MATH 367, Fall 2005
EXAM 4, In Class part

NAME:

1. (5 pts) Show that if X and Y are independent, then $E(Y|X) = E(Y)$.

2. (5 pts) The number of winter storms in a good year is a Poisson random variable with mean 4 and in a bad year is a Poisson random variable with mean 6. If the next year will be a good year with probability .35 and a bad year with probability .65, find the expected number of storms that will occur next year.

3. (10 pts) Let $m(t) = (e^t - 1)/t$ be the moment generating function of a random variable X. Find the mean and variance of X.

4. (5 pts) State De Moivre–Laplace Theorem or the Central Limit Theorem. Specify which one you will state.

5. (15 pts) X_1, X_2, X_3 are random variables with mean 2, variance 3, and covariance between any two equal to 1.

(a) Find $E(2X_1 - 3X_2 + \alpha X_3 + \pi)$
(b) Find $\text{Var}(2X_1 - 3X_2 + \alpha X_3 + \pi)$
(c) Find the correlation factor between $Y_1 = X_1$ and $Y_2 = 2X_1 - 3X_2$

MATH 367, Fall 2005
EXAM 4, Take Home part. Due Dec 9, 5pm

NAME:

1. (10 pts) Let $X_1, X_2, \ldots, X_n$ be i.i.d with mean 5 and variance 5.
   (a) Find the mean and variance of $X_1 + X_2 + \ldots + X_n$.
   (b) Let
   $$\bar{X}_n = \frac{(X_1 + X_2 + \ldots + X_n)}{n}.$$
   This is called the sample average. Find its mean $m = E(\bar{X}_n)$ and its variance $\sigma^2 = \text{Var}(\bar{X}_n)$.
   (c) What happens to the variance of $\bar{X}_n$ as $n \to \infty$? What would that tell me about $\bar{X}_n$?

2. (10 pts) In problem 2 of the in class exam we had the following situation: The number of winter storms in a good year is a Poisson random variable with mean 4 and in a bad year is a Poisson random variable with mean 6. If the next year will be a good year with probability .35 and a bad year with probability .65. Now find the variance of the number of storms that will occur next year. (You will have to condition to find $E(X^2)$.)

3. (15 pts) 10,000 coins are tossed. Use Bernoulli's Theorem or the Central Limit Theorem to estimate the following.
   (a) What is the probability that at least 4950 of them fall heads?
   (b) What is the probability that at least 5100 of them fall heads?
   (c) What is the smallest n such that the probability that at least n coins (among the 10,000) fall heads is .9937903

4. (10 pts) Toss a fair coin. If it falls head draw letters from the word ABRACADABRA with replacement until you get the first A. If it falls tails, draw letters from the word IT without replacement, untill you get a vowel. Find the expected number of draws.

5. (15 pts) Let X and Y be two random variables with joint probability mass function given by

$Y \backslash X$	0	1	2	3	
0	.15	.10	.08	.07	
1	.10	.15	.10	0	
2	.08	.10	0	0	
3	.07	0	0	0	

Compute

(a) the conditional probability of X given $Y = 1$,
(b) $E(X|Y = 1)$,
(c) $P(XY \leq 2)$,
(d) $\mathrm{Cov}(X, Y)$,
(e) the moment generating function of X.

Are X and Y independent?