The Semantics of LSL (2.1)

Each atomic sentence is supposed to stand for a simple declarative sentence. Furthermore, we’re going to assume that each such sentence has exactly one truth value, i.e. that it’s either true or false. So, we interpret sentences letters by specifying whether they are true of false, i.e. by assigning them truth values.

Such a specification for all the sentence letters occurring in a sentence or argument written in LSL is called a truth value assignment.

It remains to interpret the connectives. The English equivalents we gave earlier are only supposed to suggest how they are to be interpreted. Our official definitions of what they mean take a different form. Because these connectives are truth-functional, their meaning can be given by a truth-table.

A connective is said to be truth-functional if the truth value of a compound formed by it is completely determined by the truth values of the components.

A truth table definition of the meaning of a connective specifies, for each combination of truth values of the components, what the truth value of the compound is to be.

Here are truth table definitions of the meanings of the five connectives:

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<th>Φ</th>
<th>Ψ</th>
<th>~Φ</th>
<th>Φ&amp;Ψ</th>
<th>Φ∨Ψ</th>
<th>Φ→Ψ</th>
<th>Φ↔Ψ</th>
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To interpret a sentence of LSL is to specify whether whether it is true or false. Given the preceding definitions, we can now interpret any sentence of LSL, no matter how long or complicated, relative to a truth value assignment to its sentence letters.

The official meanings of the connectives of LSL are given by the above table. How closely do these meanings correspond to those of the English phrases that we gave as their suggested equivalents earlier? In some cases, (negation, for example) the correspondence is very close; in others, (the conditional, for example), it is less so.

Not every English sentential connective is truth functional.
Examples:

- 1.a) G.W.B. is the president because he was elected by a majority of the Electoral College.
  1.b) G.W.B. is the president because his mother is named Barbara.
- 2.a) George Washington was president before G.W.B. was born.
  2.b) G.W.B. was born before George Washington was president.

Both a) and b) have the form “True sentence because/before true sentence”, yet a) is true while b) is false. This means we cannot give a truth table definition of the meaning of the connectives because and before.

Translation (1.3)

Translating a sentence of English into LSL involves three steps:

1. Identify all the words and phrases in the sentence whose meaning can be expressed by $\sim$ (‘it’s not the case that...’), $\&$ (‘...and ...’), $\lor$ (‘either ...or...’), $\rightarrow$ (‘if...then...’), and $\leftrightarrow$ (‘...if and only if ...’).

   Difficulties may arise because of

   - the existence of stylistic variants
   - the fact that in some contexts, a word or phrase that is ordinarily translated by one of the connectives of LSL may have an entirely different meaning.

2. Set up a translation scheme: a correspondence between sentence letters of LSL and the simple’ component sentences that emerge after carrying out step 1.

3. Reconstruct the English sentence as a wff of LSL, systematically replacing simple sentences by their associated sentence letters and the words or phrases identified at step 1 by the appropriate connectives.

   This means identifying the word or phrase which corresponds to the main connective first, then analyzing the clause corresponding to the subformulas that the main connective operates on, and so on. If the sentence is long and complicated, it’s sometimes helpful to paraphrase it first so as to reveal its structure more clearly.
1. Translating with &.

The suggested English equivalent is “Both...and...” but there are numerous stylistic variants. For example:

- John loves Mary; however, she barely tolerates him. (*J&T*)
- It’s raining; nevertheless, we will go on a picnic. (*R&P*)
- John is crazy; still, I like him. (*F&L*)
- I know it’s a beautiful day, but still, I want to stay home. (*B&S*)
- John got the job, although he didn’t wear a tie to the interview.
- I won’t give up, even though it is not certain that I will succeed.
- John is a student; also, he’s rich.
- Mary likes classical music and also rock.
- Mary is not only a musician, but also a first-rate scientist.
- ”But” is truth functional, while ”because” is not.
- Anne will marry John despite the fact that he can’t cook.
- John doesn’t have a job; moreover he can’t cook.

There are also some contexts in which “and” is not used as a sentential connective at all. For example:

Bacon and eggs is a popular breakfast dish.

and some contexts where “and” is perhaps not strictly truth functional. For example:

The king committed suicide and a republic was declared.

A republic was declared and the king committed suicide.

2. Translating with ∨.

The suggested English equivalent is “Either...or ....” Notice however that, in light of the truth table for ∨, we interpret “or” in the inclusive, not the exclusive sense. For example:

- Your dinner comes with soup or a salad. (Exclusive)
- Eventually, the Republicans or the Democrats will come up with a plan to balance the budget. (Inclusive)
Stylistic variants can be formed using “unless”. For example:

- The Braves will win the pennant unless the Giants play well. 
  \((B \lor G)\)
- You’ll fail logic unless you work hard. \((F \lor H)\)

3. Translating with \(\sim\).

The suggested English equivalent is “It’s not the case that…” , but there are very many other ways of expressing negation in English. For example:

- John is not happy.
- John isn’t happy.
- John is unhappy.
- It’s impossible to buy a cheap car.
- No one has three arms.
- John was nowhere to be found.
- The IRS never leaves me alone.

By combining connectives of LSL together we can express other truth functional connectives of English: For example:

- Neither the Braves nor the Mets will win the pennant.
  \((\sim B \land \sim M)\)
- Either Arnold or (exclusive) Cruz will win the election. (i.e Exactly one of Arnold or Cruz will win the election.)
  \(((A \lor C) \land \sim (A \land C))\)

4. Translating with \(\rightarrow\).

The suggested English equivalent is “If...then...”. This is the most troublesome connective. First, you need to recognize that a sentence is a conditional. There are many stylistic variants. For example:

- There will be dirty rain if the volcanic ash cloud passes over. (Q if P)
- The cat will not scratch the furniture, provided you train her. (P provided Q)
- If John quits school, he will regret it. (If P, Q)
• Supposing that interest rates remain high, the housing industry will not recover any time soon. (Supposing that P, Q)

• Rescue teams will be launched in the event that the volcano explodes. (Q in the event that P)

• Provided that Mary goes with him, John will move to Alaska. (Provided that P, Q)

• Coming to class is a sufficient condition for passing the course. (P is sufficient for Q)

• Coming to class is a necessary condition for passing the course. (Q is necessary for P)

Next, you need to identify correctly the antecedent and the consequent. In English, the antecedent does not always come before the consequent. In each of the above examples, “P” is the antecedent and “Q” the consequent.

It's particularly important to distinguish between “if” and “only if”. For example:

• You will pass the course if you attend class.

• You will pass the course only if you attend class.

Translation Scheme

C: You pass the course.

A: You attend class.

C if A: \( (A \rightarrow C) \)

C only if A: \( (C \rightarrow A) \)