Growth 1

Chapter 6
1. Motivation - why study growth

2. Brief history of growth

3. Sources of growth - accounting

4. Solow growth model
1 Motivation - why study growth

- Per capita GDP measures the standard of living (imperfectly)

- Small changes in growth rates can have large effects on per capita GDP
  
  - Rule of 70 - The number of years for a growing variable to double in size is given by 70 divided by the growth rate expressed as a percent.

  - From 1950-1973, the US grew at 2.4% per year. It would take \( \frac{70}{2.4} = 29.167 \) years for its per capita GDP to double.

  - From 1973-2000, the US grew at 2.1% per year. It would take \( \frac{70}{2.1} = 33.33 \) years to double per capita GDP, an additional 4 years if growth is only .3 percentage points lower!
2 Brief History (Lucas 2004)

- Until about 1750 or 1800, per capita incomes were roughly constant at subsistence
  - Malthus and the ‘dismal science’ - productivity improvements supported a larger population allowing more births + survivals

- After 1800, output growth increased and population growth fell, so that per capita growth increased
  - Large divergence across countries and regions of the world
  - Some countries grew rapidly early, others grew rapidly later, and others have not yet begun to grow
• Period 1960-2000 world output-per capita increased at 2.3% per year even though some countries have not grown at all

• US per capita GDP in 1990 was 18 times larger than per capita GDP in the world’s poorest countries

• What caused some countries to leave the Malthusian trap early, late, or not yet?
3 Growth Accounting

3.1 Assumptions

- Cobb-Douglas Production function

\[ Y = AK^\alpha N^{1-\alpha} \]

- Constant returns to scale - double inputs and double outputs

\[ 2Y = A (2K)^\alpha (2N)^{1-\alpha} \]

- Capital’s share of income is \( \alpha \)

- Labor’s share of income is \( 1 - \alpha \)
3.2 Compute sources of output growth

- Empirical estimates for capital's share are about $\alpha = .3$

- Growth-accounting equation: Compute growth rate for output = percentage rate of change of output

\[
\ln Y = \ln A + \alpha \ln K + (1 - \alpha) \ln N
\]

totally differentiate with respect to $Y, A, K, N$.

\[
\frac{dY}{Y} = \frac{dA}{A} + \alpha \frac{dK}{K} + (1 - \alpha) \frac{dN}{N}
\]

- Empirically estimate percentage change in capital and employment. Empirically estimate the percentage change in output. The residual is pro-
ductivity.

\[
\frac{dA}{A} = \frac{dY}{Y} - \left[ \alpha \frac{dK}{K} + (1 - \alpha) \frac{dN}{N} \right]
\]

- Note that a given percentage change in the capital stock changes output by only \( \alpha \) while a given change in productivity changes output by the same amount. Why?
Sources of growth in US

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<tbody>
<tr>
<td>Labor growth</td>
<td>1.42</td>
<td>1.40</td>
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<td>1.34</td>
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<td>Capital growth</td>
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<td>Total input growth</td>
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<td>1.82</td>
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<td>Productivity growth</td>
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<td>1.53</td>
<td>-0.27</td>
<td>1.02</td>
<td>1.06</td>
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<tr>
<td><strong>Total output growth</strong></td>
<td><strong>2.54</strong></td>
<td><strong>3.70</strong></td>
<td><strong>1.55</strong></td>
<td><strong>2.92</strong></td>
<td><strong>3.23</strong></td>
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4 Why Did Productivity Growth Slow (temporarily) after 1973?

- Measurement - inadequate accounting for quality improvements
- Legal changes - regulations for pollution control and worker safety, crime, and declines in educational quality
- Oil prices - large increases in oil prices reduced productivity because less imported oil to work with
- New industrial revolution - learning process for information technology from 1973-1990
5 Solow Growth Model

5.1 Assumptions

- Cobb-Douglas production function

\[ Y_t = AK_t^\alpha (Q_tN_t)^{1-\alpha} \]

- time subscripts

- \( Q_t \) describes efficiency of labor - for now, set \( Q_t = 1 \). We will change this assumption later.
• No government and no foreign sector, so goods market equilibrium:

\[ S_t = I_t \]

• Constant marginal propensity to save out of income

\[ S_t = sY_t \]

• Capital accumulation

\[ K_{t+1} = K_t (1 - d) + I_t \]

where \( d \) denotes depreciation.

• Workers increase in number at rate \( n \) due to population growth

\[ N_{t+1} = N_t (1 + n) \]
5.2 Solve for long-run equilibrium in which can express long-run values as constants

- Derive capital accumulation equation as a function of output using last four equations:

\[ sY_t = K_{t+1} - K_t (1 - d) \]

\[ K_{t+1} = K_t (1 - d) + sY_t \]

- Since workers increase in number, to get variables constant in the long-run, must express all variables as per worker- let small letters denote per worker quantities - remember for now \( Q_t = 1 \).
- Per worker output

\[ y_t = \frac{Y_t}{N_t} = A \frac{K_t^\alpha N_t^{1-\alpha}}{N_t^\alpha N_t^{1-\alpha}} = Ak_t^\alpha \]

- Per worker capital

\[ \frac{K_{t+1}N_{t+1}}{N_t N_{t+1}} = \frac{K_t}{N_t} (1 - d) + s \frac{Y_t}{N_t} \]

\[ k_{t+1}(1 + n) = (1 - d) k_t + sy_t \]

\[ k_{t+1} = \frac{(1 - d) k_t + sy_t}{1 + n} \]
- Neoclassical growth equation - substitute for output - difference equation in $k_{t+1}$

$$k_{t+1} = \frac{(1 - d) k_t + sAx^\alpha}{1 + n}$$

- Long-run equilibrium - $k$ is constant - therefore, to solve for long-run equilibrium, drop the time subscripts and rearrange

$$k(1 + n) = (1 - d) k + sAx$$

$$k(n + d) = sAx^\alpha$$

Investment per worker = saving per worker - note that steady state investment per worker is that necessary to replace the capital per worker as it erodes due to growth in the number of workers and to depreciation in the physical stock.
• Graphically - draw investment per worker \((k(n + d))\) and saving per worker \((sAk^\alpha)\) as a function of capital per worker \((k)\) and set them equal - consider points away from long-run equilibrium.
5.3 Characteristics of long-run equilibrium

- saving rate $s$
- worker growth rate $n$
- depreciation rate $d$
- productivity $A$
Golden Rule level of capital -

- What saving rate maximizes per worker consumption in the long-run?
- Let’s choose \( k \), and then realize that the choice of \( k \) implies the choice of a value for \( s \).

\[
c = y - sy = Ak^\alpha - sAk^\alpha = Ak^\alpha - k(n + d)
\]

- To maximize, take the derivative with respect to \( k \):

\[
\frac{\partial c}{\partial k} = \alpha Ak^{\alpha-1} - (n + d) = 0
\]

- Marginal product of capital should be sufficient to replace the per capita capital that diminishes due to growth in workers + depreciation.

\[
\alpha Ak^{\alpha-1} = (n + d)
\]
• Graphically - plot output and saving as a function of capital and find the value of capital which maximizes the distance between the two
6 Summary

- Sources of growth include
  - Productivity growth
  - Capital accumulation
  - Population growth

- Long-run standard of living higher
  - Higher saving rate
  - Lower population growth rate
- Larger value for productivity

- Long-run per capita output growth determined by
  - Growth rate of productivity