**#1. Problems for Gamma and Chi-square- Solutions**

1. Mean=750, T is exponential with mean 750, so λ=1/750
   1. , t>0.
   2. . Solve ,

so

1. X exponential with parameter λ=7. E(X3)=derivative of the MGF evaluated at 0.

MGF . ,

, . So E(X3)=6 λ/ λ4=6/ λ3.

1. Median=4 this means P(X<4)=.5 But . Solve for lambda:

, , . Then the problem asks to compute: P(X>5)=

1. .30=P(X<50)=F(50)=. Solve for λ: So, . Now the problem asks: P(X<80)= F(80)=
2. Cost of printer=200. X=Lifetime: exponential mean=2.

|  |  |  |
| --- | --- | --- |
|  | prob | refund |
| X<1 | P(X<1)=F(1)==.3935 | 200 |
| 1<X<2 | P(1<X<2)=F(2)-F(1)==.2387 | 100 |

Expected refund for 1 printer= (.3935)(200)+.2387(100)=102.559

Let Yi=refund for printer number i. Then we want the expected value of S=Y1+Y2+…+Y100

But E(S)=100E(Y1)=100(102.559)=10255.9

1. X=losses is exponential with men 300.
   1. E((X-100)+)=I worked this problem in class one way, now I’m going to do another way, using computing expected values by conditioning. We’ll condition on X<100 and X>100. . In the first term, when X<100, then (X-100)+=0. So the expected value is 0. In the second term, when X>100, (X-100)+=X-100. Now X is exponential, and exponentials have the memory-less property. So X-100|X>100 is another exponential with mean 300. That is . So we get:
2. If the density is f(x)=x e-x/3 / 9, for x>0, then X is a Gamma random variable with α-1=1 and λ=1/3. That is α=2 and λ=1/3. So the mean is α/ λ=6 and the variance is α/ λ2=2(9)=18. So the standard deviation is . Γ
3. 1. To compute , identify the parameters: α-1=3/2 and λ=2. So the answer is =
   2. , , identify the parameters: α-1=3 and λ=1. So we get: Recall: When n is a positive integer.
4. So

so, we know , so we can figure out by using the property of the Gamma:

1. X is Gamma with parameters α=6, λ=1/2.
   1. E(X)= α/ λ=12, Var(X)= α/ λ2=24.
   2. E(X3) = third derivative of MGF or we can use the formula:
2. is Chi-squared with n degrees of freedom.
   1. When n=5, S is Chi-squared with 5 df. Var(S)=10 so and =22.36. Thus, looking at the table for the chi-square with 5 df.
   2. When n=100, we can use the CLT. Let Xi=Zi2. Then S/100=. Notice that Xi = Zi2 is chi-square with df=1. So, its mean=E(Xi)=1, and its variance Var(Xi)=2. Now, since S is chi-square with df=100, Var(S)=200 and , so . Thus