
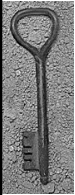


## Spatial Analysis: An Introduction




## Components of Spatial Analysis

- ◆ Exploratory Spatial Data Analysis
  - finding interesting patterns
- ◆ Visualization
  - showing interesting patterns
- ◆ Spatial Modeling
  - explaining interesting patterns



## What is Special About Spatial Data

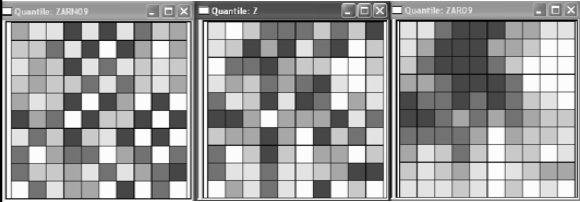
- ◆ Location, Location, Location
  - where matters
- ◆ The First Law of Geography
  - Waldo Tobler: “everything depends on everything else, but near things are more related than distant things”
- ◆ Spatial Dependence
  - distance decay
- ◆ Closeness = Similarity
- ◆ Spatial scale matters




## Spatial Autocorrelation

- ◆ Spatial Autocorrelation – correlation of a variable with itself through space.
  - If there is any systematic pattern in the spatial distribution of a variable, it is said to be spatially autocorrelated
  - If nearby or neighboring areas are more alike, this is *positive spatial autocorrelation*
  - *Negative autocorrelation* describes patterns in which neighboring areas are unlike
  - Random patterns exhibit *no spatial autocorrelation*

## Spatial Autocorrelation Pattern



Negative
None
Positive



## Spatial Autocorrelation

- ◆ It measures the extent to which the occurrence of an event in an areal unit constrains, or makes more probable, the occurrence of an event in a neighboring areal unit.
- ◆ Goals
  - Measure the strength of spatial autocorrelation in a map
  - Test the assumption of independence or randomness



### Why is spatial autocorrelation important?

- ◆ Most statistics (e.g. OLS) are based on the assumption that the values of observations in each sample are independent of one another
- ◆ Positive spatial autocorrelation may violate this, if the samples were taken from nearby areas
- ◆ If the observations are spatially correlated, the estimates will be biased and overly precise.
  - the areas with higher concentration of events will have a greater impact on the model estimate and they will overestimate precision
  - there are actually fewer number of independent observations than are being assumed.

7



### Spatial Randomness

- ◆ Null Hypothesis: No spatial autocorrelation
  - spatial randomness
  - values observed at a location do not depend on values observed at neighboring locations
  - observed spatial pattern of values is equally likely as any other spatial pattern
  - the location of values may be altered without affecting the information content of the data

8



### Spatial Randomness

- ◆ Alternative Hypothesis:
  - ◆ 1) Positive Spatial Autocorrelation
    - like values tend to cluster in space
    - neighbors are similar
    - compatible with diffusion
      - but not necessarily diffusion
  - ◆ 2) Negative Spatial Autocorrelation
    - neighbors are dissimilar
    - checkerboard pattern

9



### Spatial Objects

- ◆ Points
  - x, y coordinates
  - crimes, addresses, accidents
- ◆ Lines
  - arc: from node, to node
  - road network, transmission lines
- ◆ Polygons
  - series of connected arcs
  - administrative unites (tracts, zip zones, states)

10



### A Closer Look at Points

- ◆ Points as Events
  - crimes (addresses), accidents (locations)
  - point pattern analysis
- ◆ Points as Samples from a Surface
  - air quality monitors, housing prices
  - geostatistics
- ◆ Points as Discrete Objects
  - county centroids
  - lattice data analysis

11

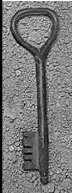


### Spatial Data and Autocorrelation

- ◆ Point Patterns
  - interest in absence of spatial randomness of locations
  - clusters, dispersion
- ◆ Continuous Surfaces (geostatistics)
  - interest in modeling spatial covariance between pairs of observations as it changes with distance
  - spatial prediction, spatial interpolation (kriging)

12





### 1. Quadrat Analysis: hypothesis test

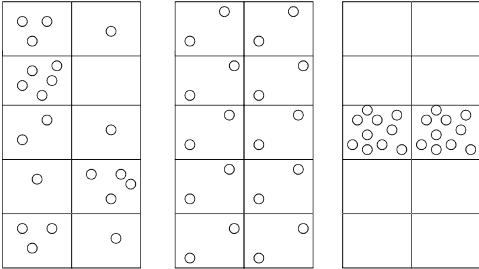
- $H_0$ : distribution is random (no pattern)
- $H_1$ : distribution is not random
- Test statistic:  $(m-1)VMR$
- If  $m < 30$ ,  $(m-1)VMR$  has a chi-squared distribution with  $df=m-1$

$$\chi^2 = (m-1)VMR = \frac{(m-1)s^2}{\bar{x}}$$

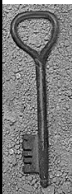
$$= \frac{(m-1) \sum (x_i - \bar{x})^2}{\bar{x}(m-1)} = \frac{\sum (x_i - \bar{x})^2}{\bar{x}}$$

- Reject the null hypothesis if  $(m-1)VMR$  is larger than the critical chi-squared value

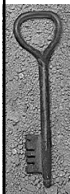
19



RANDOM                  UNIFORM                  CLUSTERED



<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>3</td><td>1</td></tr> <tr><td>5</td><td>0</td></tr> <tr><td>2</td><td>1</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>3</td><td>1</td></tr> </table>	3	1	5	0	2	1	1	3	3	1	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td>2</td></tr> </table>	2	2	2	2	2	2	2	2	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>10</td><td>10</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td></tr> </table>	0	0	0	0	10	10	0	0	0	0																																																																																
3	1																																																																																																													
5	0																																																																																																													
2	1																																																																																																													
1	3																																																																																																													
3	1																																																																																																													
2	2																																																																																																													
2	2																																																																																																													
2	2																																																																																																													
2	2																																																																																																													
0	0																																																																																																													
0	0																																																																																																													
10	10																																																																																																													
0	0																																																																																																													
0	0																																																																																																													
<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>Quadrat #</th><th>Number of Points Per Quadrat</th><th><math>x^2</math></th></tr> </thead> <tbody> <tr><td>1</td><td>3</td><td>9</td></tr> <tr><td>2</td><td>1</td><td>1</td></tr> <tr><td>3</td><td>5</td><td>25</td></tr> <tr><td>4</td><td>0</td><td>0</td></tr> <tr><td>5</td><td>2</td><td>4</td></tr> <tr><td>6</td><td>1</td><td>1</td></tr> <tr><td>7</td><td>1</td><td>1</td></tr> <tr><td>8</td><td>3</td><td>9</td></tr> <tr><td>9</td><td>3</td><td>9</td></tr> <tr><td>10</td><td>1</td><td>1</td></tr> <tr><td><b>Total</b></td><td><b>20</b></td><td><b>60</b></td></tr> </tbody> </table>	Quadrat #	Number of Points Per Quadrat	$x^2$	1	3	9	2	1	1	3	5	25	4	0	0	5	2	4	6	1	1	7	1	1	8	3	9	9	3	9	10	1	1	<b>Total</b>	<b>20</b>	<b>60</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>Quadrat #</th><th>Number of Points Per Quadrat</th><th><math>x^2</math></th></tr> </thead> <tbody> <tr><td>1</td><td>2</td><td>4</td></tr> <tr><td>2</td><td>2</td><td>4</td></tr> <tr><td>3</td><td>2</td><td>4</td></tr> <tr><td>4</td><td>2</td><td>4</td></tr> <tr><td>5</td><td>2</td><td>4</td></tr> <tr><td>6</td><td>2</td><td>4</td></tr> <tr><td>7</td><td>2</td><td>4</td></tr> <tr><td>8</td><td>2</td><td>4</td></tr> <tr><td>9</td><td>2</td><td>4</td></tr> <tr><td>10</td><td>2</td><td>4</td></tr> <tr><td><b>Total</b></td><td><b>20</b></td><td><b>40</b></td></tr> </tbody> </table>	Quadrat #	Number of Points Per Quadrat	$x^2$	1	2	4	2	2	4	3	2	4	4	2	4	5	2	4	6	2	4	7	2	4	8	2	4	9	2	4	10	2	4	<b>Total</b>	<b>20</b>	<b>40</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>Quadrat #</th><th>Number of Points Per Quadrat</th><th><math>x^2</math></th></tr> </thead> <tbody> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>2</td><td>0</td><td>0</td></tr> <tr><td>3</td><td>0</td><td>0</td></tr> <tr><td>4</td><td>0</td><td>0</td></tr> <tr><td>5</td><td>10</td><td>100</td></tr> <tr><td>6</td><td>10</td><td>100</td></tr> <tr><td>7</td><td>0</td><td>0</td></tr> <tr><td>8</td><td>0</td><td>0</td></tr> <tr><td>9</td><td>0</td><td>0</td></tr> <tr><td>10</td><td>0</td><td>0</td></tr> <tr><td><b>Total</b></td><td><b>20</b></td><td><b>200</b></td></tr> </tbody> </table>	Quadrat #	Number of Points Per Quadrat	$x^2$	1	0	0	2	0	0	3	0	0	4	0	0	5	10	100	6	10	100	7	0	0	8	0	0	9	0	0	10	0	0	<b>Total</b>	<b>20</b>	<b>200</b>
Quadrat #	Number of Points Per Quadrat	$x^2$																																																																																																												
1	3	9																																																																																																												
2	1	1																																																																																																												
3	5	25																																																																																																												
4	0	0																																																																																																												
5	2	4																																																																																																												
6	1	1																																																																																																												
7	1	1																																																																																																												
8	3	9																																																																																																												
9	3	9																																																																																																												
10	1	1																																																																																																												
<b>Total</b>	<b>20</b>	<b>60</b>																																																																																																												
Quadrat #	Number of Points Per Quadrat	$x^2$																																																																																																												
1	2	4																																																																																																												
2	2	4																																																																																																												
3	2	4																																																																																																												
4	2	4																																																																																																												
5	2	4																																																																																																												
6	2	4																																																																																																												
7	2	4																																																																																																												
8	2	4																																																																																																												
9	2	4																																																																																																												
10	2	4																																																																																																												
<b>Total</b>	<b>20</b>	<b>40</b>																																																																																																												
Quadrat #	Number of Points Per Quadrat	$x^2$																																																																																																												
1	0	0																																																																																																												
2	0	0																																																																																																												
3	0	0																																																																																																												
4	0	0																																																																																																												
5	10	100																																																																																																												
6	10	100																																																																																																												
7	0	0																																																																																																												
8	0	0																																																																																																												
9	0	0																																																																																																												
10	0	0																																																																																																												
<b>Total</b>	<b>20</b>	<b>200</b>																																																																																																												
Variance 2.222 Mean 2.000 Var/Mean 1.111	Variance 0.000 Mean 2.000 Var/Mean 0.000	Variance 17.778 Mean 2.000 Var/Mean 8.889																																																																																																												

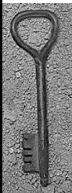


<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>3</td><td>1</td></tr> <tr><td>5</td><td>0</td></tr> <tr><td>2</td><td>1</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>3</td><td>1</td></tr> </table>	3	1	5	0	2	1	1	3	3	1	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td>2</td></tr> </table>	2	2	2	2	2	2	2	2	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>10</td><td>10</td></tr> <tr><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td></tr> </table>	0	0	0	0	10	10	0	0	0	0																																																																																
3	1																																																																																																													
5	0																																																																																																													
2	1																																																																																																													
1	3																																																																																																													
3	1																																																																																																													
2	2																																																																																																													
2	2																																																																																																													
2	2																																																																																																													
2	2																																																																																																													
0	0																																																																																																													
0	0																																																																																																													
10	10																																																																																																													
0	0																																																																																																													
0	0																																																																																																													
<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>Quadrat #</th><th>Number of Points Per Quadrat</th><th><math>x^2</math></th></tr> </thead> <tbody> <tr><td>1</td><td>3</td><td>9</td></tr> <tr><td>2</td><td>1</td><td>1</td></tr> <tr><td>3</td><td>5</td><td>25</td></tr> <tr><td>4</td><td>0</td><td>0</td></tr> <tr><td>5</td><td>2</td><td>4</td></tr> <tr><td>6</td><td>1</td><td>1</td></tr> <tr><td>7</td><td>1</td><td>1</td></tr> <tr><td>8</td><td>3</td><td>9</td></tr> <tr><td>9</td><td>3</td><td>9</td></tr> <tr><td>10</td><td>1</td><td>1</td></tr> <tr><td><b>Total</b></td><td><b>20</b></td><td><b>60</b></td></tr> </tbody> </table>	Quadrat #	Number of Points Per Quadrat	$x^2$	1	3	9	2	1	1	3	5	25	4	0	0	5	2	4	6	1	1	7	1	1	8	3	9	9	3	9	10	1	1	<b>Total</b>	<b>20</b>	<b>60</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>Quadrat #</th><th>Number of Points Per Quadrat</th><th><math>x^2</math></th></tr> </thead> <tbody> <tr><td>1</td><td>2</td><td>4</td></tr> <tr><td>2</td><td>2</td><td>4</td></tr> <tr><td>3</td><td>2</td><td>4</td></tr> <tr><td>4</td><td>2</td><td>4</td></tr> <tr><td>5</td><td>2</td><td>4</td></tr> <tr><td>6</td><td>2</td><td>4</td></tr> <tr><td>7</td><td>2</td><td>4</td></tr> <tr><td>8</td><td>2</td><td>4</td></tr> <tr><td>9</td><td>2</td><td>4</td></tr> <tr><td>10</td><td>2</td><td>4</td></tr> <tr><td><b>Total</b></td><td><b>20</b></td><td><b>40</b></td></tr> </tbody> </table>	Quadrat #	Number of Points Per Quadrat	$x^2$	1	2	4	2	2	4	3	2	4	4	2	4	5	2	4	6	2	4	7	2	4	8	2	4	9	2	4	10	2	4	<b>Total</b>	<b>20</b>	<b>40</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr><th>Quadrat #</th><th>Number of Points Per Quadrat</th><th><math>x^2</math></th></tr> </thead> <tbody> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>2</td><td>0</td><td>0</td></tr> <tr><td>3</td><td>0</td><td>0</td></tr> <tr><td>4</td><td>0</td><td>0</td></tr> <tr><td>5</td><td>10</td><td>100</td></tr> <tr><td>6</td><td>10</td><td>100</td></tr> <tr><td>7</td><td>0</td><td>0</td></tr> <tr><td>8</td><td>0</td><td>0</td></tr> <tr><td>9</td><td>0</td><td>0</td></tr> <tr><td>10</td><td>0</td><td>0</td></tr> <tr><td><b>Total</b></td><td><b>20</b></td><td><b>200</b></td></tr> </tbody> </table>	Quadrat #	Number of Points Per Quadrat	$x^2$	1	0	0	2	0	0	3	0	0	4	0	0	5	10	100	6	10	100	7	0	0	8	0	0	9	0	0	10	0	0	<b>Total</b>	<b>20</b>	<b>200</b>
Quadrat #	Number of Points Per Quadrat	$x^2$																																																																																																												
1	3	9																																																																																																												
2	1	1																																																																																																												
3	5	25																																																																																																												
4	0	0																																																																																																												
5	2	4																																																																																																												
6	1	1																																																																																																												
7	1	1																																																																																																												
8	3	9																																																																																																												
9	3	9																																																																																																												
10	1	1																																																																																																												
<b>Total</b>	<b>20</b>	<b>60</b>																																																																																																												
Quadrat #	Number of Points Per Quadrat	$x^2$																																																																																																												
1	2	4																																																																																																												
2	2	4																																																																																																												
3	2	4																																																																																																												
4	2	4																																																																																																												
5	2	4																																																																																																												
6	2	4																																																																																																												
7	2	4																																																																																																												
8	2	4																																																																																																												
9	2	4																																																																																																												
10	2	4																																																																																																												
<b>Total</b>	<b>20</b>	<b>40</b>																																																																																																												
Quadrat #	Number of Points Per Quadrat	$x^2$																																																																																																												
1	0	0																																																																																																												
2	0	0																																																																																																												
3	0	0																																																																																																												
4	0	0																																																																																																												
5	10	100																																																																																																												
6	10	100																																																																																																												
7	0	0																																																																																																												
8	0	0																																																																																																												
9	0	0																																																																																																												
10	0	0																																																																																																												
<b>Total</b>	<b>20</b>	<b>200</b>																																																																																																												
Variance 2.222 Mean 2.000 Var/Mean 1.111	Variance 0.000 Mean 2.000 Var/Mean 0.000	Variance 17.778 Mean 2.000 Var/Mean 8.889																																																																																																												

$\chi^2 = (m-1)VMR = (10-1) * 1.111 = 9.99$   
 $\chi^2 = (m-1)VMR = (10-1) * 0 = 0$   
 $\chi^2 = (m-1)VMR = (10-1) * 8.889 = 80$

$df = 10 - 1 = 9$   
 Chi-square = 16.9  
 Conclusion?

21



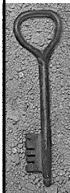
### 1. Quadrat Analysis: hypothesis test

- $H_0$ : distribution is random (no pattern)
- $H_1$ : distribution is not random
- If  $m >= 30$ ,  $(m-1)VMR$  has a normal distribution with mean  $m-1$  and variance  $2(m-1)$  when the null is true.

$$Z = \frac{(m-1)VMR - (m-1)}{\sqrt{2(m-1)}} = \sqrt{(m-1)/2} (VMR - 1)$$

- If  $z > 1.96$ , reject the null hypothesis of random distribution at 0.05 level  $\rightarrow$  clustering.
- If  $z < -1.96$ , uniform distribution

23



### Example

- $m=36, N=25$

$$\bar{x} = \frac{N}{m} = 25/36 = 0.6944$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{m-1} = 0.3325$$

$$VMR = \frac{s^2}{\bar{x}} = 0.3325 / 0.6944 = 0.4789$$

$$Z = \sqrt{(m-1)/2} (VMR - 1) = \sqrt{(36-1)/2} (0.4789 - 1) = -2.18$$

Conclusion?

$|Z| > 1.96$ , reject the null, distribution is not random  
 $Z < -1.96$ , uniform distribution

24



**Example**

$$d_o = \sum_{i=1}^n \frac{d_i}{n} = (1+1+2+3+3+3)/6 = 2.167$$

$$d_e = \frac{1}{2\sqrt{n/A}} = \frac{1}{2\sqrt{6/(6*7)}} = 1.323$$

$$NNI = \frac{d_o}{d_e} = 2.167/1.323 = 1.638 \quad \text{This shows a tendency toward uniformity}$$

**Significance Test**

$$S.E. = \frac{0.26136}{\sqrt{N^2/A}} = \frac{0.26136}{\sqrt{6*6/(6*7)}} = 0.2823$$

$$Z = \frac{d_o - d_e}{S_e} = \frac{2.167 - 1.323}{0.2823} = 2.9897$$

- ◆  $Z > 1.96$ , reject the null hypothesis of a random distribution. Instead, there is a significant tendency for uniformity.

**Issues with Near Neighbor Analysis**

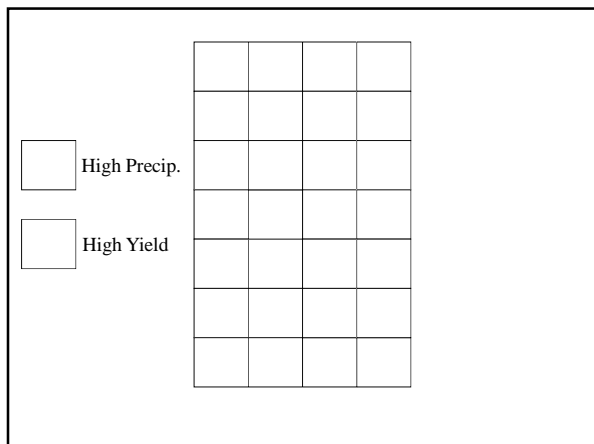
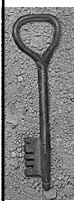
- ◆ **Boundary problem:**
  - NNI and significance test may be affected by the shape of region. Long, narrow rectangular shapes may have low NNI simply because of the shape.
  - **Solution:**
    - add a buffer area
    - Point in the buffer zone should be used

**Issues with Near Neighbor Analysis**

- ◆ **Boundary problem:**
- ◆ Clustering is only detected on a small spatial scale due to nearest neighbor distance. Can extend the approach to the second and higher-order nearest neighbors.
- ◆ Location only, no other background attribute data (e.g. population). Thus detecting clustering against the background factor is not possible

**3. Chi-Squared Test**

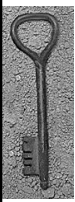
- ◆ Measures the strength of association between two spatial distributions
- ◆ **Example**
  - Relationship between wheat yield and precipitation
  - Two maps showing high and low yields and high and low precipitation

### Chi-Squared

- ◆ By combining distributions on one map we can better understand the relationship between the two distributions
- ◆ Divide the map into grids
  - The finer the grid, the more precise the measurement
- ◆ Different possibilities
  - Low rainfall, low yield
  - Low rainfall, high yield
  - High rainfall, low yield
  - High rainfall, high yield

38

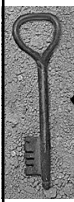


### Chi-Squared

- ◆ Record the total number of occurrences into a table of observed frequencies

		WHEAT		
		High	Low	
PRECIP.	High	8	2	10
	Low	5	13	18
		13	15	28

39



### Chi-Square

- ◆ Create a table of expected frequencies using probability statistics and table margins


		WHEAT	
		High	Low
PRECIP.	High	5	5
	Low	8	10

40

### Compute Chi-Squared Statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$


	High	Low		High	Low
High	8	2	High	5	5
Low	5	13	Low	8	10

$$\frac{\text{High/High}}{5} + \frac{\text{High/Low}}{5} + \frac{\text{Low/High}}{8} + \frac{\text{Low/Low}}{8} = 5.625$$


### Interpreting Chi Square

- ◆ Df=(2-1)\*(2-1)=1
- ◆ The critical value=3.84
- ◆ 5.6>3.84, reject the null hypothesis, thus these two distributions are associated.

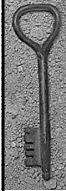
42



### 4. Moran's I Statistic

- ◆ Measure the degree of spatial autocorrelation in areal data
- ◆ One of the oldest indicators of spatial autocorrelation. Still a de facto standard for determining spatial autocorrelation
- ◆ Applied to zones or points with continuous variables associated with them.

43




### Moran's I

- ◆ Compares the value of the variable at any one location with the value at all other locations

$$I = \frac{N \sum_i \sum_j W_{ij} (X_i - \bar{X})(X_j - \bar{X})}{(\sum_i \sum_j W_{ij}) \sum_i (X_i - \bar{X})^2}$$

Where  $N$  is the number of cases  
 $X_i$  is the variable value at a particular location  $i$   
 $X_j$  is the variable value at another location  $j$   
 $\bar{X}$  is the mean of the variable  
 $W_{ij}$  is a weight that measures the spatial proximity between region  $i$  and region  $j$

44



### Moran's I

$$I = \frac{N \sum_i \sum_j W_{ij} (X_i - \bar{X})(X_j - \bar{X})}{(\sum_i \sum_j W_{ij}) \sum_i (X_i - \bar{X})^2}$$


- ◆ A simpler expression of I

$$I = \frac{N \sum_i \sum_j W_{ij} Z_i Z_j}{(N-1) \sum_i \sum_j W_{ij}}$$

Where  $N$  is the number of cases  
 $Z_i$  is the z-score of  $x$  at location  $i$   
 $Z_j$  is the z-score of  $x$  at location  $j$

$$Z = \frac{x - \bar{x}}{s}$$

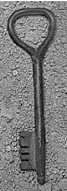
45



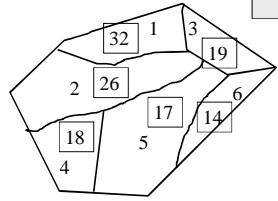
### Moran's I

- ◆  $W_{ij}$  is a contiguity matrix
  - Measures the spatial proximity between regions  $i$  and  $j$
  - If zone  $j$  is adjacent to zone  $i$  (at least one common point), the interaction receives a weight of 1, otherwise 0
  - Another option is to make  $W_{ij}$  a distance-based weight which is the inverse distance between locations  $i$  and  $j$  ( $1/d_{ij}$ )
- ◆ Moran's I is similar to correlation coefficient, it varies between  $-1.0$  and  $+1.0$ 
  - When autocorrelation is high, the coefficient is high
  - A high  $I$  value indicates positive autocorrelation
  - $I$  near  $-1$  indicates strong negative autocorrelation

46



### Example of spatial weight




$$w = \{w_{ij}\} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$\bar{x} = 21$

$\sum_i \sum_j W_{ij} = 18$

$I = 0.1488$

47

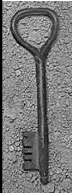


$$I = \frac{N \sum_i \sum_j W_{ij} Z_i Z_j}{(N-1) \sum_i \sum_j W_{ij}}$$

i	X	Z	$W_{ij} Z_i Z_j$	I
1	32	1.6434	0.7366	0.14881
2	26	0.7470	0.2232	
3	19	-0.2988	-0.2232	
4	18	-0.4482	-0.0670	
5	17	-0.5976	0.6250	
6	14	-1.0458	0.9375	
mean	21		2.2321	
std	6.69328			

Conclusion?  
 Use spatial analysis software such as SpaceStat and Geoda


48



## Summary


- ◆ Spatial autocorrelation
- ◆ Quadrant analysis
- ◆ Nearest neighbor analysis
- ◆ Chi-squared analysis
- ◆ Moran's I

49



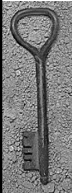
## Point Pattern Analysis

- ◆ Examine and evaluate spatial patterns and the processes of point features
- ◆ Spatial properties of the entire body of points are studied rather than the individual



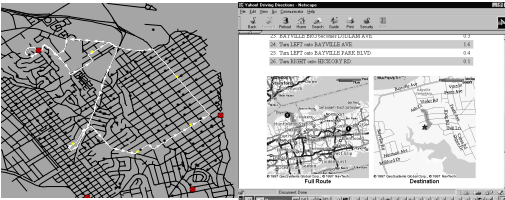
Pittsburgh Homicide Locations (Points) and Counts by Census Tract (Polygons)

50




## Network Analysis

- ◆ Designed specifically for line features organized in connected networks, typically applies to transportation problems and location analysis


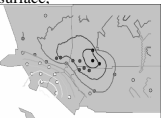
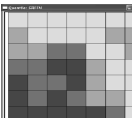


51



## Surface Analysis

- ◆ Spatial distribution of surface information in terms of a three-dimensional structure
- ◆ Mostly for Geophysical Data
  - Elevation, air quality, vegetation
  - also social science: risk surface, price surface
- ◆ Representation
  - equation for the surface
  - discrete approximation
    - grid, raster
    - contour lines

Continuous surface contours  
Los Angeles Basin Air Quality

Grid representation of  
greenness index

52