Math 220 - Determinants - Fall 2000

The DETERMINANT of and $n \times n$ matrix A is:

$$|A| = \det(A) = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \ \det(A_{ij}) = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \ M_{i,j}$$

where A_{ij} is the matrix obtained by removing the ith row and the jth column. The determinat $\det(A_{i,j}) = M_{i,j}$ is called the (i,j)–**minor**.

This formula for determinant is called the *cofactor expansion of a determinant about the ith row.* There is nothing special about row, we could have also done it using the columns instead:

$$|A| = \det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \ \det(A_{ij}) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \ M_{i,j}$$

Problem 1. let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 2 \end{bmatrix}$. a) Compute $|A|$ by hand.

b) Compute $|A|$ using maple. (enter the command: det(A);)
c) Compute $|A|$ using the cofactor expansion about the: (i) second row, (ii) the third column.

Problem 2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$.

a. Find $\det(A) =$.
b. Find $|A|$ using the cofactor expansion about (i) the first column, (ii) the fourth row.

Problem 3. a) Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, find $\det(A) =$

b) What do you conclude? The determinat of a diagonal matrix =

c) Let $B = \begin{bmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$, find $\det(B) =$

d) Let $C = \begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ e & f & c \end{bmatrix}$, find $\det(C) =$

e) What do you conclude? The determinat of an upper triangular or a lower triangular matrix =

Problem 4. For the matrices

$$\text{(i)} \quad A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \qquad \text{(ii)} \quad B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 8 \\ 3 & 2 & 7 \end{bmatrix}$$

compute: a) $|A| =$, $|B| =$

b) $|A^t| =$, $|B^t| =$

c) $|A^{-1}| =$, $|B^{-1}| =$

d) What is your conclusion?
For any matrix A, $\det(A^t) =$

For any invertible matrix A, $\det(A^{-1}) =$

e) Read Theorem 9, Section 6.2.

Problem 5. Let $A = \begin{bmatrix} 1+a & 2+b & 3+c \\ -1 & 2 & -1 \\ 3 & 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \\ 3 & 1 & 8 \end{bmatrix}$, $C = \begin{bmatrix} a & b & c \\ -1 & 2 & -1 \\ 3 & 1 & 8 \end{bmatrix}$.

a) Find: $\det(A) =$

$\det(B) =$

$\det(C) =$

$\det(B)+\det(C) =$

b) What can you conclude? Can you prove it?

c) Read the Sum of Rows (Columns) Theorem (Theorem 6) in sectiom 6.2.

Problem 6. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \\ 3 & 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} a & 2a & 3a \\ -1 & 2 & -1 \\ 3 & 1 & 8 \end{bmatrix}$, $C = \begin{bmatrix} a & 2 & 3 \\ -a & 2 & -1 \\ 3a & 1 & 8 \end{bmatrix}$.

a) Find: $\det(A) =$

$\det(B) =$

$\det(C) =$

b) What can you conclude? Can you prove it?

c) Read Theorem 1.

Problem 7. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 8 \\ 7 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 8 & 1 \\ 7 & 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 7 & 3 & 2 \\ 2 & 1 & 8 \\ 1 & 1 & 1 \end{bmatrix}$.

a) Find
$\det(A) =$

$\det(B) =$

$\det(C) =$

b) What do you conclude?

c) Read the Sign Reversal Theorem (part 2 of Theorem 1, section 6.1).

HW: Read sections 6.1 and 6.2. Do problems: Section 6.1: 3, 7, 9, 11, 15, 17, 21, 29, 31.
Section 6.2: 1, 3, 6, 7, 9, 11, 13, 15, 19, 23, 33, 34, 36, 37, 39.