#4**. The T-distribution, the F-distribution, and Confidence Intervals**

1. Let X represent the score on a standardized test. X is normal distributed. A random sample of 25 test scores is taken, with sample mean = 72.3 and sample variance 16. Calculate the 95% CI for the population mean.
2. A random sample of size 8 from a normal population X had sample variance 81. A random sample of size 13 from a normal population Y had sample variance 15. Determine the 90% confidence interval for .
3. Your scores from an afternoon of bowling were 128, 189, 156, 143, 117, 152. Your sister’s bowling scores were 204, 176, 189, 198,218, 177, 190, 203. Determine the 95% confidence interval for . Assume bowling scores are normally distributed.
4. Let U and V be independent random variables with U~χ2(m) and V~χ2(n).
   1. Use MGF to show that S=U/m is Gamma with α=m/2 and λ=m/2 and T=V/n is Gamma with α=n/2 and λ=n/2 .
   2. Let W=S/T. Use the fact that to show that , where c is a constant, α=(n+m)/2, and λ=(n+mw)/2.
5. A random sample X1,X2,…,Xn from a normal population with mean µ and standard deviation 1. Later, an additional independent observation Xn+1 is observed. What is the distribution of .
6. The following information is based on a sample of 10 observations from a normal distribution: and . Find a 95% confidence interval for σ2.
7. Let T=, where X, Y, Z and W are independent normal random variables with mean 0 and variance σ2. For what value of k, T has the t distribution? What are the degrees of freedom?
8. Suppose that X1,X2,…,X6, and Y1,Y2,…,Y9 are independent identically distributed normal random variables with mean 0 and variance σ2. What is the 95th percentile of ?
9. Find the cumulative distribution and the density function of the maximum and the minimum for a random sample from
   1. Exponential λ
   2. Uniform on [a,b]
10. Find the confidence level for the confidence interval (Y1,Yn) for a random sample of size 6 from uniform (θ-1/2,θ+1/2).
11. Find the confidence level for the confidence interval (Yn,2 Yn) for a random sample of size 6 from uniform (0,θ).