Notes - Gruber, Public Finance

Chapter 3.3

Cross-sectional regression analysis

Another way to try to figure out the relationship between TANF benefits and labor supply of single mothers would be to regress labor supply on TANF benefits. In cross-sectional regression analysis, many data points are observed at one point in time. Thus one might compare the labor hours of individuals who for some reason have different TANF benefit levels.

The simplest form of cross-sectional regression would be a regression of the form

\[ \text{HOURS}_i = \alpha + \beta \text{TANF}_i + \epsilon_i, \]

where \( \text{HOURS}_i \) are the number of hours worked in the year by person \( i \), \( \text{TANF}_i \) is the amount of TANF benefits received in the year by person \( i \), and \( \epsilon_i \) is a random error term, which can be assumed to be normally distributed. The regression line is best-fit line that can be drawn through the points on the graph of TANF benefits versus hours worked. It is the line that minimizes the sum of the errors squared. That is, it is the line \( \text{HOURS} = \alpha + \beta \text{TANF} \) such that \( \sum_{i=1}^{n} (\alpha + \beta \text{TANF}_i - \text{HOURS}_i)^2 \) is minimized over all possible \( \alpha \) and \( \beta \).

A simple cross-sectional regression of labor hours on the natural log of TANF benefits using real-world data shows a negative relationship. The regression equation is

\[ \text{HOURS}_i = \alpha + \beta \ln(\text{TANF}_i) + \epsilon_i \]

The best-fit line has a slope of -110. This tells us that when the log of TANF benefits (in dollars) increases by 1, labor hours per year can be expected to decrease by 110. So when TANF benefits are multiplied by approximately 2.718, labor hours can be expected to decrease by 110.

To determine whether there is any causal relationship, it would be better to include other explanatory variables in the cross-sectional regression analysis besides just TANF benefits. The one variable regression just shows that there is negative correlation between the two. But there could be alternative reasons other than causality of TANF on labor for the correlation. TANF benefits fall as the recipient works more (this was one of the reasons for partially replacing TANF by EITC). So it could be that desire for less labor was causing increase in TANF benefits rather than the other way around.

Using control variables can help figure out which causes which, for instance by using a variable that measures taste for leisure (time spent not working). If a variable that measures taste for leisure was included in the explanatory variables, we could compare the labor hours of single mothers with different TANF benefits but the same taste for leisure. Then if there is still a significant difference between the labor hours of those with different TANF benefits keeping taste for leisure constant, we could come closer to saying that TANF benefits
are causing changes in labor hours. Control variables try to take into account differences between individuals other than the differences in independent variable, so that the differences found among dependent variables can be assumed to be related to differences in the independent variable alone.

A problem with this is that it is difficult to measure intrinsic taste for leisure. It is often hard to get rid of bias with control variables, since the variables that can be measured are often only imperfect proxies for the actual differences between treatment and control groups. However, it is important in empirical work to make sure that there are no systematic differences between two groups that are correlated with the independent variable, as this biases results. In the example, there should be no other factor that causes individuals to supply more or less work other than differences in TANF benefits.

Quasi-experiments

Quasi-experiments are situations that arise when a policy change or other change in the environment creates similar treatment and control groups. Suppose we have a sample with a large number of single mothers in Arkansas and Louisiana in 1996 and 1998. Suppose in 1997 Arkansas cut its benefit guarantee by 20%; Louisiana’s stayed the same.

To examine the effect of TANF on labor supply, can use the difference-in-difference estimator. Take the difference between the labor supply changes in the treatment group (Arkansas) and the labor supply changes in the control group (Louisiana). Because Arkansas and Louisiana experienced otherwise similar economic conditions during that period, Louisiana can be used as a control group for Arkansas. There may have been changes in economic conditions during the period under study that affected single mothers’ labor supply, such as a general improvement in economic conditions across the country. To be able to use the difference-in-difference estimator, one must know that the improvement in economic conditions affected Arkansas and Louisiana similarly.

So we have: Hours(Arkansas, 1998) - Hours(Arkansas, 1996) = Treatment effect + bias from economic boom,
Hours(Louisiana, 1998) - Hours(Louisiana, 1996) = bias from economic boom,
Thus, (Hours(Arkansas, 1998) - Hours(Arkansas, 1996)) - (Hours(Louisiana, 1998) - Hours(Louisiana, 1996)) = Treatment effect.

Consider a set of hypothetical numbers to illustrate how the difference-in-difference estimator works. Suppose that Arkansas reduced the welfare guarantee from $5000 to $4000 between 1996 and 1998. Over that period, average hours worked per year by single mothers in Arkansas rose from 1000 to 1200. If we just used this (time-series) data, we would get an elasticity of labor hours to TANF benefits of -1 (labor hours rose by 20% when TANF benefits decreased by 20%). This elasticity is much higher than that found in the California randomized trial, -0.67.
Suppose that in Louisiana, while TANF benefits remained the same over the period 1996-1998, average hours worked by single mothers increased by 50 hours per year. This suggests that the economic boom had a role in increasing the hours worked by single mothers. Since the economic boom also affected Arkansas, the above estimate of elasticity is biased, as it includes the increase in labor hours due to the economic boom.

To get a possibly unbiased estimator, we take the difference between the difference in labor hours in Arkansas and the difference in labor hours in Louisiana. This removes the effect of the boom. The estimator is \((1200 - 1000)-(1100 - 1050) = 150\). The resulting elasticity of labor hours to TANF benefits is \(-0.75 = (150/1000)/(-1000/5000) = 0.15/ - 0.2\). Take 150 as a percentage of the original number of hours worked, and divide it by the change in benefits as a percentage of the original benefits.

By comparing the change in Arkansas with the change in Louisiana, the difference-in-difference estimator controls for other time series factors that would bias the time series analysis within Arkansas. By comparing the change within each state instead of just comparing the two states at one point in time, the estimator controls for omitted factors that bias cross-sectional analysis across the states. Cross-sectional analysis in 1998 finds that benefits were $1000 lower in Arkansas than in Louisiana, and hours worked were 100 per year higher in Arkansas than in Louisiana. This leads to an elasticity estimate of \(11/ - 20 = -0.55\), rather than \(-0.75\).

The cross-sectional estimate is biased by the fact that single mothers in Louisiana tend to work more hours regardless of the level of TANF benefits. When the levels of TANF benefits were equal in both states in 1996, the average hours of work were 50 more a year in Louisiana than in Arkansas. One way to account for this would be to find control variables that cause the higher amount of work in Louisiana. Another way is to use the difference-in-difference estimator, which uses as a control hours of work in the same state before there was a benefits change. By comparing the change within one state to the change within the other state, the difference-in-difference estimator controls for all possible differences across states that might bias the comparison.

Problems with quasi-experimental analysis.

To be able to use the difference-in-difference estimator in this example, we must know that the economic boom affected Arkansas in the same way as Louisiana. However, if it affected one state more strongly that the other, the estimator will be biased. Suppose the economic boom in 1996-1998 affected Arkansas differently from Louisiana. Then we have:

\[
\text{Hours(Arkansas, 1998)} - \text{Hours(Arkansas, 1996)} = \text{Arkansas bias from economic boom} + \text{Treatment},
\]

\[
\text{Hours(Louisiana, 1998)} - \text{Hours(Louisiana, 1996)} = \text{Louisiana bias from economic boom},
\]
so the difference equals Treatment + AR bias from economic boom - LA bias from economic boom. If these biases are not the same, then we have not found out what the effect of treatment is.

Structural modeling

Chapter 2.1
Constrained Utility maximization - budget sets and indifference curves

There are two parts to this analysis. The first part is modeling a consumer’s preferences over different bundles of goods. The second part is defining their budget constraint, and maximizing their preferences subject to their budget constraint - that is, letting the consumer choose the bundle of goods that they prefer among the bundles that are affordable.

Preferences of a consumer tell us how a consumer ranks different bundles of goods.

Let A, B, C, etc. denote bundles of goods. We assume that all consumers’ preferences satisfy certain conditions which make the preferences easier to work with:

1. Preferences are complete - given A and B, the consumer either prefers A to B, or prefers B to A, or is indifferent between the two.
2. Preferences are transitive - if a consumer prefers A to B, and prefers B to C, then that consumer prefers A to C.
3. Preferences satisfy non-satiation - If bundle A contains more of one good and at least as much of all other goods than bundle B, then any consumer strictly prefers bundle A to bundle B. We are talking about ”goods”, not ”bads”. ”Bads” are things that a consumer would prefer to have less of like air pollution.

Having imposed these assumptions of reasonableness on preferences, we can draw a consumer’s indifference curve over bundles consisting of different amounts of two goods. Given a bundle A, an indifference curve is a graphical representation of all the bundles such that the consumer is indifferent between A and them. Thus, the consumer is indifferent between all the bundles in one indifference curve.

Let bundle A be 2 CDs and 1 movie, bundle B be 1 CD and 2 movies and bundle C be 2 CDs and 2 movies. Then by the non-satiation assumption, we know that bundle C must be strictly preferred by a consumer to bundles A and B.

Also by the nonsatiation assumption, the indifference curves must be downward-sloping. If an indifference curve were upward-sloping or horizontal, that would mean that a consumer is indifferent between two bundles, one of which has more of one good and at least as much of the other good, which violates non-satiation.

Again by the non-satiation assumption, consumers prefer higher indifference
curves.

Some ("well-behaved") preferences can be represented by utility functions. A utility function is a function that assigns a real number to every bundle of goods; the real number represents the amount of benefit a consumer derives from consumption of that bundle of goods. To represent a consumer’s preferences, the utility function must assign a higher number to a bundle of goods that a consumer prefers, that is, if the consumer prefers B to A, and has preferences represented by a utility function U, then \( U(B) > U(A) \). If the consumer is indifferent between B and C, then \( U(B) = U(C) \).

Suppose Andrea has preferences over CDs and movies represented the utility function \( U(Q_C, Q_M) = \sqrt{Q_C \times Q_M} \), where \( Q_C \) is quantity of CDs she consumes and \( Q_M \) is quantity of movies she consumes. Then she is indifferent between 2 CDs and 2 movies, 4 CDs and 1 movie, and 1 CD and 4 movies. She prefers 3 CDs and 3 movies to all of these. The equation of an indifference curve corresponding to this utility function is \( Q_C = U^2/Q_M \) for some positive number \( U^2 \).

This utility function has a realistic feature of diminishing marginal utility in both CDs and movies. As the number of movies increases with number of CDs remaining fixed, the additional utility she gets from one additional CD decreases. For instance, at 1 movie and 1 CD, the additional utility she gets from an additional movie is \( \sqrt{2} - 1 \approx 0.4 \). At 4 movies and 1 CD, the additional utility she gets from an additional CD is \( \sqrt{5} - 2 \approx 0.24 < 0.4 \).

This makes sense because people tend to get the most utility from the first unit of a good consumed, and to get less utility from additional units. For movies, there must be one that you want to see most, which you will watch first. The following ones will give less additional benefit.

The diminishing marginal rate of substitution means that the indifference curves become less steep as you move to the right. At any point, the slope of her indifference curve is the number of CDs Andrea is willing to give up in order to get an additional movie. As the number of movies increases, she is willing to give up fewer and fewer CDs to get an additional movie. The same is true for CDs - as the number of CDs increases, she is willing to give up fewer and fewer movies to get an additional CD. The rate at which she is willing to trade CDs for movies is called the marginal rate of substitution (MRS). The MRS is diminishing in quantity of movies.

Budget constraints

People have limited resources, which prevents them from consuming an infinite amount of every good. The budget constraint over CDs and movies of an individual with income \( Y \) is given by \( Y = P_C Q_C + P_M Q_M \) where \( P_C \) is the price of CDs and \( P_M \) is the price of movies. \( P_C Q_C \) is her expenditure on CDs, \( P_M Q_M \) is her expenditure on movies, and the sum of these cannot be more than her income (we are ignoring the possibility of borrowing here). Since she strictly
prefers more of each good, she will choose quantities of CDs and movies such that the budget constraint is satisfied with equality.

The budget constraint can be represented as a line segment in the $Q_M, Q_C$ axis. The slope of this line segment is $-P_M/P_C$. Holding income constant, for every extra movie that she buys, she lowers the number of CDs she can buy by $P_M/P_C$.

Constrained choice

The consumer chooses the utility-maximizing bundle that she can afford. On the graph, this corresponds to choosing the highest indifference curve that touches the budget set. If the utility function is $U = \sqrt{Q_C \times Q_M}$, such an indifference curve must be tangent to the budget set. This is the most-preferred indifference curve that can be attained given the budget constraint. She then consumes the quantities of CDs and movies at the point of tangency. At this point, the MRS (the rate at which she is willing to trade CDs for movies) equals $-P_M/P_C$. If the rate at which she is willing to trade CDs for movies is greater than $-P_M/P_C$, then she would be better off by trading some CDs for movies at the going price. If $MRS < -P_M/P_C$, then she would be better off by trading some movies for CDs at the going price. Unless it is impossible to trade more movies for CDs - that would happen at the edge of the graph.

Substitution effects and Income effects

Because $MRS = -MU_M/MU_C = -P_M/P_C$, when the relative price of a good rises, then the relative quantity demanded of that good falls. When the relative price of movies rises, so that $P_M/P_C$ rises, then $MU_M/MU_C$ must also rise, and MRS must fall. For $MU_M/MU_C$ to rise, the quantity of movies relative to CDs must fall, since in our example the marginal utility of a good falls when the quantity consumed rises.

In the graph, you can see the effect of an increase in price of movies. Suppose Andrea is originally consuming at point A on her budget line. After the price change, the budget line rotates inward, so fewer movies can be purchased at every quantity of CDs than before. Andrea chooses to consume at the point C, where an indifference curve is tangent to her new budget line. She is necessarily worse off than before, because she is on a lower indifference curve.

Suppose that the government can somehow protect Andrea from her lost utility due to the change in price of movies. They compensate her with enough money so that she can move back to her original indifference curve. This is equivalent to shifting the new budget line outwards in a parallel way until it hits her first indifference curve at point B. The difference between B and A is called the substitution effect of the price change. Holding utility constant, a relative increase in the price of a good will always cause a consumer to choose less of the good.

The income effect measures how Andrea’s consumption changes due to the fact
that she is effectively poorer because of the increase in price of movies. It is given by the difference between C and B. While the substitution effect necessarily causes the amount of movies consumed to fall, the income effect could cause the number of movies consumed to rise or to fall. If the amount of a good consumed due to the income effect falls (when income falls), then the good is a normal good. If it rises, then the good is an inferior good.

Chapter 2.3
Equilibrium and Social Welfare - TANF

Continue with a TANF example. Suppose you are analyzing the question: Will labor supply of single mothers rise or not when TANF benefits are cut?

Behind this question lies another question: What is the effect on society as a whole when TANF benefits are cut?

Welfare economics studies how changes in the economy affect the total well-being of everyone in society. How to measure social welfare?

First determine demand and supply curves. The demand curve for a particular person for a particular good can be found by observing, at a given income level, what quantity of the good they demand at a given price. Theoretically, the demand curve (or demand correspondence) can be found by finding the consumer’s preferred point (or points) along each budget set as price changes.

An important property of a demand curve is its elasticity. This tells you the percentage change in quantity demanded for a percentage change in price. (Unlike slope, elasticity does not depend on the units used). \[
\epsilon_d = \frac{\frac{Q_2-Q_1}{Q_2+Q_1}}{\frac{P_2-P_1}{P_1+P_2}}
\]

Note that since quantity demanded usually falls as price rises, elasticity is usually negative (some books take absolute value to make it positive).

Also, elasticity is not constant along a typical demand curve. For a straight line demand curve, elasticity falls as you move down the curve.

A firm’s supply curve can be derived from the firm’s profit-maximization. Suppose a particular firm’s production function has the form \( q = K^{1/2}L^{1/2} \). This production function has diminishing marginal productivity of both inputs, meaning that as you increase labor input keeping capital input constant, the output increases at a decreasing rate. The same is true if you increase capital input keeping labor input constant.

The firm’s problem is to maximize profit, which is \( pq - (rK + wL) \), where \( p \) is the price of the output good, \( w \) is the wage rate and \( r \) is the rental rate of capital.

To maximize profit, the firm will produce at a level such that the marginal cost of producing an additional unit of output equals the marginal benefit of producing (and selling) an additional unit, which is the price of output. This determines \( q \) as a function of price. Thus the supply curve is (in the short-run)
the portion of the marginal cost curve above the average variable cost curve where marginal cost is not decreasing, and (in the long-run) the portion of the marginal cost curve above the average total cost curve where marginal cost is not decreasing. K and L are determined by finding the cheapest combination that is capable of producing $q$ units of output.

Equilibrium

To get market demand and market supply curves from individual demand and supply curves for a good, add horizontally the demand curves of all potential buyers of the good, and add horizontally the supply curves of all potential sellers of the good.

The intersection of the market demand and market supply curves determines the market equilibrium quantity and price for that good.

Social efficiency (in a particular market) is the sum of all benefit to society that results from trade of that good. It is the sum of consumer surplus and producer surplus. (Social efficiency is also called total surplus).

Consumer surplus is the sum of all the benefit to consumers that results from trading that good. It is the sum of the differences between willingness to pay for different quantities of the good and the price of the good. Graphically it is represented by the area between demand curve, the y-axis, and the horizontal line at price.

Given a particular equilibrium, consumer surplus rises as demand becomes more inelastic and falls as demand becomes more elastic. If there is a lack of good substitutes, so that demand is inelastic, consumer surplus due to trade of a good will be higher than if there are many good substitutes.

Producer surplus is the sum of all benefits to producers that result from trading the good. It is the sum of the differences between price and production cost for every unit of the good produced. Graphically, it is the area above the supply curve and below the horizontal equilibrium price line. Producer surplus is also equal to "variable profit", which is profit plus sunk cost ($\pi(Q) + C(0)$, where $\pi(Q)$ is profit at output $Q$).

Given an equilibrium, producer surplus decreases when elasticity of supply become higher and increases when it becomes lower.

With no externalities, perfect info, compete markets, a competitive equilibrium maximizes social efficiency. This is intuitive: a benefit is created whenever a trade occurs whose benefits to the buyer exceed its costs to the seller. If the amount traded is restricted to be lower than the equilibrium amount, this outcome is inefficient: There are potential buyers whose valuation of the good exceeds some potential sellers’ cost, but they are not trading at any price.

Similarly, if the amount traded is forced to be higher than the equilibrium amount, there are buyers and sellers who are trading, such that the buyer’s valuation is lower than the seller’s cost. Not trading would be better for both
sides.

Chapter 5.1

Externals

Definition of an externality: An externality exists when the actions of one party make another party worse or better off, but the first party does not bear the net cost imposed or receive the net benefit.

Example (negative externality): A corn farm uses fertilizer which runs off into the water, causing algae to grow and killing the fish. This negatively affects the local population, who made a living from fishing.

The corn farm owner bears some negative consequences from this activity, but not the full consequence. Bearing the full consequence would mean that the corn farm owner would pay the full loss to the fishermen.

Example (positive externality): A homeowner spends $50,000 dollars fixing up her house. This causes the values of neighboring houses to rise.

For a negative externality, whenever there is an agent who does not bear the net cost of its action, there is inefficiency: The agents who are affected by the action could theoretically compensate the first agent for reducing the amount of action taken. Thus it is possible to make some player better off without making anyone else worse off.

This is the same for a positive externality if you talk about net benefits. For a positive externality, when there is an agent who does not receive the net benefit of its action, there is inefficiency: The agents affected by the action could theoretically compensate the first agent for doing more of the action. Thus again it is possible for some player to be made better off without anyone being made worse off.