Other type of annuities

Annuities where the payments vary in an arithmetic progression:

An annuity where the first payment $= P$, second payment $= P + Q$, third payment $= P + 2Q$, ..., that is, the payments are in arithmetic progression with a constant increase of Q, has present value:

$$P\, a_{\overline{n}|} + Q\, \frac{[a_{\overline{n}|} - nv^n]}{i}$$

and future value:

$$P\, s_{\overline{n}|} + Q\, \frac{[s_{\overline{n}|} - n]}{i}$$

An **increasing annuity** is an annuity where the first payment $= 1$, second payment $= 2$, third payment $= 3$, ...

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$(Is)_{\overline{n}|} = \frac{\ddot{s}_{\overline{n}|} - n}{i} = \frac{s_{\overline{n+1}|} - (n+1)}{i}$$

$$(Ia)_{\overline{\infty}|} = \frac{1}{i} + \frac{1}{i^2}$$

A **decreasing annuity** is an annuity where the first payment $= n$, second payment $= n-1$, third payment $= n-2$, ...

$$(Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

$$(Ds)_{\overline{n}|} = \frac{n(1+i)^n - s_{\overline{n}|}}{i}$$

Payments varying in geometric progression:
annuity immediate with a term of n periods in which the first payment is 1, and successive payments increase in geometric progression with common ratio $(1+k)$. The present value when $k \neq i$ is

$$v + v^2(1+k) + v^3(1+k)^2 + \ldots + v^n(1+k)^{n-1} = \frac{1 - [\frac{1+k}{1+i}]^n}{i-k}.$$

When $k = i$ the present value is: nv.

Other payments patterns:
Learn to sum geometric sums !!!