A country participating in a monetary union is constrained by loss of control over seigniorage revenue. Once the government reaches its fiscal limit on ordinary taxation, it cannot turn to seigniorage for financing. We show that a monetary union country can increase its seigniorage revenue by reissuing its own currency even as it fully honors all outstanding debt obligations. We use a simple cash-in-advance model, with domestic currency demand motivated by the need to pay taxes, to show that this policy effectively redistributes seigniorage revenue away from other monetary union members toward the acting country. The magnitude of the seigniorage created by currency reissue is limited both by the relative size of the country and by money demand, and, therefore, by the tax base. If this seigniorage revenue is insufficient, some additional seigniorage is available by allowing the new currency to grow and depreciate over time and domestic real wages to fall.

Key Words: Monetary Union, Seigniorage, Dual Currency, Fiscal Limits
1 Introduction

In the wake of the sovereign debt crisis in Greece, many economists have suggested that Greece leave the euro and return to its own currency. The country could convert its euro liabilities and those of its banks into new drachma and allow the drachma to depreciate. This would give Greece access to seigniorage revenue, which is unavailable under the current monetary union. Additionally, it could reduce real wages and improve the current account. However, legally and practically, this is difficult. The initial decision to join the monetary union is officially "irrevocable," and replacing euro liabilities with drachma liabilities, which lose value immediately, is essentially a default.

The European Central Bank (ECB) is unlikely to provide seigniorage to help Greece with its debt. Reis (2013) explains that although a small redistribution of seigniorage could have a large effect for a small country, the ECB has no way to effect such a redistribution. He demonstrates that the ECB’s policy instrument available to reduce financial constraints on member-country governments is the overall quantity of seigniorage, an instrument they are unwilling to use for this purpose.

As an alternative, Marek Belka of the Polish Central Bank (Financial Times 2012) suggested that Greece reissue drachma without leaving the euro. The purpose of this paper is to explore such a dual currency option. Could a monetary union country, struggling with high government debt, obtain seigniorage revenue to enable it to honor its debt, denominated in the common currency, by reissuing its own currency? The simple answer is that reissuing a national currency can generate seigniorage, but the quantity of seigniorage
the country would be able to raise is limited. The limits are less severe the smaller the fraction of monetary-union GDP the home country comprises. Since Greece is a small economy, it could potentially secure a relatively large quantity of seigniorage this way. The limits are also less severe the greater the demand for domestic currency the acting country is able to generate. We motivate this demand with tax payments, implying that seigniorage would be limited by the size of the ordinary tax base.

We also address the issue of competitiveness. To what extent can the reissuance of a domestic currency, which is allowed to depreciate over time, reduce domestic real wages, thereby allowing an increase in competitiveness and greater net exports?

The paper is organized as follows. Section 2 contains a description of the model of the small open economy together with an initial equilibrium in which government debt is growing. Section 3 introduces money into the model and characterizes currency flows in the initial equilibrium. Section 4 describes the dual-currency equilibrium after the reintroduction of the domestic currency together with equilibrium in the period of domestic currency reissuance. Section 5 provides quantitative implications, and Section 6 concludes.

2 Model

2.1 Setting the Stage

We need a model in which rational optimizing agents, operating in a small open economy, generate an equilibrium with explosive external government debt. In a standard representative agent model, Ricardian Equivalence holds. Any increase in borrowing on the part of the government is met by an increase in savings by domestic agents, as they prepare for
future tax obligations, and external debt does not increase. Therefore, our model must not exhibit the standard Ricardian Equivalence. And not only must domestic agents fail to save to acquire growing government debt – foreign agents must be willing to acquire the growing debt of the small open economy.

We make three modifications to the standard model. First, we introduce fiscal limits. The government faces fiscal limits on the quantity of tax revenues it can raise by increasing the tax rate (Bi (2011), Sims (1999), Davig, Leeper, and Walker (2011), Daniel and Shiamptanis (2012)). We assume that there is a maximum above which the tax rate cannot rise and continue to generate additional tax revenue. The fiscal limit on the tax rate could be at the top of the Laffer curve or it could be determined by political will to raise taxes. We assume here that the fiscal limit on the tax rate is determined by tax evasion. As the tax rate rises, incentives to evade taxes increase, and revenue falls below the tax rate multiplied by income quickly, limiting the tax rate to \( \tau \leq \bar{\tau} \). To simplify, we assume that there is no tax evasion for \( \tau \leq \bar{\tau} \), and that once the tax rate exceeds \( \bar{\tau} \), evasion limits the effective tax rate to \( \bar{\tau} \). Tax revenues are at their fiscal limit.

We also impose a fiscal limit on the quantity of the public good that government workers produce. We assume that there is a lower bound on the quantity of the government good relative to consumption that agents are willing to allow. To simplify, we set this lower bound at the optimal value of the government good relative to consumption.

Second, we need to generate an unwillingness by domestic agents to purchase government debt and a foreign willingness. We follow Engel and Rogers (2006), who hypothesized that the large US current account deficit is due to the expectation that future US output growth would be high relative to that of other countries. Using this idea, we assume
that the advent of monetary union for Greece creates the expectation of high future productivity relative to that in the other EMU countries. This expectation implies that domestic residents would like to borrow against their higher expected future income, while foreign agents are willing to lend based on the same expectation. Third, we add borrowing constraints on private domestic agents, implying that the domestic government can borrow, while agents cannot. Since domestic agents want to borrow, but cannot, they spend their incomes.

Using these three modifications, we set up an initial equilibrium at fiscal limits. The initial equilibrium has constant government debt, but is fragile. A temporary negative shock to tax revenue increases government debt and sets it off along an explosive path.

2.2 Initial Equilibrium at Fiscal Limits

In this section, we first characterize a stationary equilibrium. Then we let debt increase until the tax rate reaches its fiscal limit. This yields a stationary equilibrium with the government operating at its fiscal limit. This equilibrium is sustainable as long as nothing happens to increase government debt, and hence is fragile.

The economy produces two goods, a private consumption good \((Y)\) and a government good \((G)\), each using a constant returns to scale production function with labor allocated to sector \(i \in \{Y, G\}\) as the only input. The private good is tradeable and subject to the law of one price, while the government good is not tradeable and is freely distributed. The supply of labor is normalized at unity. Denoting \(N\) as labor allocated to the production of the private good, production of the consumption good is given by

\[ Y = \alpha N, \] (1)
with production of the government good given by

\[ G = \alpha (1 - N). \quad (2) \]

The government collects taxes to finance production of the government good. We assume that taxes are levied on sales of the private good such that the euro price of the private good is \(1 + \tau\), where \(\tau\) is the tax rate. This tax is essentially a value-added tax. Consistent with actual practice, we assume that the value-added tax is rebated on exports and levied on imports, yielding tax revenue on net exports \((X)\) of \(-\tau X\). This tax policy assures that residents of a single country face the same price for domestic and foreign goods, independent of domestic and foreign tax rates.

Given constant returns to scale, gross wages in the private sector are determined to exhaust the value of output. Therefore, workers in the private sector, equivalently private sector producers, receive \((1 + \tau)Y\) for the production of the good, and pay \(\tau Y\) in taxes to the government, leaving them with total wage payments net of taxes of \(Y\). Letting \(W\) denote payments to producers net of taxes, essentially wages net of taxes, wage payments to private producers are given by

\[ WN = (1 + \tau) Y - \tau Y = Y. \]

Given that \(N\) agents together receive net income of \(Y\), the wage is given by

\[ W = \frac{Y}{N} = \alpha \]

where the second equality uses equation (1). Labor is perfectly mobile across sectors, implying that government workers also receive a wage of \(\alpha\).
The assumptions that future output is expected to be higher, and that agents face borrowing constraints, which prevent them from borrowing against future income, imply that consumption of the private good equals disposable income.\(^1\) Producers together receive and spend \(Y = \alpha N\), while government workers together receive and spend \(\alpha (1 - N)\).

Due to the value added tax, the price of a good is \(\frac{1}{1+\tau}\), yielding consumption as

\[ C = \frac{\alpha}{1+\tau}. \] (3)

Net exports \((X)\) in the small open economy are given by production of the private good less consumption, yielding

\[ X = Y - C = Y - \frac{\alpha}{1+\tau}. \] (4)

In the public sector, the government uses tax revenue \((\tau Y)\) plus seigniorage from the central bank \((\psi)\) to pay government workers \((W (1 - N))\), to pay interest on debt \((rb)\), and to rebate value-added taxes on net exports of goods \((\tau X)\). Defining \(b\) as domestic debt held by the public, \(b^c\) as domestic debt held by the central bank, and using primes \((')\) to denote one-period-ahead values, the government’s budget constraint can be expressed as

\[ b' + b^{c'} = (1 + r) (b + b^c) + \tau X + W (1 - N) - \tau Y - \psi. \] (5)

We impose an initial equilibrium in which government debt is constant. To simplify, we assume \(\psi = rb^c\), and that this quantity is constant over time, yielding the simplified government budget constraint as

\[ b' = (1 + r) b + \tau X + W (1 - N) - \tau Y. \] (6)

\(^1\) Magnitudes do matter here. We assume that expected future productivity is high enough that agents would like to borrow. The closest they can get is to consume all of income.
Therefore, government wage payments in the initial equilibrium are the residual of tax revenue less interest on government debt and tax rebates to net exports, yielding

\[ W(1 - N) = \tau (Y - X) - rb = \frac{\alpha \tau}{1 + \tau} - rb. \]  

(7)

Labor is freely mobile across sectors and is allocated such that after-tax wages are equal. Equating after-tax wages per worker in each sector requires

\[ W = \frac{\alpha \tau - rb (1 + \tau)}{(1 - N)(1 + \tau)} = \frac{Y}{N} = \alpha. \]  

(8)

Solving equation (8) for the allocation of labor across sectors yields

\[ N = \frac{\alpha + rb (1 + \tau)}{\alpha (1 + \tau)} \quad 1 - N = \frac{\alpha \tau - rb (1 + \tau)}{\alpha (1 + \tau)}. \]  

(9)

Substituting sectorial employment into equations (1) and (2), yields solutions for output of each good as

\[ Y = \alpha N = \frac{\alpha}{1 + \tau} + rb, \]  

(10)

\[ G = \alpha (1 - N) = \frac{\alpha \tau}{1 + \tau} - rb, \]  

(11)

with net exports given by

\[ X = Y - C = rb. \]  

(12)

The final element of the initial equilibrium is determination of the tax rate. We assume that the government chooses the tax rate to maximize utility of the representative agent, given by

\[ U = \ln C + \lambda \ln G, \]

yielding the relationship between consumption and government spending at the optimum as

\[ G^* = \lambda C^*, \]  

(13)
with the optimal tax rate given by

$$\tau^* = \frac{\alpha \lambda + rb}{\alpha - rb}. \quad (14)$$

The optimal tax rate is increasing in the level of debt. This is because at a given tax rate, the increase in government debt reduces available resources to attract government workers for production of the government good. Maintenance of government spending at the optimal ratio of private spending requires an increase in the tax rate.

Using equations (9) and (14), we can solve for the optimal allocation of labor to each sector as

$$N^* = \frac{\alpha + \lambda rb}{\alpha (1 + \lambda)} \quad 1 - N^* = \frac{\lambda (\alpha - rb)}{\alpha (1 + \lambda)}. \quad (15)$$

Equations (1) to (4), together with equation (14) yield values in the initial equilibrium at optimal tax rates as

$$Y^* = \frac{\alpha + \lambda rb}{1 + \lambda} \quad (16)$$

$$G^* = \frac{\lambda (\alpha - rb)}{1 + \lambda} \quad (17)$$

$$C^* = \frac{\alpha - rb}{1 + \lambda}. \quad (18)$$

$$X^* = Y^* - C^* = rb \quad (19)$$

At the optimal tax rate, as debt increases, labor is allocated away from government toward the private sector. This allows larger exports of the private good, while maintaining the ratio of consumption to government spending constant. Note that in the stationary equilibrium, net exports equal interest on debt, assuring current account balance.

We assume that the economy has reached an initial equilibrium in which the increase in debt has pushed the tax rate to its fiscal limit of $\tilde{\tau}$. Additionally, $G$ is constrained not
to fall below $\lambda C$. We can use equation (14) to solve for the upper bound on debt at $\bar{r}$, yielding

$$ r\bar{b} = \frac{\alpha (\bar{r} - \lambda)}{1 + \bar{r}}. $$

Substituting into equations (15)-(19) yields values for employment in each sector, output in the private sector, production and consumption of government goods, consumption of private-sector goods, and net exports, all in the initial equilibrium at which the tax rate and debt have reached upper bounds.

$$ \bar{N} = \frac{1 + \bar{r} - \lambda}{1 + \bar{r}} \quad 1 - \bar{N} = \frac{\lambda}{1 + \bar{r}} \quad (20) $$

$$ \bar{Y} = \alpha \left[ \frac{1 + \bar{r} - \lambda}{1 + \bar{r}} \right] $$

$$ \bar{G} = \frac{\alpha \lambda}{1 + \bar{r}} \quad (21) $$

$$ \bar{C} = \frac{\alpha}{1 + \bar{r}}. $$

$$ \bar{X} = \bar{Y} - \bar{C} = \frac{\alpha (\bar{r} - \lambda)}{1 + \bar{r}} = r\bar{b} $$

We can verify that the government budget constraint in this initial equilibrium is satisfied at constant debt by substituting into equation (6) to yield

$$ \bar{b} = (1 + r)\bar{b} + \bar{r}\bar{X} + \bar{W} (1 - \bar{N}) - \bar{r}\bar{Y} \quad (22) $$

$$ = \bar{b} + \frac{\alpha (\bar{r} - \lambda)}{1 + \bar{r}} + \frac{\bar{r}\alpha (\bar{r} - \lambda)}{1 + \bar{r}} + \frac{\alpha \lambda}{1 + \bar{r}} - \alpha\bar{r} \left[ \frac{1 + \bar{r} - \lambda}{1 + \bar{r}} \right] $$

$$ = \bar{b}, $$

This equilibrium is sustainable indefinitely as long as nothing happens to increase government debt. However, we illustrate below that, once government debt increases beyond $\bar{b}$, debt becomes explosive. Therefore, this initial equilibrium is fragile.
2.3 Explosive Government Debt

In this section we introduce a one-period transitory shock which serves to place debt on an explosive path. We assume that for a single period, productivity in the private sector declines to $\tilde{\alpha} < \alpha$, where a tilde is used to denote values in the explosive equilibrium. Government tax revenue falls during this period due to the fall in the production of private goods. The government is at fiscal limits and is therefore unable to raise the tax rate. Additionally, we assume that the government is under political pressure to sustain payment of government wages as though tax revenue had not declined. It therefore pays workers as though tax revenues had not fallen, yielding total payments to workers from equation (21) as

$$\tilde{W} \left(1 - \tilde{N}\right) = \tilde{G} = \tilde{G} = \frac{\alpha \lambda}{1 + \tau}.$$

Effectively, tax revenue falls, due to the decline in productivity, but the government does not reduce spending in line with the fall in tax revenue. This is possible because foreign agents are willing to lend to the government based on the expectation that future productivity will be even higher than its normal value of $\alpha$.

The productivity shock reduces the after-tax wage in the private sector to $\tilde{\alpha}$. We can solve for the equilibrium allocation of workers across sectors in the period of low productivity by equating after-tax wages using

$$\frac{\alpha \lambda}{(1 + \tau) \left(1 - \tilde{N}\right)} = \tilde{\alpha},$$

to yield

$$\tilde{N} = \frac{\tilde{\alpha} (1 + \tau) - \alpha \lambda}{\tilde{\alpha} (1 + \tau)}, \quad 1 - \tilde{N} = \frac{\alpha \lambda}{\tilde{\alpha} (1 + \tau)}.$$

10
Even though the government maintains its total spending at $\bar{G}$, the reallocation of workers toward the government sector, together with fixed total wage payments, forces it to reduce the wage payment to each worker. The reallocation continues until net wages are equal.

Equilibrium values are obtained as before, using the production functions, equations (1) and (2), together with assumptions which imply that both types of agents spend all of their incomes, to yield

$$\bar{Y} = \frac{\bar{\alpha} (1 + \bar{\tau}) - \alpha \lambda}{1 + \bar{\tau}}$$

$$\bar{G} = \frac{\alpha \lambda}{1 + \bar{\tau}}$$

$$\bar{C} = \frac{\bar{\alpha}}{1 + \bar{\tau}}.$$  

$$\bar{X} = \bar{Y} - \bar{C} = \frac{\bar{\alpha} \bar{\tau} - \alpha \lambda}{1 + \bar{\tau}}.$$  

Substituting into the government budget constraint yields

$$b' = (1 + r) \bar{b} + \bar{\tau} \bar{X} + \bar{W} \left(1 - \bar{N}\right) - \bar{\tau} \bar{Y}$$

$$= \bar{b} + \frac{\alpha (\bar{\tau} - \lambda) + \bar{\tau} (\bar{\alpha} \bar{\tau} - \alpha \lambda) + \alpha \lambda - \bar{\tau} [\bar{\alpha} (1 + \bar{\tau}) - \alpha \lambda]}{1 + \bar{\tau}}$$

$$= \bar{b} + \epsilon,$$

where

$$\epsilon = \frac{(\alpha - \bar{\alpha}) \bar{\tau}}{1 + \bar{\tau}} > 0.$$  

In the period after the shock, productivity and all values except debt return to their initial equilibrium values. The government must spend more on interest payments on debt, and with the tax rate and government spending at fiscal limits, they borrow to do
so. The budget constraint is given by

\[ b' = (1 + r)(\bar{b} + \epsilon) + \tau\bar{X} + \bar{W}(1 - \bar{N}) - \tau\bar{Y} \]

\[ = \bar{b} + (1 + r)\epsilon. \]

Debt is on an explosive path. The problem is due to two fiscal limits, one on the tax rate and one on wage payments to government workers. If both become fixed with government debt above \( \bar{b} \), then debt becomes explosive. Foreign agents are willing to lend to allow accumulating debt because all expect higher future productivity in the country with growing indebtedness. The growing government debt reflects a current account deficit. Therefore, the policy must also address the current account deficit, possibly through a reduction in domestic wages, which would increase competitiveness, as originally argued by Belka.

This shock is meant to illustrate how a policy that governments often follow, maintaining government spending in the presence of a recession, could create explosive debt. Any other shock which raises debt above its fiscal limit would also create explosive debt.

3 Money in Equilibrium

In the next section, we introduce a dual currency solution to the problem of explosive government debt. To consider this solution, we must introduce seigniorage and money into the model. Consider, first, the role of seigniorage.

The European Central Bank (ECB) buys government bonds from member-country banks under a repo arrangement and earns interest on these assets. This interest is distributed to member countries as seigniorage revenues, using a complicated formula.
based on GDP and population, but not on individual-country levels of government debt. Letting $b$ denote Greek government bonds and $B$ denote government bonds issued by all other monetary union countries together, the base money supply in the monetary union, $E$, can be represented as

$$E = B^c + b^c,$$

where the superscript $c$ denotes central bank holdings.\(^2\) When the quantity of euro is not changing, total seigniorage for the monetary union is the interest earned on these government bonds. This seigniorage is higher when the monetary authority holds more government bonds, supporting a larger euro money supply.

There is an important difference between the production and distribution of seigniorage in a monetary union and in a sovereign nation with its own money supply. A sovereign nation can choose to create more seigniorage by having its central bank purchase additional government bonds. The interest on these bonds is returned to the fiscal authority. A country in the monetary union has no way to convince the monetary union to purchase more bonds and grant it more seigniorage. Even if the ECB purchased additional Greek bonds and sold others to keep the supply of euro fixed, Greece would still pay interest on central bank holdings of its bonds and its seigniorage revenues would not increase. The Greek government would see no benefit in its budget constraint.

Additionally, it is necessary to motivate demand for money if the introduction of a new money is to have real effects on the country’s accumulation of debt. We motivate money demand by cash-in-advance. Payments by private-sector agents must be made with cash held in advance. Payments by the government can be made either with issuance of new

\(^2\) We abstract from a banking system, implying that base money is equivalent to the money supply.
debt or with cash held in advance.

The assumption on timing is as follows. Each period is divided into three subperiods. The cash-in-advance constraint requires that agents acquire the cash they need for payments prior to the subperiod in which payments are made. In the first subperiod, producers pay taxes to government with cash carried over from the end of the period. The middle sub-period contains government financial transactions, including wage payments to government workers, interest payments on debt, and new debt issuance if necessary. Payments by the government are made from tax revenue collected in the first sub-period and possibly from issuance of new government debt.\textsuperscript{3} The third subperiod contains goods market transactions. Government workers purchase goods with wages paid in the middle subperiod; producers make purchases with euro carried over from the end of the previous period; foreign creditors use some of the interest payments received in the previous sub-period to purchase goods; and the government makes necessary export-tax rebates using a combination of euro from taxes in the first subperiod and new bond issuance in the second.

We characterize equilibrium flows in temporary equilibrium with explosive government debt and taxes at their fiscal limit. Table 1 contains these budget constraints for government ($G$), private producers (Pro), government workers (GW), and foreign agents (F) in rows. The first two columns and the final one list initial and end-of-period assets from government bonds ($-b$) and euro ($E$). End of the period asset values are denoted with primes. Since end-of-period euro equal beginning of period euro, the end-of-period euro

\textsuperscript{3} The government can make payments with newly issued debt in the same subperiod, not subject to a cash-in-advance requirement. Governments do often issue new debt to service existing debt.
column is omitted. Euro flows are described by middle columns with payments of euro receiving a negative sign and receipts receiving a positive sign. The vertical sum of entries in flow columns must be zero since payments must equal receipts. The budget constraints are read horizontally as end of period assets, given by bonds in the final column plus euro in the second (since final euro equal initial euro), must equal beginning of period assets, given by the sum of the first two entries, plus flows, given by the sum of the middle entries. We label values which are changing over time with time subscripts $t$.

The cash-in-advance constraint requires that all payments be preceded by a receipt in a previous subperiod large enough to make the payment in a subsequent subperiod. For example, the producer must acquire euro of $\bar{\tau}Y$ prior to the subperiod in which taxes are paid and euro of $Y$ prior to the subperiod in which goods are purchased. Effectively, cash-in-advance requires that payments (flow entries with a negative sign) be preceded by receipts (flow entries with a positive sign) of equal or greater magnitude in a prior subperiod. The constraint does not apply to government payments made by issuing new debt, but applies to all others.

Table 1: Euro Flows at Initial Explosive Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$b$</th>
<th>$E$</th>
<th>tax</th>
<th>$W(1-N)$</th>
<th>int</th>
<th>goods</th>
<th>$b'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>$-b_{t-1}$</td>
<td>0</td>
<td>$\bar{\tau}Y$</td>
<td>$-[\bar{\tau}Y - (1 + \bar{\tau}) r\bar{b}]$</td>
<td>$-rb_{t-1}$</td>
<td>$-\tau r\bar{b}$</td>
<td>$-b_t$</td>
</tr>
<tr>
<td>Pro</td>
<td>0</td>
<td>$(1 + \bar{\tau})Y$</td>
<td>$-\bar{\tau}Y$</td>
<td>0</td>
<td>0</td>
<td>$-Y + (1 + \bar{\tau}) Y$</td>
<td>0</td>
</tr>
<tr>
<td>GW</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\bar{\tau}Y - (1 + \bar{\tau}) r\bar{b}$</td>
<td>0</td>
<td>$-[\bar{\tau}Y - (1 + \bar{\tau}) r\bar{b}]$</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>$b_{t-1}$</td>
<td>$E'$</td>
<td>0</td>
<td>0</td>
<td>$rb_{t-1}$</td>
<td>$-r\bar{b}$</td>
<td>$b_t$</td>
</tr>
</tbody>
</table>

The government budget constraint is the row labeled G in Table 1. The government
carries over \(-b_{t-1}\) bond assets from the end of period \(t-1\). In the first subperiod, it receives \(\bar{r}Y\) euro in tax payments. In the second subperiod, it uses these euro and new government debt \((b_t - b_{t-1})\) to pay \(\bar{r}Y - (1 + \bar{r})r\bar{b}\) in wages to government workers and \(rb_{t-1}\) in interest on government debt, saving \(\bar{r}r\bar{b}\) euro to make export tax rebates in the third subperiod. Final assets are given by the sum of all rows, yielding \(b_t = b_{t-1} + r\left(b_{t-1} - \bar{b}\right)\) in debt and no euro. Under the assumption that \(r\left(b_{t-1} - \bar{b}\right) > 0\), the home government is increasing its indebtedness while foreign agents are increasing their assets.

The budget constraint for producers of final goods is given in the row labeled Pro. Producers begin the period with \((1 + \bar{r})Y\) euro, which they have carried over from sales at the end of last period. In the first subperiod, they spend \(\bar{r}Y\) in payment of taxes to the government. In the third subperiod, they spend \(Y\) on purchases of the private good from other producers. They receive \(Y\) euro from sales to producers, \(\bar{r}Y - (1 + \bar{r})r\bar{b}\) euro from sales to government workers, \(r\bar{b}\) from sales to foreign creditors, and \(\bar{r}r\bar{b}\) from government export tax rebates. At the end of the period, they have \((1 + \bar{r})Y\) euro, which they carry into the next period.

The budget constraint for government workers is given in the row labeled GW in Table 1. Government workers have no initial assets. They receive \(\bar{r}Y - (1 + \bar{r})r\bar{b}\) euro in wages from the government in the second subperiod and use all of these euro to purchase goods from producers in the third subperiod. They end the period with no assets.

The budget constraint for foreign creditors is described by the row labeled F in Table 1. They hold initial assets of \(b_{t-1}\) Greek bonds and \(E^f\) euro, carried forward from the end of period \(t-1\). In the middle subperiod they receive \(rb_{t-1}\) in interest. Of the total payment, \(r\bar{b}\) is paid in euro and the remainder \(r\left(b_{t-1} - \bar{b}\right)\), in bonds, as the government
issues additional debt which raises bonds to \( b_t \). In the third subperiod, foreign creditors use the \( r\bar{b} \) euro to purchase goods from domestic producers, and the quantity of euro held by foreign creditors returns to its initial value.

We can check that the domestic goods market clears at a euro price of \( (1 + \bar{\tau}) \). Producers spend \( Y \) euro, government workers spend \( \bar{\tau}Y - r(1 + \bar{\tau})\bar{b} \) euro, and foreign creditors together with government export-tax rebates spend \( r(1 + \bar{\tau})\bar{b} \) euro for a total expenditure on \( Y \) goods of \( Y (1 + \bar{\tau}) \), yielding production multiplied by price.

We can also check that some one in the economy is always holding the \( (1 + \bar{\tau})Y \) euro and satisfying their cash-in-advance constraints. At the end of the period, producers hold all of the \( (1 + \bar{\tau})Y \) euro and carry them to the beginning of the next period. In the first subperiod, producers use \( \bar{\tau}Y \) of them to pay taxes, such that at the end of the first subperiod, euro held by producers is \( Y \) and by government is \( \bar{\tau}Y \). At the end of the middle period, producers retain their \( Y \) euro to satisfy their cash-in-advance constraint in the third subperiod, and government workers have \( \bar{\tau}Y - r (1 + \bar{\tau})\bar{b} \) from wage payments. Foreign creditors have \( r\bar{b} \) from interest payments, and the government retains \( r\bar{\tau}\bar{b} \). Goods market transactions in subperiod 3, with purchases made using the euro acquired in previous subperiods, return all \( (1 + \bar{\tau})Y \) euro to producers.

4 Dual Currency Solution to Explosive Debt

Assume that in period \( T \), the government uses period \( T \) euro tax revenue to pay down debt and issues drachma to pay government workers. It announces that taxes going forward are payable only in drachma and promises to operate a foreign exchange market in drachma and euro at the end of each period. Consider, first, the steady-state equilibrium after
the period in which the government uses the euro tax revenue to repurchase some of its bonds. Subsequently, we consider the equilibrium in the period of policy change.

4.1 Ex Post Budget Constraints

We assume an equilibrium after policy change in which euro-denominated government debt remains constant at $b_T$, with $b \leq b_T < b_{T-1}$, the tax rate at its fiscal limit of $\bar{\tau}$, and government spending set such that $G_T = \lambda C_T$. We denote equilibrium values after the debt buyback with subscript $T$. The elimination of government borrowing effectively places the government under the same cash in advance constraint as the private sector since the government can no longer make payments with new debt.

If the bond buyback leaves $b_T > \bar{b}$, then the new equilibrium must be characterized by lower domestic consumption and/or higher production of the private good to allow net exports to increase from $r\bar{b}$ to $r b_T$, thereby balancing the current account. Alternatively, if $b_T = \bar{b}$, then no reduction in domestic spending or in government wage payments is necessary since the government debt buyback alone is able to restore a non-explosive path for debt.

Consider the case in which ex-post debt is high enough to require drachma inflation and domestic spending reduction. The drachma inflation tax reduces net wages in the private sector. Therefore, the government can reduce total wage payments in the government sector while maintaining the optimal ratio of $G_T$ to $C_T$. We model this reduction in government wage payments as the subtraction of $z$ from initial government wage payments of $\bar{\tau}Y_T - r\bar{b}(1 + \bar{\tau})$. Therefore, we assume that the government pays workers

$$W_T (1 - N_T) = P_t (\bar{\tau}Y_T - Z),$$
where

\[ Z = (r \bar{b} (1 + \bar{r}) + z), \]

and determine the value of \( z \) later in the analysis to maximize welfare.

The government budget constraint in euro is given by the row labeled G in Table 2A. The government enters each period with bond debt of \( b_T \) and with euro equal to \( rb_T (1 + \bar{r}) \). In the second subperiod, the government pays \( rb_T \) euro in interest to foreign creditors. In the third subperiod, it pays \( \bar{r} rb_T \) toward the purchase of goods as a rebate of taxes on exports. It ends the period with \( b_T \) debt and no euro.

<table>
<thead>
<tr>
<th></th>
<th>b</th>
<th>E</th>
<th>int</th>
<th>goods</th>
<th>b'</th>
<th>E'</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>(-b_T)</td>
<td>(rb_T (1 + \bar{r}))</td>
<td>(-rb_T)</td>
<td>(-\bar{r} rb_T)</td>
<td>(-b_T)</td>
<td>0</td>
</tr>
<tr>
<td>Pro</td>
<td>0</td>
<td>(Y_T)</td>
<td>0</td>
<td>(-Y_T + rb_T (1 + \bar{r}) + Y_T)</td>
<td>0</td>
<td>(Y_T + rb_T (1 + \bar{r}))</td>
</tr>
<tr>
<td>F</td>
<td>(b_T)</td>
<td>(E_T^f)</td>
<td>(rb_T)</td>
<td>(-rb_T)</td>
<td>(b_T)</td>
<td>(E_T^f)</td>
</tr>
</tbody>
</table>

The government’s drachma budget constraint is given in row G of Table 2B. It begins the period with no drachma and in the first subperiod, receives drachma taxes of \( P_t \bar{r} Y_T \), where \( P_t \) is value of drachma/euro in period \( t \), equivalently the drachma price of euro. The government uses these drachma taxes to pay wages to government workers in the second subperiod. After payment of government wages, the government retains \( P_t Z = P_t (r \bar{b} (1 + \bar{r}) + z) \) drachma to carry to the end of the period.
Table 2B: Drachma Flows Ex Post Equilibrium with Euro Gov Debt Constant at $b_T$

<table>
<thead>
<tr>
<th></th>
<th>$D$</th>
<th>taxes</th>
<th>$W (1 - N)$</th>
<th>goods</th>
<th>$D'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0</td>
<td>$P_t\bar{\tau}Y_T$</td>
<td>$-P_t(\bar{\tau}Y_T - Z)$</td>
<td>0</td>
<td>$P_tZ$</td>
</tr>
<tr>
<td>Pro</td>
<td>$P_t\bar{\tau}Y_T$</td>
<td>$-P_t\bar{\tau}Y_T$</td>
<td>0</td>
<td>$P_t(\bar{\tau}Y_T - Z)$</td>
<td>$P_t(\bar{\tau}Y_T - Z)$</td>
</tr>
<tr>
<td>GW</td>
<td>0</td>
<td>0</td>
<td>$P_t(\bar{\tau}Y_T - Z)$</td>
<td>$-P_t(\bar{\tau}Y_T - Z)$</td>
<td>0</td>
</tr>
</tbody>
</table>

Producers operate in both euro and drachma, given by the row labeled Pro in Table 2A and Table 2B. They begin the period with $Y_T$ euro and with $P_t\bar{\tau}Y_T$ drachma. In the first subperiod, they pay $P_t\bar{\tau}Y_T$ drachma to the government in taxes, and in the third, they spend the $Y_T$ euro on goods. They produce $Y_T$ goods and in the third subperiod, they sell them to government workers for $P_t(\bar{\tau}Y_T - Z)$ drachma, to other producers for $Y_T$ euro, and to foreign agents for $rb_T(1 + \bar{\tau})$ euro, inclusive of the government export-tax rebate. Producers are willing to accept drachma because they will need them at the beginning of next period to pay taxes, and they accept euro because they can use them to buy goods next period. Producers have $Y_T + rb_T(1 + \bar{\tau})$ euro and $P_t(\bar{\tau}Y_T - rb(1 + \bar{\tau}) - z)$ drachma at the end of the period.

The budget constraint for foreign creditors is in euro and is presented in the row labeled F of Table 2A. They begin the period with assets of $b_T$ Greek government bonds and $E^f_T$ euro. They receive $rb_T$ in interest income and spend this on goods from domestic producers. When $b_T > \bar{b}$, foreign consumption of domestic goods, equivalently net exports, is higher in the new equilibrium. End-of-period assets are unchanged from the beginning of the period.

Government workers transact in drachma only. Their budget constraint is described in Table 2B. They begin the period with no assets. They receive $P_t(\bar{\tau}Y_T - rb(1 + \bar{\tau}) - z)$
drachma in wages and spend everything on goods from producers, leaving themselves with no currency to carry to the end of the period. They are willing to accept payment of wages in drachma because they can use the drachma to buy goods, and drachma wages have equal purchasing power with euro wages in the private sector.

At the end of the period, the government operates a foreign exchange market in which it exchanges drachma for euro. Table 2C describes transactions in the foreign exchange market.

### Table 2C: Foreign Exchange Market in Ex post Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$D + \Delta D$</th>
<th>$E'$</th>
<th>$D'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0</td>
<td>$P_t \left( \bar{r}b (1 + \bar{\tau}) + z \right) + (P_{t+1} - P_t) \bar{\tau}Y_T$</td>
<td>$rb_T (1 + \bar{\tau})$</td>
<td>0</td>
</tr>
<tr>
<td>Pro</td>
<td>$Y_T + rb_T (1 + \bar{\tau})$</td>
<td>$P_t \left( \bar{\tau}Y_T - \bar{r}b (1 + \bar{\tau}) - z \right)$</td>
<td>$Y_T$</td>
<td>$P_{t+1} \bar{\tau}Y_T$</td>
</tr>
</tbody>
</table>

Producers enter the foreign exchange market with $Y_T + rb_T (1 + \bar{\tau})$ euro and with $P_t \left( \bar{\tau}Y_T - \bar{r}b (1 + \bar{\tau}) - z \right)$ drachma. They must sell euro and exit with $P_{t+1} \bar{\tau}Y_T$ drachma to pay taxes next period. The government enters the foreign exchange market with no euro and with $P_t \left( \bar{r}b (1 + \bar{\tau}) + z \right)$ drachma from undisbursed drachma tax collections. The government must sell existing drachma and create new ones to sell in exchange for sufficient euro to pay interest on debt and rebate export taxes to foreign agents for a total of $rb_T (1 + \bar{\tau})$.

Therefore, existing drachma of $P_t \left( \bar{r}b (1 + \bar{\tau}) + z \right)$ plus new drachma created of $(P_{t+1} - P_t) \bar{\tau}Y_T$ must be exchanged for $rb_T (1 + \bar{\tau})$ euro at price $P_t$.

$$P_t \left( \bar{r}b (1 + \bar{\tau}) + z \right) + (P_{t+1} - P_t) \bar{\tau}Y_T = P_t rb_T (1 + \bar{\tau}),$$
Solving for price inflation in drachma, and equivalently drachma depreciation yields

\[
\frac{P_{t+1} - P_t}{P_t} \tau Y_T = \pi \tau Y_T = r (1 + \bar{\tau}) (b_T - \bar{b}) - z. \tag{23}
\]

Domestic producers end the period with \(Y_T\) euro and with \(P_{t+1} \tau Y_T\) drachma. The government ends with \(rb_T (1 + \bar{\tau})\) euro. This is consistent with our initial assumptions, implying a steady-state equilibrium with unchanged real assets.

Equation (23) reveals that if the debt buyback is large enough that \(b_T = \bar{b}\), then the government can maintain wage payments at their initial value by setting \(z = 0\), and there is no drachma inflation after drachma are issued. If not, then either inflation \((\pi)\), or government wage reduction \((z)\), or both are necessary to produce the revenue to pay interest on euro debt and make export tax rebates.

We can check that the cash-in-advance constraints always hold by observing that all flow payments (negative sign) are preceded in a prior subperiod by a flow receipt of equal or greater magnitude (positive sign). Begin with euro cash-in-advance constraints in Table 2A. The government begins the period with \(r (1 + \bar{\tau}) b_T\) euro and disburses exactly that amount over the middle and final subperiods. It replenishes its euro in the foreign exchange market at the end of the period, by swapping drachma, and begins again. Producers begin the period with \(Y_T\) euro and use these for expenditures on goods in the third subperiod. They receive euro of \(Y_T + r (1 + \bar{\tau}) b_T\) in the third subperiod of which they retain \(Y_T\) to carry into the next period and use \(r (1 + \bar{\tau}) b_T\) to trade for drachma to carry into the next period. Foreign creditors receive \(rb_T\) euro as interest on debt in the middle subperiod and use this in the final subperiod to purchase goods.

Now, consider drachma cash-in-advance constraints using Table 2B. In the first subpe-
riod, the government receives \( P_t \bar{\tau}Y_T \) in drachma taxes and uses \( P_t (\bar{\tau}Y_T - Z) \) of these to pay government workers in the middle subperiod. It carries \( P_t Z \) drachma to the foreign exchange market to exchange for the euro it will need in the next period. Producers bring \( P_t \bar{\tau}Y_T \) in drachma to the beginning of the period from the foreign exchange market. They use them fully in the first subperiod to pay taxes, and replenish drachma from the sale of goods in the middle subperiod and foreign exchange transactions at the end. Government workers receive drachma wages in the middle subperiod and use them in their entirety in the third subperiod to purchase goods.

Note that total demand for money in the domestic economy is now \( P_t \bar{\tau}Y_T \) drachma plus \( Y_T + r (1 + \bar{\tau}) b_T \) euro, compared with total demand in the initial equilibrium of \( (1 + \bar{\tau}) Y \) euro. The increase in total money demand by \( (rb_T + Y_T - Y) (1 + \bar{\tau}) \) is due both to the government’s need to obtain euro in the foreign exchange market to make its payment obligations in euro, and to producers’ need to hold more money to purchase the larger quantity of private good.

4.2 Equilibrium Allocations and Prices

To complete the solution, we must solve for \( z \). The inflation tax on producers reduces producer after-tax wages, yielding the incentive for producers to move to the government sector. The government chooses \( z \), effectively choosing how much to reduce government spending, to maintain the optimal ratio of \( G_T \) to \( C_T \). The reduction in wage payments to government workers reallocates labor toward the private sector. Labor, allocated according to

\[
N_T = \frac{\alpha + \lambda r b_T}{\alpha (1 + \lambda)} \quad 1 - N_T = \frac{\lambda (\alpha - r b_T)}{\alpha (1 + \lambda)},
\]

(24)
maintains the ratio of government spending to consumption at $\lambda$, yielding the optimal allocation. Production of each good is given by $\alpha$, multiplied by employment, implying that the change in production of the private good is given by

$$\Delta Y = Y_T - Y = \alpha (N_T - N) = \frac{\lambda}{1 + \lambda} r (b_T - \bar{b}),$$

where $\Delta$ denotes change. Consumption is output of the private good less net exports of $rb_T$, yielding

$$C_T = \frac{\alpha - rb_T}{1 + \lambda}.$$

Note that for $b_T > \bar{b}$, production of the private good is higher and consumption is lower, allowing the increase in net exports.

The government chooses $z$ to effect this allocation. Equating wages in each sector after substituting from equation (23) for $\pi \bar{r} Y_T$ yields

$$\frac{\pi Y_T - r (1 + \bar{r}) \bar{b} - z}{1 - N_T} = \frac{Y_T - [r (1 + \bar{r}) (b_T - \bar{b}) - z]}{N_T}.$$  

Substituting from equation (24) for the allocation of labor and using equation (14) to express $1 + \bar{r}$ in terms of $\bar{b}$, yields the solution for the optimal value for $z$ as

$$z = \left( \frac{(1 + \lambda)^2 \alpha - (\alpha + \lambda rb_T)}{(1 + \lambda) (\alpha - r \bar{b})} \right) r (b_T - \bar{b}).$$

Substituting into the either side of equation (26) yields equilibrium wages as

$$W_T = \alpha \left( \frac{\alpha - rb_T}{\alpha - r \bar{b}} \right).$$

Note that if the government can get debt down to $\bar{b}$, then wages are unchanged from the initial equilibrium. This is because the seigniorage generated by the debt buyback is sufficient to eliminate both explosive government debt and current account imbalance such
that no wage reduction is needed. However, if government debt after the buyback remains higher than \( \bar{b} \), then producer wages fall by the inflation tax, allowing the government to reduce wages to government workers without changing the ratio of consumption to government spending.

Substituting equation (27) into equation (23) allows us to solve for drachma inflation in the dual currency equilibrium as

\[
\pi_T = \frac{r (b_T - \bar{b})}{\alpha \lambda + r \bar{b}},
\]

implying positive drachma inflation only when the debt buyback is not large enough to bring debt down to \( \bar{b} \).

We can use the results above to consider the effect of the introduction of the second currency on competitiveness and on net exports in the two cases. If the debt buyback is large enough to put debt at its fiscal limit, then the buyback alone eliminates the current account deficit without additional drachma inflation or a fall in wages. When debt and the tax rate are both at their fiscal limits, net exports are large enough to pay interest on government debt, such that the current account is balanced. There is no increase in net exports in equilibrium, and no increase in competitiveness.

However, when the debt buyback is not large enough and debt remains above its fiscal limit, then drachma inflation is necessary. In the new equilibrium, wages in both sectors fall and production is reallocated away from the government good toward the private good. The supply of the private good increases while consumption falls. Typically, these two forces would create a fall in the price of the good. However, with the small-country assumption of infinitely elastic demand, price does not fall, but net exports do
rise, reflecting the increased competitiveness.

Additionally, it is important to note that in the case of \( b_T > \bar{b} \), the explosive government debt problem is not solved with new seigniorage alone. Since money growth acts as an inflation tax on producers, the government is able to reduce wage payments to workers without reducing the ratio of \( G \) to \( C \) required by the fiscal limits. And the reduction in government wages shifts workers from the government sector to the private sector, enabling more tax revenue for a given tax rate. Therefore, the government balances its budget with the optimal combination of a reduction in expenditures on government workers, a shift in production creating increased tax revenue, and increased seigniorage.

4.3 Currency Flows in the Period \( T \) of Policy Change

Consider budget constraints in euro and drachma in the period of policy change, labeled \( T \). We assume that the government changes payments to workers in period \( T \), thereby reallocating labor toward the private good, increasing its production from \( Y \) to \( Y_T \). We continue to denote values in the initial equilibrium without subscripts and those in the period of policy change and beyond with the subscript \( T \). Values which are changing over time continue to receive time subscripts.

We describe euro flows in Table 3A and drachma flows in Table 3B. At the beginning of the period of policy change, period \( T \), the government collects taxes from producers of private goods in euro equal to \( \tau Y \). It uses some of these euro to pay interest on debt \( (rb_{T-1}) \), and rebate taxes on exports\(^4\) of \( \left[ \frac{\tau (r\bar{b} + \frac{\Delta Y + z}{1+r})}{1+\tau} \right] \), but it refrains from using any to pay government workers. Instead, much of the euro tax revenue is used to buy back

\(^4\) We derive the value for exports below.
government bonds. However, the government cannot use all of the net tax revenue for bond buybacks. The government must save some of the current euro tax revenue for use next period because, from Table 2A, the government will need a total of $rb_T (1 + \bar{\tau})$ euro in period $T + 1$ to pay interest on euro-denominated debt and make export tax rebates. The government will receive no euro tax revenue next period, and it cannot obtain all of these euro in the foreign exchange market in period $T$.

The government is limited in the amount of euro it can obtain in the foreign exchange market to the quantity that domestic producers want to sell. This is because foreign agents have no use for drachma. We show below that domestic producers want to sell $r \bar{b} (1 + \bar{\tau}) + z$ euro. Therefore, the government must retain the difference between its euro obligations in period $T + 1$ of $rb_T (1 + \bar{\tau})$ and the quantity it can obtain from the foreign exchange market at the end of period $T$ for a total of $r(b_T - \bar{b}) (1 + \bar{\tau}) - z$ end-of-period euro. The quantity of bonds outstanding after the bond buyback is $b_T$, and is implicitly given by the budget constraint in row G of Table 3A, which can be expressed as

$$-(b_T - b_{T-1}) = \bar{\tau}Y - r (1 + \bar{\tau}) b_T - \Delta Y - r (b_{T-1} - \bar{b}) + \frac{\Delta Y + z}{1 + \bar{\tau}}. \quad (30)$$

<table>
<thead>
<tr>
<th></th>
<th>$b$</th>
<th>$E$</th>
<th>tax</th>
<th>int</th>
<th>goods</th>
<th>$b'$</th>
<th>$E'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>$b_{T-1}$</td>
<td>0</td>
<td>$\bar{\tau}Y$</td>
<td>$-rb_{T-1}$</td>
<td>$-\bar{\tau} \left( r\bar{b} + \frac{\Delta Y + z}{1 + \bar{\tau}} \right)$</td>
<td>$-b_T$</td>
<td>$r (1 + \bar{\tau}) (b_T - \bar{b}) - z$</td>
</tr>
<tr>
<td>Pro</td>
<td>0</td>
<td>$(1 + \bar{\tau}) Y$</td>
<td>$-\bar{\tau}Y$</td>
<td>0</td>
<td>$r (1 + \bar{\tau}) \bar{b} + \Delta Y + z$</td>
<td>0</td>
<td>$Y_T + r (1 + \bar{\tau}) \bar{b} + z$</td>
</tr>
<tr>
<td>F</td>
<td>$b_{T-1}$</td>
<td>$E'$</td>
<td>0</td>
<td>$rb_{T-1}$</td>
<td>$-r\bar{b} - \frac{\Delta Y + z}{1 + \bar{\tau}}$</td>
<td>$b_T$</td>
<td>$E' + \bar{\tau}Y - r (1 + \bar{\tau}) b_T - \Delta Y$</td>
</tr>
</tbody>
</table>

The government meets its obligations to pay government workers by issuing drachma. We assume that the government issues drachma in the quantity that producers would
need to pay their taxes next period if $P_{T+1} = P_T$, yielding $P_T \bar{\tau} Y_T$, the first entry in Table 3B.\footnote{We show below that in equilibrium $P_{T+1} = P_T$.} The government uses $P_T \left( \bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z \right)$ of these drachma to pay workers, retaining $P_T \left( r \bar{b} (1 + \bar{\tau}) + z \right)$ drachma to trade for euro in the foreign exchange market at the end of the period.

Table 3B: Drachma Flows in the Period of Policy Change

<table>
<thead>
<tr>
<th></th>
<th>$\Delta D$</th>
<th>$W (1 - N)$</th>
<th>goods</th>
<th>$D'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>$P_T \bar{\tau} Y_T$</td>
<td>$-P_T \left( \bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z \right)$</td>
<td>0</td>
<td>$P_T \left( r \bar{b} (1 + \bar{\tau}) + z \right)$</td>
</tr>
<tr>
<td>Pro</td>
<td>0</td>
<td>0</td>
<td>$P_T \left( \bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z \right)$</td>
<td>$P_T \left( \bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z \right)$</td>
</tr>
<tr>
<td>GW</td>
<td>0</td>
<td>$P_T \left( \bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z \right)$</td>
<td>$-P_T \left( \bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z \right)$</td>
<td>0</td>
</tr>
</tbody>
</table>

Producers operate in both euro and drachma. Producers enter the period with $(1 + \bar{\tau}) Y$ euro as shown in Table 3A, row Pro. They pay current taxes in euro of $\bar{\tau} Y$, receive euro in expenditures by producers of $Y$ and make the same expenditures themselves. Foreign creditors purchase the excess supply of domestic goods,\footnote{Government workers spend the euro value of their wages, given by $\bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z$, and private producers spend $Y$. Since the euro price of goods is $\frac{1}{1 + \tau}$, domestic agents purchase $\frac{Y_T - r b (1 + \tau) - z + Y}{1 + \tau}$ goods. Excess supply, and equivalently purchases by foreign creditors, is given by the difference between total production of $Y_T$ and domestic spending, yielding $\frac{\Delta Y + z}{1 + \tau} + r \bar{b}$.} given by $\frac{\Delta Y + z}{1 + \tau} + r \bar{b}$, and the government makes tax rebates on foreign purchases of of $\bar{\tau} \left( \frac{\Delta Y + z}{1 + \tau} + r \bar{b} \right)$. Producers end the period with $Y$ euro from other producers and with $r \bar{b} (1 + \bar{\tau}) + \Delta Y + z$ from foreign sales for the total given in the $E'$ column of Table 3A. Producers receive $(\bar{\tau} Y_T - r \bar{b} (1 + \bar{\tau}) - z)$ drachma from workers and end the period with these drachma, Table 3B, row Pro.

The budget constraint for foreign creditors in given row F of Table 3A. We make the small open economy assumption that the foreign country accommodates the home
country’s policy with no change in world interest rates. Therefore, in the middle subperiod, they purchase the quantity of bonds that the home country wants to sell, given by equation (30) for $b_T - b_{T-1}$. In the middle subperiod, foreign creditors also receive $rb_{T-1}$ in interest on outstanding bonds. They spend $r\bar{b} + \frac{\Delta Y + z}{1 + \bar{\tau}}$ on domestic goods in the third subperiod, leaving them with additional euro at the end of the period of $\bar{\tau}Y - rb_T (1 + \bar{\tau}) - \Delta Y$. Effectively, the debt buyback has transferred $\bar{\tau}Y - rb_T (1 + \bar{\tau}) - \Delta Y$ euro to foreign creditors.

Row GW of Table 3B contains the budget constraint for government workers. They receive $P_T (\bar{\tau}Y_T - r\bar{b} (1 + \bar{\tau}) - z)$ drachma as wages and use them in their entirety for purchases of goods from producers.

The government operates a foreign exchange market at the end of the period, enabling producers to meet their drachma tax obligations at the beginning of the next period and enabling itself to meet its euro bond interest and export tax rebate obligations. Table 3C lists quantities of drachma and euro which producers and the government bring into the foreign exchange market at the end of the period together with the quantities that they want after engaging foreign exchange transactions.

Table 3C: Foreign Exchange Market in the Period of Policy Change

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$D$</th>
<th>$E'$</th>
<th>$D'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>$r (b_T - \bar{b}) (1 + \bar{\tau}) - z$</td>
<td>$P_T (r\bar{b} (1 + \bar{\tau}) + z)$</td>
<td>$rb_T (1 + \bar{\tau})$</td>
<td>0</td>
</tr>
<tr>
<td>Pro</td>
<td>$Y_T + r\bar{b} (1 + \bar{\tau}) + z$</td>
<td>$P_T (\bar{\tau}Y_T - r\bar{b} (1 + \bar{\tau}) - z)$</td>
<td>$Y_T$</td>
<td>$P_{T+1}\bar{\tau}Y_T$</td>
</tr>
</tbody>
</table>

The government would like to purchase the difference between its exit value of euro of $rb_T (1 + \bar{\tau})$ and its entrance value of $r (b_T - \bar{b}) (1 + \bar{\tau}) - z$, yielding desired government purchases of $r\bar{b} (1 + \bar{\tau}) + z$ euro. Producers would like to sell the difference between
their entry value of \( Y_T + r\tilde{b}(1 + \tau) + z \) and their exit value of \( Y_T \), yielding the same value for desired euro sales. Now consider drachma. The government would like to sell all \( P_T (r\tilde{b}(1 + \tau) + z) \) drachma, while producers would like to buy \( (P_{T+1} - P_T)\tau Y_T + P_T (r\tilde{b}(1 + \tau) + z) \). The two are equal when there is no inflation in the initial period after the policy change. Therefore, \( r\tilde{b}(1 + \tau) + z \) euro trade for \( P_T (r\tilde{b}(1 + \tau) + z) \) drachma, yielding an initial exchange rate of unity.

Adding end-of-period euro held by the government and the private sector, using entries in rows G and Pro in column \( E' \) of Table 3A yields \( Y_T + rb_T(1 + \tau) \). Adding end-of-period euro held by the government and producers from Table 1, prior to the policy change, yields \( (1 + \tau)Y \), implying that the change in euro held domestically is \(-\tau Y + rb_T (1 + \tau) + \Delta Y \). The reduction in euro in Greece due to the bond buyback equals the increase in euro in the rest of the monetary union.

Foreign agents have no additional demand for euro. Therefore, they sell their additional euro to the ECB in exchange for government bonds. The ECB buys the \( \tau Y - rb_T (1 + \tau) - \Delta Y \) additional euro as it conducts open-market operations to keep the euro price level constant. Since the ECB has fewer bonds, it has less seigniorage. Since Greece is a small country in the EMU, its seigniorage receipts fall by only a small fraction of the total reduction in ECB seigniorage revenue. In the model, we have set this amount to zero using the small country approximation.

In contrast, the Greek central bank buys \( \tau Y_T \) new government drachma bonds, allowing the drachma money supply to increase by \( \tau Y_T \). The Greek central bank’s purchase of government bonds creates seigniorage. Effectively, the ECB has lost seigniorage due to a reduction in central bank bond holdings of \( \tau Y - rb_T (1 + \tau) - \Delta Y \), while the Greek central
bank has gained seigniorage due to an increase in its bond holdings of $\tilde{\tau} Y_T$. Total seigniorage has increased by the relatively small increase in total central-bank bond holdings of $(rb_T + \Delta Y) (1 + \tilde{\tau})$, due to the increase in total money demand in Greece. Greece has succeeded in redistributing seigniorage, something the ECB cannot do alone. In the goods market, this redistribution of seigniorage implies that Greece will never supply goods in exchange for the reduction in bonds, supplying euro instead.

Could all countries participate in creating more seigniorage for themselves? If all countries reissued their own currencies, then the ECB would have to sell the bonds that the aggregate of individual-country central banks purchase. The ability to earn seigniorage would be transferred from the ECB to individual-country central banks by the change in ownership of bonds. If all countries reissued their own currencies, proportionate to size, then each would lose ECB seigniorage roughly equal to the own seigniorage that it gains. Some second order changes would occur due to the ECB’s seigniorage allocation formula, but gains and losses among countries would offset each other and be small. A small open economy can effect a redistribution of seigniorage, but since the policy is effectively a redistribution, all countries together cannot redistribute seigniorage to their own advantage.

If this seigniorage is not large enough to eliminate the government and current account deficit, Greece can create additional seigniorage by allowing drachma inflation accompanied by drachma depreciation. Drachma inflation reduces the after-tax (inflation and value-added tax) wages to producers, allowing the government to reduce wage payments to government workers while keeping $G_T = \lambda C_T$. The domestic euro price of the good is unchanged at $1 + \tilde{\tau}$, while foreign agents pay domestic agents the same initial price
of unity due to the export tax rebate. With the cash-in-advance constraint, the higher inflation does not reduce the real demand for drachma. Therefore, the implications of this model are reasonable only when the equilibrium value of the drachma inflation is low. This additional seigniorage due to drachma inflation has no implications for overall euro seigniorage in the monetary union.

5 Quantitative Implications

How much seigniorage could Greece have raised with a dual currency? Explicitly, if Greece had switched to a dual currency, could the country have avoided the partial default that followed the crisis in early 2010? The model presented above is highly-stylized and can therefore only be a first step in addressing this question. However, the model does imply that a monetary-union government can redirect its tax receipts at a point in time from making expenditures toward buying down debt. Then it can print its own currency to make payments as long as it generates demand for this currency by requiring redenomination of tax payments in the new currency. This does translate into something a real-world government could do. The question about magnitudes hinges in part on how large a fraction of annual tax receipts the government would have available to use for the debt buy-down at a particular point in time.

The less frequently the government collects taxes, the larger the tax take at each collection point in time (and the more cash agents must acquire to pay taxes), the larger the debt buy-down can be. In reality, governments receive tax revenues almost continuously, so it is not obvious how to calibrate a period. The same is true for agents receiving and making payments in money, but we abstract from this by modeling cash-in-advance using
periods. Our cash-in-advance model is meaningful quantitatively only if the abstraction of treating tax payments as occurring in periods, prior to which agents must hold cash, is reasonable. With this assumption, we make some rough calculations allowing the period to be a quarter, and others allowing the period to be a month.

We take data from Eurostat. Debt data is quarterly over the period 2000Q1 to 20013Q2. Gross Greek government debt to GDP at the beginning of the world-wide financial crisis in 2008Q4 is 1.129. This value matches the largest quarterly value in the sample, that of 2000Q1. Since Greek government debt did not embark on an explosive path at this value of debt, we set \( \bar{b}/\alpha = 1.129 \), where \( \alpha = Y + G \), equivalently production of the private good plus production of the government good, yielding GDP. Debt/GDP increases in 2009Q1 to 1.213, and by 2009Q4, it has risen to 1.297. The Greek debt crisis began in January 2010 with the extreme widening in interest spread. We calibrate \( \lambda \) as the average value of government spending relative to consumption over the period 2006-2010, yielding \( \lambda = 0.205 \). We set the upper bound on the tax rate as the largest value of government revenue relative to GDP over the same period, yielding a tax rate on GDP, which we denote as \( \hat{\tau} \), of 0.407. Our measure for \( \hat{\tau} \) is for tax revenue relative to production in the private sector where \( \hat{\tau} (Y + G) = \hat{\tau} Y \) such that \( \hat{\tau} \frac{Y}{\alpha} = \hat{\tau} \), and substituting from equation (16), \( \hat{\tau} = \hat{\tau} \frac{1 + \lambda}{1 + \lambda \bar{b}/\alpha} \). Additionally, since Greece is a growing economy, not a static one as in the model, the interest rate should be the growth-adjusted interested rate. We let the growth-adjusted interest rate be one percent, which could be high, had Greece been able to restore substantial growth by avoiding default.

These data points do not satisfy the budget constraint in the model since the model does not explicitly include transfers. Therefore, to make quantitative assessments, we
must add transfer payments to the model. In the appendix, we add transfers by assuming
that the tax rate is high enough to finance transfers to both types workers in equal
amounts. Under the assumption that the government’s budget is balanced in the initial
equilibrium at \( \hat{\tau} = 0.407 \) and \( \hat{b} = 1.129 \), the required value for transfers relative to GDP is
0.224. In the equilibrium with transfers, allocations are unchanged, but prices including
inflation, do change. The size of the bond buyback does not change since transfers are paid
in drachma after enactment of the policy. Analytical solutions are given in the appendix.

Consider whether Greece could have staved off the crisis by acting at the end of
2009Q1. We can write equation (30) as a quadratic equation in the difference between
post-policy debt relative to GDP \((b_T/\alpha)\) and its upper bound \((\bar{b}/\alpha)\) by substituting for
\( z \) from equation (33) in the appendix, and for \( \Delta Y \) from equation (25) and dividing by \( \alpha \)
to yield

\[
0 = -\left( \frac{r}{1 + \lambda} \right)^2 \lambda \left( \frac{b_T}{\alpha} - \frac{\bar{b}}{\alpha} \right)^2 + \left[ 1 - r \left( \hat{\tau} + \frac{\hat{\tau} \lambda}{(1 + \hat{\tau})(1 + \lambda)} + \frac{1 + \lambda r \bar{b}}{(1 + \lambda)^2} \right) \right] \left( \frac{b_T}{\alpha} - \frac{\bar{b}}{\alpha} \right) \\
+ \frac{\hat{\tau} Y}{\alpha} + (1 - r \hat{\tau}) \frac{\bar{b}}{\alpha} - (1 + r) \frac{b_{T-1}}{\alpha}.
\]

The final adjustment we must make is to adjust flow values from annual to either quarterly
or monthly. We adjust flow values for production \((Y)\) and the interest rate \((r)\) to quarterly
(monthly) values by multiplying by 1/4 (1/12).

Table 4A contains a summary of results, letting the cash-in-advance period be both
one quarter and one month. The value of debt relative to GDP in the period of the
buyback is given by \( b_{T-1}/\alpha \), and the level after the buyback is given by \( b_T/\alpha \). We measure
inflation at annual rates.

Table 4A: Debt and Inflation after Policy Implementation \((r = 0.01)\)
<table>
<thead>
<tr>
<th>date</th>
<th>period</th>
<th>$b_{T-1}/\alpha$ (%)</th>
<th>$b_T/\alpha$ (%)</th>
<th>$\pi$ (%)</th>
<th>$W_T/W^*$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009Q1</td>
<td>quarter</td>
<td>121.3</td>
<td>111.6</td>
<td>0.0</td>
<td>100.00</td>
</tr>
<tr>
<td>2009Q1</td>
<td>month</td>
<td>121.3</td>
<td>118.1</td>
<td>0.13</td>
<td>99.95</td>
</tr>
<tr>
<td>2009Q4</td>
<td>quarter</td>
<td>127.9</td>
<td>118.2</td>
<td>0.14</td>
<td>99.94</td>
</tr>
<tr>
<td>2009Q4</td>
<td>month</td>
<td>127.9</td>
<td>124.7</td>
<td>0.30</td>
<td>99.88</td>
</tr>
<tr>
<td>2013Q2</td>
<td>quarter</td>
<td>169.1</td>
<td>159.7</td>
<td>1.19</td>
<td>99.50</td>
</tr>
<tr>
<td>2013Q2</td>
<td>month</td>
<td>169.1</td>
<td>166.0</td>
<td>1.35</td>
<td>99.45</td>
</tr>
</tbody>
</table>

With a period of one quarter, had Greece acted in 2009Q1, the debt buyback would have been 9.7% of GDP. This would have brought debt down to 111.6% of GDP. Debt would have been below its upper bound, and in the absence of additional negative shocks, Greek debt would have been on a sustainable path. With debt below the upper bound, there is no need for inflation or for a fall in wages, and the tax rate can fall below its upper bound. However, if the period is only a month, then the debt buyback would have been smaller, bringing debt down to 118.1% of GDP and requiring a combination of additional seigniorage revenue and reduction in government spending on workers to put Greece on a sustainable path. Inflation would have risen to 0.13%.\(^7\)

However, there were more shocks to come, and Greece did not act in the first quarter when debt initially became unsustainable. If Greece had acted at the end of 2009Q4, then the magnitudes of the debt buybacks would have changed little, but the post-buyback values for debt would have risen to 118.2% of GDP if the period is a quarter and to 124.7% if the period is one month. The necessary drachma inflation would have risen

\(^7\) The magnitude would be slightly smaller to the extent that all government debt is not held abroad and that not all taxes are value-added taxes. Both reduce the magnitude of tax rebates on exports.
from zero to 0.14% for a period of one quarter, and from 0.13% to 0.30% for a period of one month. Were Greece to try to solve its debt problem now, with debt reaching 169.1% of GDP in 2013Q2, then for period length of one quarter, inflation would be 1.19%, while for period length of one month, inflation would rise to 1.35%.

These inflation rates do not appear intolerably high. In particular, they do not appear high enough for agents to seriously economize on their holdings of drachma, invalidating the cash-in-advance assumption. Required inflation is low primarily because the government revenue short-fall is small due to a small value for the real growth-adjusted interest rate. But, inflation rates are also kept down because optimal policy in the dual-currency regime requires that Greece solve its revenue problem not only by increasing seigniorage, but also by reducing payments to government workers to prevent the government sector from becoming too large relative to domestic consumption. Under the alternative scenarios above, wages do fall, but by little, since the revenue short-fall is small due to the low value of the growth-adjusted interest rate. Using equation (34) in the appendix, our calibration yields values for wages in the new equilibrium of 100% to 99.45% of their initial values.

There are scenarios in which these magnitudes are too small. If Greece is not successful in restoring growth, then the inflation rates would be higher due to a larger growth-adjusted interest rate, yielding a larger revenue short-fall. And if Greece is not successful in convincing the world that it will not default, interest costs would carry a default-risk premium, implying substantially higher inflation. If we raise the growth-adjusted interest rate to 0.03, inflation rates remain less than one percent, had Greece acted in 2009, but
rise closer to 4% for recent values of debt.\(^8\) Values for the higher growth-adjusted interest rate are contained in Table 4B.

Table 4B: Debt and Inflation after Policy Implementation \((r = 0.03)\)

<table>
<thead>
<tr>
<th>date</th>
<th>period</th>
<th>(\frac{b_{T-1}}{\alpha}) (%)</th>
<th>(\frac{b_T}{\alpha}) (%)</th>
<th>(\pi) (%)</th>
<th>(\frac{W_T}{W}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009Q1</td>
<td>quarter</td>
<td>121.3</td>
<td>112.4</td>
<td>0.0</td>
<td>1.00</td>
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<tr>
<td>2009Q1</td>
<td>month</td>
<td>121.3</td>
<td>118.4</td>
<td>0.42</td>
<td>99.83</td>
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<tr>
<td>2009Q4</td>
<td>quarter</td>
<td>127.9</td>
<td>119.2</td>
<td>0.48</td>
<td>99.94</td>
</tr>
<tr>
<td>2009Q4</td>
<td>month</td>
<td>127.9</td>
<td>125.0</td>
<td>0.92</td>
<td>99.62</td>
</tr>
<tr>
<td>2013Q2</td>
<td>quarter</td>
<td>169.1</td>
<td>161.1</td>
<td>3.68</td>
<td>99.50</td>
</tr>
<tr>
<td>2013Q2</td>
<td>month</td>
<td>169.1</td>
<td>166.4</td>
<td>4.09</td>
<td>98.31</td>
</tr>
</tbody>
</table>

Finally, we can address the quantitative implications for the loss of seigniorage in the monetary union. The reduction in the quantity of euro in the system is given by \(\tau Y_{T-1} - rb_T (1 + \tau) - \Delta Y\), which is between 2.95% and 9.76% of Greek GDP in our calibrations. However, since Greece has such a small share of total monetary union GDP, with a recent peak value in 2009 of 2.6%, this loss would be between 0.08% and 0.25% of EMU GDP, and the EMU experiences this loss for one year only. Therefore, the currency swap due to the debt buyback has a negligible effect on euro-area seigniorage. There are no effects of Greece’s drachma inflation on euro-area seigniorage.

\(^8\) With the higher interest rate, the value of transfers relative to GDP which balances the budget falls to 0.195.
6 Conclusion

A small monetary-union country, which is experiencing explosive growth in external government debt, needs additional revenue to restore government budget balance and possibly additional exports to restore current account balance. In the paper, we label the monetary union the EMU and the small country Greece, but the arguments apply in general. We assume that the small country has raised the tax rate to the fiscal limit, such that further increases yield no additional tax revenue. They have also reduced spending on government workers such that provision of the government good has fallen to its fiscal limit. We consider a policy whereby the country uses the bulk of its current euro tax revenue to pay down debt, continues to fully service its remaining euro obligations, reissues its own currency to pay government workers, and begins collecting taxes only in domestic currency. Tax collection in domestic currency generates demand to hold the newly-issued domestic currency. There is no default on euro-denominated assets.

We demonstrate that such a policy effectively transfers seigniorage away from other monetary-union countries toward the small country enacting the policy. The act of reissuing domestic currency together with the assumed ECB policy of fixing the price level, effectively transfers some ability to earn seigniorage revenue away from the ECB toward the Greek central bank. The Greek government is able to fully appropriate this seigniorage revenue accruing to its own central bank. If government debt is not initially too high, then this seigniorage revenue can be large enough to restore government budget and current account balance.

Such a policy is less effective when practiced by a large country and is not effective at
all when practiced by all countries. Issuance by a large country of its own currency would require a larger sale of bonds by the ECB as it maintains a fixed price level, reducing seigniorage accruing to all, including the country issuing the new currency. And if all countries reissued their currencies, proportionate to their size, then there would be no first order redistribution because aggregate euro seigniorage would fall by the amount that the aggregate of each country’s own seigniorage from its central bank increased. Redistribution of seigniorage would be second-order small, with winners and losers, based on the ECB’s seigniorage allocation formula.

When post buyback government debt is higher than the fiscal limit based solely on ordinary tax revenue, the government needs additional seigniorage revenue on an ongoing basis. And the country needs additional exports to restore current account balance. Additional revenue can be generated by allowing the domestic currency to grow after its initial issue, and its exchange rate to depreciate. This currency depreciation adds an inflation tax to the price of goods in the domestic country, reducing the purchasing power of private-sector wages.

The reduction in after-tax wages for producers allows the government to reduce its expenditures on government workers while maintaining the optimal ratio of G to C. Wages, net of both ordinary and inflation taxes, fall in both sectors, and employment is directed away from the government sector towards the private sector. The fall in wages reduces domestic consumption which, together with the increase in output of the private good, raises net exports. The depreciating domestic currency does not make Greek goods relatively cheaper, as suggested by some accounts of leaving the euro, because the law of one price continues to hold in the model. However, it does increase competitiveness and
net exports through wage reduction and reallocation of labor toward the production of the private good. There are no consequences of this additional seigniorage for total euro seigniorage in the monetary union.

Rough quantitative calculations suggest that Greece would have been able to eliminate its explosive debt problem had it acted at the end of 2009Q4 and had it received no additional negative shocks, at the cost of permanent drachma inflation of less than one percentage point. Debt is so large now that the inflation costs could exceed 1% a year, possibly reaching 4%.

7 Appendix: Dual Currency Equilibrium with Transfers

We add transfer payments to the model by assuming that the tax rate is large enough to finance transfer payments to both types of workers of equal amounts in the middle subperiod. With this assumption, equilibrium allocations are unchanged, but prices do change.

Transfer payments reduce revenues available to pay government wages at each value of government debt. This changes the optimal value of the tax rate at each value of government debt. To solve for the tax rate which yields the optimal allocation of labor, first solve for consumption. Let $T$ denote transfers per worker such that producers receive $TN$ while government workers receive $T(1 - N)$. Producers spend wages net of taxes plus transfer payments $(\alpha + T)N$, while government workers also spend wages plus transfer payments $((\alpha + T)(1 - N))$. Adding expenditures and realizing that the price of consumption is
$1 + \bar{\pi}$ yields consumption

$$C = \frac{\alpha + T}{1 + \bar{\pi}}.$$  

The optimal value for consumption with debt at its upper bound is given by setting $b = \bar{b}$ in equation (18). Equating these two values for consumption and solving for the tax rate yields

$$1 + \bar{\pi} = \frac{(\alpha + T)(1 + \lambda)}{\alpha - \bar{r}b}. \quad (31)$$

We derive $z$ by equating wages across sectors with labor allocated optimally according to equations (24). Wages available to pay government workers are reduced by the need to make transfer payments. Equating wages requires

$$\frac{\bar{\pi}Y - r(1 + \bar{\pi})\bar{b} - T - z}{1 - N} = \frac{Y - \left[r(1 + \bar{\pi})(b_T - \bar{b}) - z\right]}{N}. \quad (32)$$

Substituting from equation (24) for the allocation of labor and from equation (31), for $1 + \bar{\pi}$ yields the solution for the optimal value for $z$ as

$$z = \left(\frac{(1 + \lambda)^2 \alpha - (\alpha + \lambda r b_T)}{(1 + \lambda)(\alpha - \bar{r}b)}\right) \left(\frac{\alpha + T}{\alpha}\right) r (b_T - \bar{b}). \quad (33)$$

Substituting into the either side of equation (32) yields equilibrium wages as

$$W = \frac{\alpha(\alpha - r b_T) - r(b_T - \bar{b})T}{\alpha - \bar{r}b}. \quad (34)$$

Substituting equation (33) into equation (23) allows solution for drachma inflation as

$$\pi = \frac{(1 + \bar{T})}{\lambda \alpha + \bar{r}b + T(1 + \lambda)} \cdot r (b_T - \bar{b}). \quad (35)$$
References


