Math 220 - Rank and Nullity - Fall 2000

1. Find a basis for Null(A) where

\[
A = \begin{bmatrix}
1 & -1 & 2 & 3 & 0 \\
1 & 0 & -4 & 3 & -1 \\
2 & -1 & 6 & 0 & 1 \\
1 & 2 & 0 & -1 & 1
\end{bmatrix}
\]

Find the dimension of Null(A).

2. The matrix A takes a vector in \( \mathbb{R}^5 \) and returns a vector in what space?

Definition: the nullity of a matrix A is the dimension of Null(A).

Definition: The Column space of A (Col(A)), is the space spanned by the columns of the matrix A. The rank of A is the dimension of Col(A). The Row space of A is the space spanned by the rows of A.

3. For the above matrix, find a basis for Col(A) and the rank of A.

4. Let \( e_1, e_2, e_3, e_4, e_5 \) be the standard basis of \( \mathbb{R}^5 \). Find \( Ae_1, Ae_2, Ae_3, Ae_4 \) and \( Ae_5 \). What did you get? What is span(\( Ae_1, Ae_2, Ae_3, Ae_4, Ae_5 \))? 

Theorem 1. \( \dim \, \text{Col}(A) = \dim \, \text{Row} \,(A) \)

This is because \( \dim(\text{Col} \, A) \) is the number of pivot columns of A, which is the same as the number of pivot rows of A.

Corollary 2. A and \( A' \) have the same rank.

Theorem 3. (Rank Theorem) \( \text{Rank}(A) + \text{Nullity}(A) = \text{number of columns of } A \).

5. Find basis for Null(A) and Col(A), and verify the Rank Theorem for

\[
A = \begin{bmatrix}
1 & 2 & 2 & -1 \\
1 & 3 & 1 & -2 \\
1 & 1 & 3 & 0 \\
0 & 1 & -1 & -1
\end{bmatrix}
\]

Homework Sec 4.6: Read Section 4.6, do problems: 1–9 odds, 19–23 odds, 31, 33, 37, 39, 41, 45, 52, 53.