Other types of Annuities

Problems from actuarial exams

1. Jeff and Jason spend $X$ dollars to purchase an annuity. Jeff buys a perpetuity-immediate, which makes annual payments of 30. Jason buys a 10-year annuity-immediate, also with annual payments. The first payment is 53, with each subsequent payment $k\%$ larger than the previous year's payment. Both annuities use an annual effective interest rate of $k\%$. Calculate $k$.

2. John receives a perpetuity paying 2 at the end of the 4th year, 4 at the end of the 6th year, 6 at the end of the 8th year, etc. The present value of this perpetuity at an annual effective interest rate of 10% equals $X$. Calculate $X$.

3. Bill deposits money into a bank account at the end of each year. Bill's deposits in year $t$ is equal to $100t$, $t = 1, 2, 3, \ldots$. The bank credits interest at an annual effective rate of $i$. The amount of interest earned in Bill's account during the 11th year is equal to 500. Calculate $i$.

4. The present value of a perpetuity of 6,500 paid at the end of each year plus the present value of a perpetuity of 8,500 paid at the end of every 5 years is equal to the present value of an annuity of $k$ paid at the end of each year for 25 years. Interest is 6% convertible quarterly. Calculate $k$.
   
   (A) 10,340, (B) 11,340, (C) 12,340, (D) 19,370, (E) 19,560

5. Mary purchases an increasing annuity-immediate for 50,000 that makes twenty annual payments as follows:
   
   (i) $P, 2P, \ldots, 10P$ in years 1 through 10, and
   
   (ii) $10P(1.05), 10P(1.05)^2, \ldots, 10P(1.05)^{10}$ in years 11 through 20.
   
   The annual effective interest rate is 7% for the first 10 years and 5% thereafter. Calculate $P$.
   
   (A) 564, (B) 574, (C) 584, (D) 594, (E) 604

6. Perpetuity $X$ has annual payments of 1, 2, 3, 4, \ldots at the end of each year. Perpetuity $Y$ has annual payments of $q, q, 2q, 2q, 3q, 3q, \ldots$ at the end of each year.

   The present value of $X$ is equal to the present value of $Y$ at an annual effective interest rate of 10%. Calculate $q$.
   
   (A) 1.1, (B) 1.3, (C) 1.5, (D) 1.7, (E) 1.9

7. Jane receives a 10-year increasing annuity-immediate paying 100 the first year and increasing by 100 each year thereafter. Mary receives a 10-year decreasing annuity-immediate paying $X$ the first year and decreasing by $X/10$ each year thereafter.
At an effective annual interest rate of 5%, both annuities have the same present value. calculate X.
(A) 860, (B) 864, (C) 868, (D) 872, (E) 876

8. Stan elects to receive his retirement benefit over 20 years at the rate of 2,000 per month beginning one month from now. The monthly benefit increases by 5% each year. At a nominal interest rate of 6% convertible monthly, calculate the present value of the retirement benefit.
(A) 244,800, (B) 294,850, (C) 394,000, (D) 419,250, (E) 444,300