies fixed at the optimal rate.
conditional on the current tax rates.

\[ \text{[Figure 14 inserted here]} \]

\[ \text{[Figure 15 inserted here]} \]

4.2.3 Learning-by-doing

Learning-by-doing adds labor hours as an input in the human capital production function. Since an increase in labor taxes reduces labor hours, then the labor tax Laffer curve should be flatter on this account. However, if we also include expenditures on education as an input in the human capital production function, then the Laffer curve should show the same relative slopes with and without productive government spending, as in the earlier analysis. To check the robustness of our results for “learning-by-doing”, we augment the human capital production function in Trabandt and Uhlig (2011) with education expenditures and recompute the Laffer curves.\(^{14}\)

\[
h_{t+1} = (1 - \delta_h)h_t + \epsilon_t^{\omega_e} h_t^{\omega_h} \left[ z_t^{1-\omega_e-\omega_h} (n_{h,t} + q n_{w,t}) \right]^{1-\omega_e-\omega_h}. \quad (43)
\]

The introduction of learning-by-doing changes the first order condition with respect to \(n_{w,t}\) in equation (14)

\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \lambda_t w_t (1 - \tau_{n,t}) + q \mu_t z_t (1 - \omega_e - \omega_h) \epsilon_t^{\omega_e} h_t^{\omega_h} (n_{h,t} + q n_{w,t})^{-\omega_e - \omega_h}. \quad (44)
\]

The additional term in the new Euler equation describes the marginal benefit of producing human capital through learning-by-doing. The steady state of equation (44) can be simplified using equation (9) and written as:

\[
\frac{\alpha_n}{1 - n_{w} - n_{h}} = (1 - \theta_h - \theta_k) \frac{\tilde{y}}{\tilde{c}} \frac{1}{n_{h}} + q (1 - s) \frac{1 - \omega_e - \omega_h}{\omega_e} \frac{\hat{c}/\hat{y}}{\hat{c}/\hat{y} n_{h} + q n_{w}}. \quad (45)
\]

The remaining steady state Euler equations follow those in the baseline model.

\(^{14}\)This specification assumes that education expenditures complement not only schooling time but also labor hours. A function that treats the human capital investment from schooling and from work as separate and additive does not have closed form solutions and might not be compatible with a balanced growth path. We use the present functional form as a robustness check on whether the addition of productive government spending affects the shape of the Laffer curve as we claim, even when the human capital production function contains labor hours.
Calibration of the learning-by-doing model requires knowledge of \( q \). Mincer (1994) estimates that workers obtain an average of five weeks (200 hours) of training (formal and informal, or learning on the job) per year. That implies that approximately 10\% of work time is devoted to skills formation. Therefore, we calibrate \( q = 0.1 \), and solve for the parameters \( \{ \omega_e, \omega_h, \theta_h, \alpha_n \} \) from equations (23) and (45), the steady state of (15), and the restriction \( \omega_e + \omega_h = 0.5 \), so that equilibrium values relative to output, school and labor hours match data. \( \phi \) is assumed to be the same as in the benchmark. This yields \( \{ \omega_e = 0.09, \omega_h = 0.41, \theta_h = 0.26, \alpha_n = 1.35 \} \). The remaining parameters are the same as in the baseline calibration.

Laffer curves for labor and capital taxes with learning-by-doing are computed and shown in Figure 16 and 17. The dotted line is the benchmark Laffer curve with the subsidy fixed. The solid line is the Laffer curve with marginal tax revenue optimally allocated between productive government spending and consumption. The shapes of the Laffer curves are very similar to those in the baseline model – peaks are higher and occur at larger tax rates when marginal tax revenues are optimally allocated. Productive government spending has a significant effect on the shape of Laffer curves, and is robust to whether or not learning-by-doing is included in the human capital production function.

\[ \text{Figure 16 inserted here} \]
\[ \text{Figure 17 inserted here} \]

### 4.2.4 The Constant Frisch Elasticity (CFE) Preferences

We consider an alternative preference specification featuring a lower Frisch elasticity (CFE) of labor supply than implied by the Cobb-Douglas specification, as in Trabandt and Uhlig (2011). We modify our preference specification to be

\[
U(c_t, n_{w,t}, n_{h,t}) = \log c_t - \kappa (n_{w,t} + n_{h,t})^{1+1/\eta} + \alpha_g \log g_{c,t}. \tag{46}
\]

The first order conditions with respect to \( n_{w,t} \) and \( n_{h,t} \) yield

\[
\kappa (1 + 1/\eta) (n_{w,t} + n_{h,t})^{1/\eta} = \lambda_t w_t (1 - \tau_{n,t}), \tag{47}
\]
\[ \kappa(1 + 1/\eta) (n_{w,t} + n_{h,t})^{1/\eta} = \mu t (1 - \omega_e - \omega_h) e^{\omega_e} h^{\omega_h} z^{1 - \eta_k - \eta_h - \theta_e - \theta_h} n_{h,t} - \omega_e - \omega_h. \] (48)

The remaining first order conditions, as well as equations for \( n_h/n_w \) and \( \tilde{e}/\tilde{y} \) are the same as in the baseline model. We calibrate \( \kappa \) so that work and school hours match the observed data, yielding \( \kappa = 3.29 \). We retain the remaining parameters from the benchmark model.

Figure 18 and 19 depict the Laffer curves for labor and capital taxes with CFE preferences. They are similar to our baseline case with a higher labor supply elasticity. For labor taxes, the peak in both cases occurs at a tax rate of 0.72. For the case with the lower labor supply elasticity, the peak generates an additional 88.7% increase in tax revenue compared with a 71% increase in the benchmark case. This is because the lower labor supply elasticity implies that a tax increase creates a smaller reduction in labor supply. Capital-tax Laffer curves are unchanged. Therefore, the effect of productive spending on the shape of the Laffer curve is robust to a lower labor supply elasticity.

\begin{itemize}
  \item [Figure 18 inserted here]
  \item [Figure 19 inserted here]
\end{itemize}

### 4.3 Welfare

In this section, we consider the constrained optimal values for labor and capital tax rates in the steady state. The constraint is that the government must raise taxes through distortionary labor and capital taxes to finance exogenous transfer payments, the exogenous component of government spending, and government debt, all of which are growing at the rate of growth of the economy. From this minimum value for taxes, the government could choose to raise additional tax revenue. If so, we assume that the marginal tax revenue is allocated optimally between utility-enhancing government spending and subsidies to education.

Caveats to our welfare calculations are necessary. Our interest is in understanding how the introduction of productive government spending alters welfare under alternative tax rates. The exogeneity of some expenditures together with the exogeneity of debt implies that we are not calculating fully optimal tax rates, but instead optimal tax rates under the constraints imposed by the exogeneous variables. Additionally, since we make
steady-state calculations only, we are not claiming particular changes in welfare as a result of moving from current tax rates to our constrained optimal tax rates.

First, we present welfare curves, which we define as welfare as a function of the tax rates, under the two alternative assumptions about the allocation of marginal tax revenue. Figure 20 graphs the welfare curve for the labor tax rate when the capital tax rate is fixed at 0.36. Welfare is maximized at a labor tax rate of 0.22, smaller than the current rate of 0.28. Figure 21 shows that welfare is maximized at capital tax rate of 0.10, given the labor tax of 0.28. Note that under optimal allocation of marginal tax revenue, a small change in either tax rate, from an initially low value, creates a very small change in welfare. A reduction in the labor tax rate from 0.28 to its constrained optimal value of 0.22, a 21% decrease in the labor tax rate, increases welfare by only 1.5% in terms of consumption.  

To further understand the welfare implications of labor and capital taxes with productive government spending, we find the combination of labor and capital tax rates that maximizes welfare of the representative agent in the steady state. We emphasize that this is not an optimal taxation calculation because government debt, transfers, and exogenous government spending, all as a fraction of GDP, are held constant. The results are summarized in Table 3. We find that the optimal capital tax is zero, as in the optimal tax literature, and the optimal labor tax is higher at 0.32. Productive government spending does not change the basic result from the optimal tax literature that the capital tax should be zero. The gain in welfare, moving from the current set of taxes to the optimal set is 5% in terms of consumption. At optimal tax rates and optimal allocation, the ratios of aggregates to GDP are lower, except for capital and investment. However, the lower consumption relative to GDP is due to the rise in GDP since consumption actually rises. The remaining aggregates are lower under our constrained optimum.

The last column in Table 3 displays the optimal tax rates and aggregate ratios when the government makes no subsidies to education, equivalently when all government spend-

---

15Following Lucas (1987), the magnitude of welfare costs is measured as the percentage decrease in consumption.
Table 3: Optimal Taxation

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Optimal</th>
<th>Optimal (s = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_k$</td>
<td>0.36</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>0.28</td>
<td>0.32</td>
<td>0.29</td>
</tr>
<tr>
<td>$s$</td>
<td>0.72</td>
<td>0.56</td>
<td>0</td>
</tr>
<tr>
<td>$g_c/y$</td>
<td>5.7%</td>
<td>3.5%</td>
<td>3.5%</td>
</tr>
<tr>
<td>$g_h/y$</td>
<td>5.2%</td>
<td>2.4%</td>
<td>0</td>
</tr>
<tr>
<td>$e/y$</td>
<td>7.2%</td>
<td>4.3%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$i/y$</td>
<td>19.7%</td>
<td>25.8%</td>
<td>25.8%</td>
</tr>
<tr>
<td>$c/y$</td>
<td>66.9%</td>
<td>65.9%</td>
<td>68.2%</td>
</tr>
<tr>
<td>Welfare($\Delta c%)$</td>
<td>-5.1%</td>
<td>0</td>
<td>-1.9%</td>
</tr>
<tr>
<td>Output($y%)$</td>
<td>93.0%</td>
<td>100%</td>
<td>95.8%</td>
</tr>
</tbody>
</table>

ing is non-productive. The optimal labor tax rate falls from 0.32 to 0.29. In the absence of government subsidies to education, total expenditure on education falls sharply to 2.0%, and output falls relative to the that with optimal subsidies. Consumption-to-GDP rises to 68.2% but the level of consumption falls due to the fall in GDP. Welfare is substantially lower with no subsidies to education.

5 Concluding Remarks

We calculate steady-state Laffer curves for the US under the assumption that marginal tax revenue is optimally allocated between utility-enhancing government consumption and productive subsidies to human capital investment. We find that Laffer curves for both labor and capital taxes have steeper slopes at low tax rates and higher peaks, compared with Laffer curves along which all marginal tax revenue is allocated to government consumption. Additionally, revenue-maximizing tax rates are higher. Therefore, if government optimally allocates marginal tax revenue between productive spending and utility-enhancing spending, then a government’s ability to raise taxes is less constrained than in the absence of productive spending.

For labor taxes, these numbers are large. The peak of the labor-tax Laffer curve occurs at an additional 79% in tax revenue. This additional revenue is 45% higher than could be achieved with all marginal spending on public consumption. The revenue-maximizing tax rate is 0.70, compared with a rate of 0.65 in the absence of productive spending. The
Laffer curve for the capital tax is relatively flat. Therefore, even though tax revenues are 50% higher with optimal allocation than with allocation to government consumption, they are only 12% higher than their current level at the peak of the Laffer curve. The revenue-maximizing tax rate with productive government spending is 0.67 compared with 0.63 in its absence.

Sensitivity analysis with varying values for the externality associated with human capital, with learning-by-doing in the human capital production, and with preferences using a lower Frisch elasticity for labor supply, confirm the general results about the changes in the shape of the Laffer curve with productive government spending. And our analysis includes only one type of productive government spending, implying that Laffer curve slopes at low tax rates and peaks could be even higher.

The ability to raise additional tax revenue does not mean that raising tax revenue would increase welfare. Under the assumption that government debt, transfer payments, and defense spending are exogenous, we compute optimal values for the labor and capital tax rates with marginal tax revenue allocated optimally to government consumption and to education subsidies. At the optimum, the government should reduce the capital tax rate to zero, consistent with the optimal tax literature, and raise the labor tax rate to provide subsidies to human capital and utility-enhancing government spending. The inclusion of human capital subsidies substantially raises the optimal labor tax. The optimum labor tax rate is four percentage points higher with productive government spending than without in our calibration, while the optimal capital tax remains at zero.

This analysis highlights the importance of productive government spending in determining both the government’s ability to raise tax revenue and the welfare effects of such behavior. The standard assumption, that government spending is not productive, leads to a smaller government than is optimal when productive government spending is allowed. Additionally, since subsidies to education are only one form of productive government spending, we view our calibrations as a lower bound on the effects of including productive government spending on the Laffer curve and welfare.
References


Figure 1: Laffer Curves for Labor Taxes

Figure 2: Optimal Subsidy Rate - Labor Taxes
Figure 3: Production - Labor Taxes

Figure 4: Effects of Labor Taxes
Figure 5: Government Expenditures - Labor Taxes

Figure 6: Laffer Curves for Capital Taxes
Figure 7: Optimal Subsidy Rate - Capital Taxes

Figure 8: Production - Capital Taxes
Figure 9: Effects of Capital Taxes

Figure 10: Government Expenditures - Capital Taxes
Figure 11: Laffer Curve Surface

Figure 12: Sensitivity Analysis for Externalities: Laffer Curves for Labor Taxes
Figure 13: Sensitivity Analysis for Externalities: Laffer Curves for Capital Taxes

Figure 14: Sensitivity Analysis for Human Capital: Laffer Curves for Labor Taxes

Figure 15: Sensitivity Analysis for Human Capital: Laffer Curves for Capital Taxes
Figure 16: Laffer Curves for Labor Taxes With Learning-by-doing

Figure 17: Laffer Curves for Capital Taxes With Learning-by-doing

Figure 18: Laffer Curves for Labor Taxes With CFE Preferences
Figure 19: Laffer Curves for Capital Taxes With CFE Preferences

Figure 20: Welfare for Labor Taxes

Figure 21: Welfare for Capital Taxes
Appendix A. Stationary Equilibrium

Along a balanced growth path, \( n_{w,t}, n_{h,t}, R_t, R_{b,t}, s_t, \tau_{n,t}, \tau_{k,t} \) and \( s_t \) are constant. All other variables grow at rate

\[
\psi = \xi^{1/(1-\theta_h-\theta_w-\phi)}.
\]

In order to obtain stationary solutions, all the growing variables are detrended. Define the detrended variables along the balanced growth path as \( \tilde{x} = x_t/\psi^t \). Since variables are constant along the balanced growth path, we drop the time subscripts.

In the economy with the representative agent, \( h_{a,t} = h_t \), implying that the aggregate per worker production function is given by

\[
y_t = z_t k_t^{\theta_k} \left( h_{w,t}^{1-\theta_k-\theta_h} h_t^{\theta_h} + \phi \right).
\]

Dividing both sides of the production function by \( z_t = \psi^t \) yields

\[
\tilde{y}_t = \tilde{k}_t^{\theta_k} \left( n_{w,t}^{1-\theta_k-\theta_h} h_t^{\theta_h} + \phi \right).
\]

The detrended consumer budget constraint is obtained by dividing equation (4) by \( \psi^t \) to yield

\[
\tilde{c}_t + (1-s_t)\hat{c}_t + \hat{t}_t + \psi \tilde{b}_{t+1} = (1-\tau_{n,t})(\tilde{w}_t n_{w,t} + w_{h,t} \tilde{h}_t) + [(1-\tau_{k,t})(d_t - \delta) + \delta] \tilde{k}_t + \tilde{T} \tilde{R}_t + R_{b,t} \tilde{b}_t + \tilde{m}_t.
\]

Dividing equations (5), (6), and (4) by \( \psi^t \) yields detrended versions of the physical capital and human capital accumulation equations and the government budget constraint

\[
\psi \tilde{k}_{t+1} = (1-\delta) \tilde{k}_t + \tilde{t}_t,
\]

\[
\psi \tilde{h}_{t+1} = (1-\delta_h) \tilde{h}_t + \tilde{c}_t \tilde{h}_{t+1}^{\omega_h} \left( n_{h,t}^{1-\omega_h-\omega_h} \right),
\]

\[
\hat{g}_t + \hat{g}_{c,t} + \tilde{T} \tilde{R}_t + s_t \hat{e}_t + R_{b,t} \tilde{b}_t = \tilde{T}_t + \psi \tilde{b}_{t+1},
\]

49
with detrended tax revenues from equation (17) given by

\[ \tilde{T}_t = \tau_{n,t} (1 - \theta_k) \tilde{y}_t + \tau_{k,t} \theta_k \tilde{y}_t - \tau_{k,t} \delta \tilde{k}_t. \]  

(53)

Adding the consumer and government budget constraints and imposing equation (3) yields the detrended resource constraint as

\[ \tilde{y}_t + \tilde{m}_t = \tilde{c}_t + \tilde{\iota}_t + \tilde{e}_t + \tilde{g}_{c,t} + \tilde{g}_t. \]  

(54)

Along the balanced growth path, the Lagrange multipliers shrink at the rate of growth. Therefore, define \( \tilde{\lambda}_t, \tilde{\mu}_t \) as the detrended Lagrange multiplier along the balanced growth path, where

\[ \tilde{\lambda}_t = \lambda_t \psi^t, \quad \tilde{\mu}_t = \mu_t \psi^t. \]

Since the detrended multipliers must be constant along the balanced growth path, \( \lambda_{t+1}/\lambda_t = \mu_{t+1}/\mu_t = \psi^{-1} \).

The detrended first order conditions with respect to \( \tilde{c}_t, \tilde{\iota}_t, \tilde{e}_t, \tilde{b}_t+1, \tilde{k}_t+1, \tilde{h}_t+1 \) are derived by multiplying equations (7), (8), (9), (10), (11), and (12) by \( \psi^t \) to yield

\[ \tilde{\lambda}_t = \frac{1}{\tilde{c}_t}, \]  

(55)

\[ \tilde{\lambda}_t = \tilde{\zeta}_t, \]  

(56)

\[ (1 - s_t) \tilde{\lambda}_t = \tilde{\mu}_t \omega_e \tilde{c}_t \tilde{e}_t^{-1} \tilde{h}_t n_{h,t} n_{w,t}^{-\omega_e - \omega_h}, \]  

(57)

\[ \tilde{\lambda}_t = \beta E_t \left\{ \psi^{-1} \tilde{\lambda}_{t+1} R_{b,t+1} \right\}, \]  

(58)

\[ \tilde{\zeta}_t = \beta \psi^{-1} E_t \left\{ \tilde{\lambda}_{t+1} \left[ (1 - \tau_{k,t+1}) \left( \theta_k \frac{\tilde{y}_{t+1}}{\tilde{k}_{t+1}} - \delta \right) + \delta \right] + \tilde{\zeta}_{t+1} (1 - \delta) \right\}, \]  

(59)

\[ \tilde{\mu}_t = \beta \psi^{-1} E_t \left\{ \tilde{\lambda}_{t+1} \theta_h (1 - \tau_{n,t+1}) \frac{\tilde{y}_{t+1}}{\tilde{h}_{t+1}} + \tilde{\mu}_{t+1} \left[ 1 - \delta_h + \omega_h \tilde{e}_{t+1} \tilde{h}_{t+1}^{\omega_h} n_{h,t+1}^{-\omega_h - \omega_h} \right] \right\}. \]  

(60)

Next, detrending the first order conditions with respect to \( n_{w,t} \) and \( n_{h,t} \), equations (14) and (15), yields

\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \tilde{\lambda}_t (1 - \theta_k - \theta_h) (1 - \tau_{n,t}) \tilde{y}_t / n_{w,t}, \]  

(61)
\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \tilde{\mu}_t (1 - \omega_e - \omega_h) e_t^\omega e_{t}^\omega h_{t}^\omega n_{h,t}^{-\omega_e - \omega_h}. \] (62)

**Appendix B. Central Planner’s Problem**

The representative agent fails to account for the effect of his choice of human capital on average human capital, implying an externality. Prices do not incorporate the externalities, resulting in underproduction of human capital. The planner is able to internalize this externality. The planner chooses optimal paths of \(c_t, g_{c,t}, e_t, n_{w,t}, n_{h,t}, i_t, k_{t+1}, h_{t+1}\) to maximize the expected discounted sum of utility, subject to the resource constraint and physical and human capital accumulation equations. We assume that \(z_t = \xi^t\), and that \(m_t\) and \(g_t\) grow exogenously along the balanced growth path.

\[
\max_{c_t, g_{c,t}, e_t, n_{w,t}, n_{h,t}, i_t, k_{t+1}, h_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log c_t + \alpha_n \log (1 - n_{h,t} - n_{w,t}) + \alpha_g \log g_{c,t} \}
\]

subject to

\[
c_t + e_t + g_{c,t} + g_t + k_{t+1} - (1 - \delta)k_t = y_t + m_t,
\]

\[
y_t = z_t k_t^{\theta_k} n_{w,t}^{1-\theta_k-\theta_h} h_t^{\theta_h+\phi},
\]

\[
k_{t+1} = (1 - \delta)k_t + i_t,
\]

\[
h_{t+1} = (1 - \delta_h)h_t + e_t^\omega e_t^\omega h_{t}^\omega n_{h,t}^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t}^{1-\omega_e-\omega_h}.
\]

Letting \(\mu_t\) and \(\lambda_t\) be Lagrange multipliers on the human capital equation and the resource constraint, respectively, the first order conditions with respect to \(c_t, g_{c,t}, e_t, k_{t+1}, h_{t+1}, n_{w,t}, n_{h,t}\) yield

\[
1/c_t = \lambda_t,
\]

\[
\alpha_g/g_{c,t} = \lambda_t,
\]

\[
\lambda_t = \mu_t (\omega_e e_t^\omega e_t^\omega h_{t}^\omega n_{h,t}^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t}^{1-\omega_e-\omega_h}),
\]

\[
\lambda_t = \beta E_t [\lambda_{t+1} (1 - \delta + \theta_k y_{t+1}/k_{t+1})],
\]

\[
\mu_t = \beta E_t \{ \mu_{t+1} [1 - \delta_h + \omega_h e_{t+1}^\omega e_{t+1}^\omega h_{t+1}^{\omega_h} (z_t^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t}^{1-\omega_e-\omega_h})] + \lambda_{t+1} (\theta_h + \phi) y_{t+1}/h_{t+1} \},
\]

51
\[
\frac{\alpha_n}{1 - n_{h,t} - n_{w,t}} = \lambda_t (1 - \theta_k - \theta_h) y_t / n_{w,t},
\]
\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \mu_t (1 - \omega_e - \omega_h) e^{\omega_t} h_t^{\omega_h} z_t^{(1-\omega_e-\omega_h)/(1-\theta_k-\theta_h-\phi)} n_{h,t}^{-\omega_e-\omega_h}.
\]

Along the balanced growth path \(\lambda_t\) and \(\mu_t\) grow at rate \(\psi^{-1}\). All other variables, except \(n_{w,t}\) and \(n_{h,t}\), grow at rate \(\psi\). Denote the central planner’s detrended variables as \(\hat{x}_t\), and their steady states as \(\hat{x} = \hat{x}_{t+1} = \hat{x}_t\). In the steady state, the human capital accumulation equation becomes

\[
(\psi - 1 + \delta_h) \hat{h}^{1-\omega_h} = \epsilon^{\omega_t} n_h^{1-\omega_e-\omega_h}.
\]

The resource constraint in steady state is

\[
\hat{c} + \hat{i} + \hat{e} + \hat{g}_c + \hat{g} = \hat{y} + \hat{m},
\]

where

\[
\hat{y} = \hat{k}^{\theta_k} n_w^{1-\theta_k-\theta_h} \hat{h}^{\theta_h+\phi}.
\]

The steady state of the first order conditions can be written as

\[
1/\hat{c} = \hat{\lambda},
\]
\[
\alpha_g / \hat{g}_c = \hat{\lambda},
\]
\[
\hat{\lambda} = \hat{\mu} \omega_e \epsilon^{\omega_t} \hat{h}^{\omega_h} n_h^{1-\omega_e-\omega_h},
\]
\[
\psi / \beta = 1 - \delta + \theta_k \frac{\hat{y}}{\hat{k}},
\]
\[
\hat{\mu} \psi / \beta = \hat{\mu} [1 - \delta_h + \omega_h \epsilon^{\omega_t} \hat{h}^{\omega_h} n_h^{1-\omega_e-\omega_h}] + \hat{\lambda} (\theta_h + \phi) \frac{\hat{y}}{\hat{h}},
\]
\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \hat{\lambda} (1 - \theta_k - \theta_h) \frac{\hat{y}}{n_{w,t}},
\]
\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \hat{\mu} (1 - \omega_e - \omega_h) \epsilon^{\omega_t} \hat{h}^{\omega_h} n_h^{1-\omega_e-\omega_h}.
\]

The steady state values for \(k/y\), \(e/y\), \(c/y\), \(n_w\) and \(n_h\) are given by equations (35) to (40) in the text.
Appendix C. Data Details

The data sets for calibration are from National Income and Product Accounts Tables (NIPA) by the Bureau of Economic Analysis, Federal Reserve Economic Data (FRED), the World Bank Data (WBD), and the Digest of Education Statistics (DES). All the data below except for real interest rates are denominated in US dollars.

**GDP**: Gross domestic product (Table 1.1.5, NIPA)

**Investment**: Gross domestic investment (Table 5.1, NIPA)

**Capital Stock**: Fixed assets (private and public) (Table 1.1, Fixed Assets Accounts Tables, NIPA)

**Consumption**: Personal consumption expenditures (Table 1.1.5, NIPA)

**Government Debt**: Federal debt held by the public (FYGFDUN, FRED)

**Total Government Consumption Expenditures**: Government consumption expenditures (Table 3.1, NIPA)

**Government Defense Expenditures**: Defense expenditures (Table 3.15.5, NIPA)

**Government Subsidies to Education**: Government education expenditures (Table 3.15.5, NIPA)

**Net Import**: Net exports of goods and services (Table 1.1.5, NIPA)

**Education Expenditures (% of GDP)**: Expenditures of educational institutions as a percent of GDP - all institutions (Table 28, 2011 Tables and Figures, DES)

**Real GDP Growth Rate**: Percent change from preceding period in real gross domestic product (Table 1.1.1, NIPA)

**Real Interest Rate**: Real interest rate, World Bank Data, available at: 
http://data.worldbank.org/indicator/FR.INR.RINR

Appendix D. Estimation of Human Capital Technology

As an alternative to the benchmark calibration, we follow Gao (2014) and estimate the coefficients on education expenditures ($\omega_e$) and human capital ($\omega_h$). First, Gao (2014) performs a cross-country regression to estimate the coefficients of the production function
in a Solow model augmented with human capital. The aggregate production function is given by

\[ Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} L_t^{1-\theta_k-\theta_h} h_{a,t}^\phi, \]  

(63)

where \( L_t \) denotes the aggregate labor force. Due to lack of data availability in a large cross section of countries for labor hours and for labor force participation, we assume that labor hours are fixed and compute per capita values instead of per worker values. In order to obtain estimated coefficients consistent with our model, we assume there is an externality from human capital, given by \( h_{a,t}^\phi \). Dividing equation (63) by \( L_t \) yields the per capita production function

\[ y_t = z_t K_t^{\theta_k} H_t^{\theta_h} h_{a,t}^\phi. \]

In equilibrium, per capita human capital is equal to the agent’s own human capital, \( h_t = h_{a,t} \). Therefore,

\[ y_t = z_t K_t^{\theta_k} H_t^{\theta_h} h_t^{\phi}. \]

(64)

After rearrangement, equation (64) can be rewritten as

\[ y_t = z_t^{1-\theta_k-\theta_h-\phi} (k_t/y_t)^{\theta_k/(1-\theta_k-\theta_h-\phi)} (h_t/y_t)^{\theta_h/(1-\theta_k-\theta_h-\phi)}. \]

(65)

Per capita output is now a function of an unobserved factor, \( z_t^{1-\theta_k-\theta_h-\phi} \), and two capital intensities.

Next, Gao (2014) uses capital and human capital accumulation technologies from equations (5) and (6) to derive expressions to substitute for \( k_t/y_t \) and \( h_t/y_t \) in equation (65). To obtain coefficients consistent with our model, we assume that human and physical capital both depreciate at the same rate of \( \delta = \delta_h \). Define \( A_t = z_t^{1-\theta_k-\theta_h-\phi} \). Assume that \( A_t \) grows exponentially at rate \( \psi \) and that labor grows exponentially at rate \( n \), such that \( A_t = A_0 e^{\psi t} \) and \( L_t = L_0 e^{nt} \). In steady state,

\[ k/y = \frac{i/y}{\psi + \delta + n}, \]

(66)

\[ h/y = (e/y)^{\omega_h/(1-\omega_h)} \left( \frac{n_h}{y_t/A_t} \right)^{1-\omega_k-\omega_h} (\psi + \delta + n)^{-1/(1-\omega_h)}, \]

(67)

where we have dropped the time subscript to denote the steady state. Note that along
the balanced growth path, $y_t/A_t$ is the detrended output per capita and, therefore, is constant.

Next, substitute the expressions for $k/y$ and $h/y$, from equations (66) and (67), into equation (65) and rearrange to yield

$$y_t = A_t(y + \delta + n) - \left(1 - \omega_h\right)\theta_k + \left(\theta_h + \phi\right)\left(1 - \omega_e - \omega_h\right)\left(\theta_h + \phi\right)\log(n_h),$$

where $\Upsilon = (1 - \omega_h)(1 - \theta_k) - \omega_c(\theta_h + \phi)$. Next, take the logarithm to yield

$$\log y_t = \log A_t - \frac{(1 - \omega_h)\theta_k + (\theta_h + \phi)}{\Upsilon} \log(y + \delta + n) + \frac{(1 - \omega_h)\theta_k}{\Upsilon} \log(i/y)$$

$$+ \frac{\omega_e(\theta_h + \phi)}{\Upsilon} \log(e/y) + \frac{(1 - \omega_e - \omega_h)(\theta_h + \phi)}{\Upsilon} \log n_h.$$

Following Mankiw, Romer, and Weil (1992), we assume that $A_t$ is randomly distributed across countries. We estimate parameters using a single cross-country regression for the log of each country’s per capita output regressed on logs of each country’s investment-to-GDP ratio, education expenses-to-GDP ratio, education enrollment, and the sum of exogenous rates. We exclude oil producers, and use the largest possible remaining sample of countries, 78 countries over our sample period of 1995 - 2007. Details of the cross-country regression are in Gao (2014). The parameters $\omega_c$ and $\omega_h$ are calculated from the estimated coefficients. The regression results are summarized in Table 4.
Table 4: **Single Cross-Country Regression**

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>log ( y ) in 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countries:</td>
<td>78</td>
</tr>
<tr>
<td>Constant</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>(2.02)</td>
</tr>
<tr>
<td>( \log(i/y) )</td>
<td>0.83***</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
</tr>
<tr>
<td>( \log(\psi + \delta + n) )</td>
<td>-3.96***</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
</tr>
<tr>
<td>( \log(n_h) )</td>
<td>0.86***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
</tr>
<tr>
<td>( \log(e/y) )</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Implied Coefficients:**

- Implied \( \omega_e \): 0.11
- Implied \( \omega_h \): 0.61

---

^a The standard errors are in parentheses immediately below.
^b *, **, *** indicate significance at 10%, 5% and 1% level, respectively.
^c \( \log(y_t) \): log real GDP per working-age population in 2007, calculated by dividing real GDP by working-age population (15-64), from Summers-Heston PWT 7.1 and World Development Indicators, respectively.
^d \( n_h \): the ratio of secondary and tertiary school enrollment to the working-age population, from UNESCO.
^e \( e/y \): public spending on education-to-GDP ratio, World Bank Data
^f \( i/y \): Investment-to-GDP ratio, from Summers-Heston PWT 7.1.
^g \( n \): working-age population growth rate, from World Development Indicators.
^h The investment, working-age population growth rates and education enrollment rates are averaged over 1995-2007; \( (\psi + \delta) \) is assumed to be 0.05.