Math 220 - Exam IV Part I - Fall 2000
Take home due Mon. Dec. 11

NAME:

1. (10 pts) Let $T : P_2 \to \mathbb{R}^3$ be defined by

$$T(a + bx + cx^2) = \begin{bmatrix} a + 2b \\ a + b - c \\ c + 2a \end{bmatrix}$$

a. Let β_1 = standard basis of P_2, and β_2 = standard basis of $\mathbb{R}^3$. Find $[T]_{\beta_1,\beta_2}$.
b. Decide whether T is an isomorphism. Give an explanation for you answer.

2. (15 pts) Let Let $T : P_3 \to P_2$ be defined by $T(a + bx + cx^2 + dx^3) = (a + b) + (c - a)x + (b + c + d)x^2$.

a. Let S_3 and S_2 be the standard basis of P_3 and P_2 respectively. Find $[T]_{S_3 S_2}$.

b. Let $\beta = \{-x + x^2, 1 + x, x\}$, and $\beta' = \{-x + x^3, 1 + x^2, x, -1 + x\}$. Find $[T]_{\beta'\beta}$.

3. (10 pts) Use the definition of determinant (cofactor expansion) to find the determinant of the following matrix:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & \\ 4 & 0 & 1 & 0 & 0 \\ 5 & 1 & 2 & 0 & -1 \\ 7 & 0 & 3 & 1 & 1 \\ -8 & 1 & 4 & 1 & 0 \end{bmatrix}$$

4. (10 pts) Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ be a matrix with determinant -3. Find the determinant of the following matrices:

a. $B = \begin{bmatrix} a_3 & a_1 & a_2 \\ b_3 & b_1 & b_2 \\ c_3 & c_1 & c_2 \end{bmatrix}$

b. $C = \begin{bmatrix} a_1 & a_2 & a_3 \\ 10b_1 & 10b_2 & 10b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

c. A^2, AA^t, A^{-3}.

d. $D = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 - 3a_1 & b_2 - 3a_2 & b_3 - 3a_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

e. $E = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1 + b_1 & a_2 + b_2 & a_3 + b_3 \end{bmatrix}$

5. (25 pts) A group of people buys cars every four years from one of three automobile manufactures, A, B and C. The transition probability of switching from one manufacturer to another is given by the matrix:

$$P = \begin{bmatrix} .5 & .4 & .6 \\ .3 & .4 & .3 \\ .2 & .2 & .1 \end{bmatrix}$$

a. Find the characteristic polynomial $p(\lambda)$, and the eigenvalues.
b. Find the eigenvectors of P. (Note: maple doesn't do a good job for this matrix. Do it yourself by hand.)
c. Diagonalize P, that is, find an invertible matrix Q and a diagonal matrix D, such that $D = Q^{-1}PQ$.
d. Compute D^2, D^3. Find an expression for D^n and then find $\lim_{n\to\infty} D^n$.
e. From (c) we have $P = QDQ^{-1}$. Use this expression to find P^2 in terms of D^2, P^3 in terms of D^3. Find an expression for P^n in terms of D^n. Then find $\lim_{n\to\infty} P^n$.
f. Will one of the manufacturers eventually dominate the market, no mater what the initial sales?