REINHOLD MATH 112 CALCULUS I

LAB 8: RIEMANN SUMS

Before starting this lab, please read section 4.5.
The goal of the lab is to use Riemann sums to approximate the definite integral of a function.

1. Riemann Sums for the function $f(x) = x^3$ in the interval [0,5].
Compute (a) the upper Riemann sum Uf, (b) the lower Riemann sum Lf and (c) the middle Riemann sum Mf of the function in the interval [0,5] by dividing the interval into $N = 500$ equal pieces. (d) Compare the results to the actual integral If.

How to do it:

```
> f:=x->x^3;
```

$$f := x \to x^3$$

```
> N:=500;
```

$$N := 500$$

```
> dx:=evalf(5/N);
```

$$dx := .01000000000$$

```
> x:=0;       # initialization of variable
```

$$x := 0$$

We will use the variable S for the Riemann sums

```
> S:=0;
```

$$S := 0$$

(a) Upper sums:

```
> for k from 1 to N do
> x:=x+dx:
> S:=S+f(x)*dx:
> od:   # the colon makes this loop silent, that is, it computes with out dumping the
> #values out.
> Uf:=S;       # upper Riemann sum
```

$$Uf := 156.8756252$$

(b) Lower sums:

```
> x:=0;
```

$$x := 0$$

```
> S:=0;
```

$$S := 0$$

```
> for k from 1 to N do
> S:=S+f(x)*dx;
> x:=x+dx;
> od:
> Lf:=S;    # lower Riemann sum
```

$$Lf := 155.6256252$$

(c) Middle sum:

```
> x:=0;
```

$$x := 0$$

```
> S:=0;
```

$$S := 0$$

```
> for k from 1 to N do
> y:=x+dx/2;
> S:=S+f(y)*dx;
> x:=x+dx;
> od:
> Mf:=S;
```

$$Mf := 156.2496874$$

(d) compare it to the value of the integral of f between 0 and 5

```
> x:='x';       # after the above loop, x=5, so we need to make it a variable again
```

$$x := x$$

```
> Int(f,x=0..5);
```

$$\int_0^5 f\,dx$$

```
> If:=evalf(");
```

$$If := 156.2500000$$

```
> # note that   Lf  < If < Uf
```

To get better accuracy in the approximation, make N larger and $dx = 5/N$

2. Riemann Sums for the function $f(x) = x^3$ in the interval $[0, X]$.
Repeat the above procedure (using just one sum, either the lower sum or the upper sum) with $N = 100$ and $dx = X/N$.