1. Good $Y$ is a gross substitute for good $X$ if an increase in the price of $Y$ causes an increase in demand for good $X$. Good $X$ is a gross complement for good $Y$ if an increase in the price of $X$ causes a decrease in demand for $Y$. It is possible to draw a price increase in $Y$ causing an increase in demand for $X$ and a price increase in $X$ causing a decrease in demand for $X$ with indifference curves that could come from the same consumer.

2. Show graphically.

3. a. The amount spent on good $X$ equals $P_X X$, where $P_X$ is the price of good $X$ and $X$ is the quantity purchased of good $X$. This equals $I/10$, where $I$ is income. Therefore $X = \frac{I}{10P_X}$.

b. The elasticity of demand for good $X$ with respect to income is $\frac{dX}{dI} I X$. We have $\frac{dX}{dI} = \frac{1}{10P_X}$. So the elasticity of demand for good $X$ with respect to income is $(\frac{1}{10P_X})(\frac{1}{I})(10P_X) = 1$. The elasticity of demand for good $X$ with respect to $P_X$ is $(\frac{dX}{dP_X})(\frac{P_X}{X})$. Since $\frac{dX}{dP_X} = -\frac{I}{10P_X^2}$, and $\frac{P_X}{X} = 10P_X^2/I$, we have $(\frac{dX}{dP_X})(\frac{P_X}{X}) = -1$. The elasticity of demand for $X$ with respect to the price $P_Y$ of another good $Y$ is 0. Since the consumer always spends $1/10$ of his income on $X$, changes in the price of any other good do not affect his demand for good $X$.

4. a. Laspeyre price index equals (amount spent on base year bundle with current year prices)/(amount spent on base year bundle with base year prices). This is $(2 \cdot 2 + 3 \cdot 4)/(2 \cdot 2 + 2 \cdot 4) = 16/12 = 1.33$.

b. It is not possible to tell that the consumer was better off in 1991. The amount spent in 1991 was $4 \cdot 2 + 1 \cdot 3 = 11$. With an income of $11$, the consumer could not purchase the bundle $(2, 4)$ at 1991 prices. So we cannot say that the consumer was better off in 1991.

c. In 1990 the consumer spent $12$ on the market basket $(2, 4)$. At 1990 prices, the market basket $(4, 1)$ cost only $10$ and thus was in the interior of the budget set. Therefore $(2, 4)$ must be strictly preferred to $(4, 1)$, so the consumer was better off in 1990.

d. The budget line for 1991 is $2X + 2Y = 12$. The budget line for 1990 is $3X + 2Y = 14$. The two budget lines intersect at $(2, 4)$. If the consumer had chosen $(2, 4)$ in 1990 and $(4, 1)$ in 1991, it would mean that they chose $(2, 4)$ when $(4, 1)$ was affordable and then chose $(4, 1)$ when $(2, 4)$ was affordable. This is impossible if the consumer had the same standard preferences in the two years.