Investment and Goods Market Equilibrium
Chapter 4
1. Investment

2. Goods market equilibrium
1 MPK Theory of Investment

1.1 Desired capital stock

- Assumptions

  - Capital stock in the current period is fixed
  - Acquisition of new capital takes at least one period
  - Firms choose next period’s capital stock to maximize the value of real profits in the future
  - Real profits are determined by
* real output

* real labor costs

* real capital costs
  
  - user cost of capital = real cost of using one unit of capital for a period = \((r + d) p_k\) where \(p_k\) is the price of capital relative to the price of the firm’s production good and \(d\) is depreciation, that is the fraction of the capital good that will wear out over the period

\[
AF(K, N) - \omega N - ucK
\]

• Maximization
- mathematically - take the partial derivative of profits with respect to capital and set equal to zero.

\[ A \frac{\partial F(K, N)}{\partial K} - uc = 0 \]

marginal product of capital (MPK) must equal the user cost

- graphically - plot MPK and user cost as a function of capital

- Theory of demand for capital (desired capital stock)

- Demand for capital for a given user cost is the marginal product of capital (MPK)

  - higher user cost (interest, price of capital, depreciation) reduces desired capital stock
Distortionary taxes - tax value of production so that firm only receives the after-tax value of sales

- Maximization problem becomes

\[
\max_K [(1 - t) AF (K, N) - \omega N - ucK]
\]

yielding

\[
(1 - t) A \frac{\partial F (K, N)}{\partial K} - uc = 0
\]

- after-tax MPK must equal user cost

- graphically, after-tax MPK is lower with the tax

- need to consider effect on other components of government budget constraint and their effects on desired capital to get full answer
1.2 Investment and the desired capital stock

- Investment is the means of raising capital to its desired level

  - Capital stock increases as firms purchase new capital stock (gross investment $= I$)

  - Capital stock decreases as existing capital depreciates ($dK_t$)

  - net investment is the increase in the capital stock

    \[ K_{t+1} - K_t = I_t - dK_t \]

  - rewriting, gross investment ($I_t$) equals net investment ($K_{t+1} - K_t$) plus depreciation ($dK_t$)

    \[ I_t = K_{t+1} - K_t + dK_t \]
• Time to adjust

  – One period: $K_{t+1}^* = K_{t+1}$, implying that investment increases whenever $K_{t+1}^*$ increases above $K_t$

    \[ I_t = K_{t+1}^* - K_t + dK_t. \]

  – Multiple periods: an increase in $K_{t+1}^*$ creates a smaller increase in $I_t$, but it is sustained over more periods

  – Once capital reaches its desired level, investment is necessary only to replace capital which wears out.
2 Tobin’s Q Theory of Investment

- When capital is at its desired level, the market value of the firm’s capital stock should equal its replacement cost.

- If the market value of the firm’s capital is higher than its replacement cost, invest.

  – Market value of capital is represented by the firm’s stock market value $V$

  – Replacement cost of capital is its price $(p_k)$ times the quantity of capital $(K)$

$$q = \frac{V}{p_k K}.$$
When $q > 1$, the firm should invest to increase the capital stock.

- Implies that investment and the stock market should move together.

- Check to make sure that this theory of investment is consistent with the MPK theory.
  - effect of interest rate on $q$?
  - effect of MPK on $q$?
3 Aggregate Desired Investment

- Add up all firms’ desired investment

- Aggregate investment is decreasing in the real interest rate

- Other determinates of investment
  - MPK (expected future)
  - price of capital
  - tax rate
  - depreciation rate
4 Goods Market Equilibrium

- Output equals desired spending on goods

\[ Y = C^d + I^d + G \]

or rearranging

\[ Y - T - C^d + T - G = I^d \]

\[ S^d = I^d \]

- Real interest adjust to equate desired savings and investment (graph)

- Anything which shifts \( I^d \) or \( S^d \) will change the equilibrium real interest rate
- Why are real interest rates so low now?
5 Summary

- Demand for capital next period \((K_{t+1})\) shifts right when
  - user costs, determined by interest \((r)\), price of capital \((p_k)\) and depreciation \((d)\), falls
  - the tax rate on production falls
  - expected future marginal product of capital rises due to expected increase in technology or employment

- Investment demand \((I_t)\) is the demand for additional capital
  \[
  I_t = K_{t+1} - (1 - d)K_t
  \]
• Investment is a downward sloping function of the interest rate
  
  – Shifts right when demand for capital next period increases
    
    * reductions in price of capital \( (p_k) \), tax rate on production \( (t) \)
    
    * increase in expected future MPK due to increase in future technology or employment
    
    * depreciation is ambiguous as it raises user costs reducing \( K_{t+1} \), but raises investment necessary to keep capital from falling as more capital wears out

• Tobin’s Q implies that the stock market and investment should move together
• Graph of goods market equilibrium
  – Investment is downward sloping function of interest
  – Savings is upward sloping function of interest
  – Goods market equilibrium occurs at interest rate which equates savings and investment