Optimal recovery of the solution of the second kind Fredgolm equation

Olga Agareva, Russian State Technology University

The problem of optimal recovery of the solution of the Fredgolm integral equation of the second kind with difference kernel is considered. We assume that the first N Fourier coefficients of the function from the right side of the equation are known with some error. We find the error of optimal recovery and a best method of recovery.

Analytic discs, linking numbers and CR-functions

Mark Agranovsky,Bar-Ilan University

We are interested in the following general question: given a smooth real manifold, Λ, of a complex manifold, to what extent existence of a nontrivial parametric family of one-dimensional complex submanifolds (analytic discs), attached by their boundaries to Λ, does induce complex structure on Λ? We give an answer in terms of lower estimates for the dimension of maximal complex tangent subspace of the tangent space $T(\Lambda)$. This result implies solution, in real-analytic category, of two open problems about characterization of holomorphic functions and their boundary values (the strip-problem and Globevnik-Stout conjecture), or, more generally, CR-functions and CR manifolds, in terms of vanishing of certain complex moments.Surprisingly, the entire problem turnes out to be rather of topological nature and in a certain sense can be viewed as a version of multidimensional argument principle for Shilov boundaries.

Approximation by polynomials and rational functions in weighted Smirnov-Orlicz classes defined on Dini-smooth curves

Ramazan Akgun, Balikesir University
Daniyal M. Israfilov, Balikesir University

Let $\Gamma \subset \mathbb{C}$ be a closed, bounded Dini-smooth curve and let $G := int\Gamma, G^- := ext\Gamma$ and ω be a weight defined on Γ. In this work, we prove the following direct and converse approximation theorems in weighted Smirnov-Orlicz classes $E_M(G,\omega)$ and $\tilde{E}_M(G^-,\omega)$.

Theorem 1 *Let$L_M(\Gamma)$ be a reflexive Orlicz space and $\omega \in A_{\alpha_M}(\Gamma) \cap A_{\beta_M}(\Gamma)$. If $f \in E_M(G,\omega)$ then for ever natural number n*

$$E_n(f,G)_{M,\omega} \preceq \Omega^r_{\Gamma,M,\omega}(f,\frac{1}{n+1}) \preceq (n+1)^{-2r}\{E_0(f,G)_{M,\omega} + \sum_{k=1}^{n+1} k^{2r-1}E_k(f,G)_{M,\omega}\}$$

for r=1, 2, 3, ..., with the constants in the inequalities independent of n.

Theorem 2 *Let$L_M(\Gamma)$ be a reflexive Orlicz space and $\omega \in A_{\alpha_M}(\Gamma) \cap A_{\beta_M}(\Gamma)$. If $f \in \tilde{E}_M(G^-,\omega)$ then for every natural number n*

$$\tilde{E}_n(f,G^-)_{M,\omega} \preceq \tilde{\Omega}^r_{\Gamma,M,\omega})(f,\frac{1}{n+1}) \preceq (n+1)^{-2r}\{\tilde{E}_0(f,G^-)_{M,\omega} + \sum_{k=1}^{n+1} k^{2r-1}\tilde{E}_k(f,G^-)_{M,\omega}\}$$

for r = 1, 2, 3, ..., with constants in the inequalities independent of n.

Extremal problems for vector potentials and their applications to the asymptotics of Hermite-Pade approximants

Alexander I. Aptekarev, Keldysh Institute of Applied Mathematics

There is a vector of measures supported on the fixed system of curves and arcs. Extremal problem of minimization of logarithmic potential energy for the system of measures with a given interaction matrix is considered. Then doing variation of the system of curves and arcs we find locations for the curves where the local maxima of the energy functional is achived. As a result an extremal system of curves and arcs appears. These extremal contours and the extremal measures on them play a crucial role for investigation of the asymptotics of Hermite-Pade approximants. The application of the Matrix Riemann-Hilbert problem method for the asymptotic analysis uses the extremal system of contours for finding an approprate deformation of the boundaries for the solution of BVP for matrix analytic functions.

About interpolatin sequences for the Dirichlet space

Nicola Arcozzi, University of Bologna

Marshall and Sundberg, and independently C. Bishop, characterized the sequences in the unit disc which are interpolating for the Dirichlet space. The problem considered in this talk is understanding when the interpolating operator associated to a sequence is onto, although not necessarily into. Some necessary and some sufficient conditions, obtained in collaboration with R.Rochberg and E.Sawyer, are given.

Directional regularity and directional metric regularity

A.V.Arutyunov, Peoples Friendship University of Russia
E.R. Avakov, Institute for Control Problems RAS
A.F. Izmailov, Moscow State University

For general constraint systems, we present the directional stability theorem based on the appropriate generalization of directional regularity condition. This theorem contains Robinson's stability theorem but does not reduce to it. Furthermore, we develop the related concept of directional metric regularity which is stable subject to small Lipschitzian perturbations of the constraint mapping, and which is equivalent to directional regularity for sufficiently smooth mappings. These results enable unification of some diverse ideas in optimization theory and variational analysis, and can serve as a basis for sensitivity analysis of variational and optimization problems, including MPECs. The latter are notorious for the lack of standard regularity of constraints, but can have directional regularity properties.

Necessary optimality conditions in abnormal extremum problems

A.V. Arutyunov, Peoples Friendship University of Russia
D.Yu. Karamzin, Computing Center of RAS

An abnormal minimization problem with equality constraints and a finite-dimensional image is examined. Second-order necessary conditions for this problem are given that strengthen previously known results.

Necessary optimality conditions for discrete optimal control problems

A.V. Arutyunov, People's Friendship University of Russia
Boban Marinkovic, Faculty of Mining and Geology, Serbia

Consider the following discrete optimal control problem:

$$\sum_{i=0}^{N-1} f_i(x_i, u_i) \to \inf$$

$$x_{i+1} = \varphi(x_i, u_i), i = \overline{0, N-1}$$

where

$$f_i(x, u) : R^n \times R^r \to R, \varphi(x, u) : R^n \times R^r \to R^n$$

$$K_1(x_0, x_N) : R^n \times R^n \to R^{k_1}, K_2(x_0, x_N) : R^n \times R^n \to R^{k_2}$$

are twice continuously differentiable functions. For the preceding problem we shall obtain first and second-order optimality conditions which are meaningful without *a priori* normality assumptions.

Nondegenerate second-order necessary conditions of optimality for nonlinear optimization problems

Aram Arutyunov, Peoples Friendship University of Russia
Fernando Lobo Pereira, Porto University

In this article, we present second-order necessary conditions of optimality for the following general nonlinear optimization problem

$$(P) \text{ Minimize } f(x) \text{ subject to } F_1(x) \leq 0, F_2(x) = 0, x \in \mathbb{C}$$

Where X is a vector space, $C \subseteq X$ is a given closed set, $f : X \to \mathbb{R}, F_1 : X \to \mathbb{R}^{k_1}$ and $F_2 : X \to \mathbb{R}^{k_2}$ are given smooth mappings, and k_1 and k_2 are also given positive integers. The fact that our optimality conditions remain informative without Robinson's constraint qualication condition (i.e.,, even for abnormal points, [1]) is an important advantage relatively to some well established optimization literature, [3, 4]. The basic idea consists in using additional information from second-order conditions in order to select an appropriate subset of the set of multipliers satisfying the local necessary conditions of optimality. We also point out that, by taking into account the second order effect of the curvature of the set in the inclusion constraints (see [5, 4]), the range of applications of our conditions match that of the current state of the art under similar assumptions. Our method of proof draws heavily from the perturbation methods developed in [1].

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A Bloch estimate for automorphic functions

Rauno Aulaskari, University of Joensuu
Peter Lappan, Michigan State University

Let $\mathcal{D} = \{\ddagger : |\ddagger| < \infty\}$ be the unit disk in the complex plane and let Γ denote a Fuchsian group acting on $\mathcal{D}$. Let F denote a fundamental region for Γ for which the area of ∂F, the boundary of F, is zero. A function f, analytic in $\mathcal{D}$, is an automorphic function relative to Γ if

$$f(\gamma(z)) = f(z)$$

for each $\gamma \in \Gamma$ and each $z \in \mathcal{D}$. A function f analytic in $\mathcal{D}$ is said to be a Bloch function if

$$M = \sup_{z \in \mathcal{D}} (1 - |z|^2)|f'(z)| < \infty$$

and we denote the class of Bloch functions by $\mathcal{B}$.

For an analytic automorphic function f relative to Γ we consider, for $p \geq 2$, the integral

$$I = \iint_F (1 - |z|^2)^{p-2} |f'(z)|^p \, dxdy$$

and give the upper bound for M by a suitable power of I which improves the earlier known estimate [1].

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Stability for 2-regular general constraint systems

E.R. Avakov, Institute for Control Problems RAS

We present the stability theorem for general constraint systems near a point violating Robinson's constraint qualification. This result implies the inverse function theorem and the error bound theorem for the 2-regular case.

Estimates of norms of subsequent derivatives of r-monotone functions

Vladislav Babenko, Dnepropetrovsk National University
Yuliya Babenko, Sam Houston State University

We shall discuss some extremal properties of derivatives of r-monotone functions. This will lead to new results related to the Kolmogorov problem on estimating the norms of subsequent derivatives of an r-monotone function $x \in L^{r,r}_{\infty,\infty}(\mathbb{R}_-)$.

We will present the following

Theorem Let d and $0 < k_1 < k_2 < \cdots < k_{d-1} < k_d = r$ be given integers. If $x \in L^{r,r}_{\infty,\infty}(\mathbb{R}_-)$ and parameters of ϕ_r are chosen so that

$$(-1)^i \|\phi_r^{(k_i)}\|_\infty \geq (-1)^i \|x^{(k_i)}\|_\infty, \qquad i = 1, \ldots, d-2,$$

$$\text{and} \qquad \|\phi_r^{(r)}\|_\infty \geq \|x^{(r)}\|_\infty,$$

then $\forall k < k_1$

$$\|\phi_r^{(k)}\|_\infty \leq \|x^{(k)}\|_\infty.$$

Kolmogorov-type inequalities for the derivatives of functions of two variables

V.F.Babenko, Dnepropetrovsk National University
S.A.Pichugov, Dnepropetrovsk National University

Let $C(\mathbb{R}^2)$ be the space of bounded continuous functions $x : \mathbb{R}^2 \to \mathbb{R}$ with the norm $\|x\|_\infty := \sup\{|x(u)| \ : \ u \in \mathbb{R}^2\}$. Set $x^{(2,0)} := \frac{\partial^2 x}{\partial u_1^2}$, $x^{(0,2)} := \frac{\partial^2 x}{\partial u_2^2}$ and $x^{(1,1)} := \frac{\partial^2 x}{\partial u_1 \partial u_2}$. Denote by $C^2(\mathbb{R}^2)$ the space of all functions $x \in C(\mathbb{R}^2)$ such that derivatives $x^{(2,0)}, x^{(0,2)}, x^{(1,1)}$ are continuous.

For a vector $t = (t_1, t_2) \in \mathbb{R}^2$ denote by $\Delta_{te_j} x(u)$ the difference of a function $x(u)$ in variable u_j with the step t_j, i.e. $\Delta_{te_j} x(u) := x(u + te_j) - x(u)$, where $\{e_1, e_2\}$ is the standard basis in $\mathbb{R}^2$. Given $\beta_j \in (0, 1]$, $j = 1, 2$, set

$$H_j^{\beta_j} := \left\{ x \in C(\mathbb{R}^2) \ : \ \|x\|_{H_j^{\beta_j}} = \sup_{t \neq 0} \frac{\|\Delta_{te_j} x(\cdot)\|_\infty}{|t|^{\beta_j}} < \infty \right\}.$$

Let $\beta_1, \beta_2 \in (0, 1]$, and let $x \in C^2(\mathbb{R}^2)$ be such that $x^{(2,0)} \in H_1^{\beta_1}$, $x^{(0,2)} \in H_2^{\beta_2}$. Then

$$\|x^{(1,1)}\|_\infty \le 2^{3\gamma_0 - 1} \frac{\gamma_1^{\gamma_1} \gamma_2^{\gamma_2}}{\gamma_0} \|x\|_\infty^{\gamma_0} \left\|x^{(2,0)}\right\|_{H_1^{\beta_1}}^{\gamma_1} \left\|x^{(0,2)}\right\|_{H_2^{\beta_2}}^{\gamma_2},$$

where

$$\gamma_0 = 1 - \gamma_1 - \gamma_2, \ \gamma_1 = \frac{1}{\beta_1 + 2}, \ \gamma_2 = \frac{1}{\beta_2 + 2}.$$

The constant in this inequality is the best possible.

This theorem interpolates known results of Konovalov and Timoshin.

Let $\beta_1, \beta_2 \in (0, 1]$, and let $x \in C^2(\mathbb{R}^2)$ be such that $x^{(1,1)} \in H^{\beta_1} \cap H^{\beta_2}$. Then

$$\|x^{(1,1)}\|_\infty \le \frac{1 + 1/\beta_1 + 1/\beta_2}{(1 + 1/\beta_1)^{\gamma_1} (1 + 1/\beta_2)^{\gamma_2}} \|x\|_\infty^{\gamma_0} \left\|x^{(1,1)}\right\|_{H_1^{\beta_1}}^{\gamma_1} \left\|x^{(1,1)}\right\|_{H_2^{\beta_2}}^{\gamma_2},$$

where

$$\gamma_0 = \frac{1}{1 + 1/\beta_1 + 1/\beta_2}, \ \gamma_1 = \frac{1/\beta_1}{1 + 1/\beta_1 + 1/\beta_2}, \ \gamma_2 = \frac{1/\beta_2}{1 + 1/\beta_1 + 1/\beta_2}.$$

The constant in this inequality is the best possible.

This theorem generalizes known result of Babenko.

Optimal interval quadrature formulae for classes of differentiable periodic functions

V.F.Babenko, Dnepropetrovsk National University
D.S.Skorokhodov, Institute of Applied Mathematics and Mechanics

Let L_p, $1 \le p \le \infty$, be the space of 2π-periodic functions $f : \mathbb{R} \to \mathbb{R}$ with usual norm $\|\cdot\|_p$. Let $n \in \mathbb{N}$ and $0 < h < \pi/n$. Denote by K_n^h the set of all possible interval quadrature formulae of the form

$$\kappa(f) = \sum_{j=1}^{n} a_j \frac{1}{2h} \int\limits_{x_j-h}^{x_j+h} f(t)\,dt,$$

where $x_1 < x_2 < \ldots < x_n < x_1 + 2\pi$ and $a_j \in \mathbb{R}$. Let

$$\kappa_n^h(f) = \frac{2\pi}{n} \sum_{j=1}^{n} \frac{1}{2h} \int\limits_{2\pi j/n-h}^{2\pi j/n+h} f(t)\,dt$$

For a nonnegative function $f \in L_1$ let us denote by $P(f,t)$ the decreasing rearrangement of the restriction of f to $[0,2\pi)$. If g is an arbitrary function from L_1, then set

$$\Pi(g,t) = P(g_+,t) - P(g_-,2\pi - t),$$

where $g_\pm(t) = \max\{\pm g(t);0\}$. Let F be a subset of L_1 such that $\{f \in F \ : \ f \perp 1\} \ne \emptyset$. The set F is called rearrangement invariant or, shortly, Π-invariant if conditions $f \in F$ and $\Pi(g) = \Pi(f)$ imply $g \in F$. Denote by W^rF the class of functions f that have locally absolutely continuous derivative $f^{(r-1)}$ and such that $f^{(r)} \in F$. Note that if F is a unit sphere in L_p then $W^rF = W_p^r$.

For $f \in W^rF$ and $\kappa \in K_n^h$ set

$$R(f,\kappa) = \int\limits_0^{2\pi} f(t)\,dt - \kappa(f).$$

Let

$$R^\pm(W^rF,\kappa) = \sup\{\pm R(f,\kappa) \ : \ f \in W^rF\},$$
$$\mathcal{R}^\pm(W^rF,K_n^h) = \inf\{R^\pm(W^rF,\kappa) \ : \ \kappa \in K_n^h\}.$$

We have proved the following

Theorem. *Let $n,r \in \mathbb{N}$ and $0 < h < \pi/n$. Then for an arbitrary Π-invariant set F*

$$\mathcal{R}^\pm(W^rF,K_n^h) = R^\pm(W^rF,\kappa_n^h)$$

This result was obtained for the class W_1^r by V. F. Babenko in 1984, for the class W_∞^r by V. P. Motornyi in 1998, and for the class W^rF by S. V. Borodachov in 2000.

Optimal recovery of the solution of the Poisson equation from inaccurate information

Elena Balova, Russian State Technological University

The optimal recovery of the solution of the generalized Poisson equation on $\mathbb{S}^{d-1}$ is considered. We assume that the right side of the Poisson equation is the function from the Sobolev class $W_2^{\beta}(\mathbb{S}^{d-1})$. The finite number of Fourier coefficients of this function are given with some error. We calculate the error of optimal recovery of the solution and find an optimal method of recovery.

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On the growth properties of a product of solutions to complex differential equations

Djamal Benbourenane, United Arab Emirates University

One of the fundamental achievements in Analysis in the first part of the last century was the theory of value distribution of meromorphic functions created by Rolf Nevanlinna. This theory studies asymptotic properties of meromorphic functions, with special emphasis on the asymptotic distribution of their a-points. We will study complex differential equations in the unit disk with coefficients of finite order using extensively Nevanlinna theory but relying on a generalization by H. Cartan of the second main theorem in the plane. This theorem has many applications in value distribution theory and in number theory. Using results developed by the author on solutions to differential equations in the unit disk, we will investigate some growth properties of a certain product of the fundamental system of solutions.

Extremal problems in H^p for $0 < p < 1$.

Catherine Bénéteau

In this talk, I will survey some results on linear extremal problems for functions in the Hardy spaces H^p when $0 < p < 1$. Because these spaces are not normed spaces, the standard duality approach fails. I will discuss the work of S. Ya. Khavinson and V. Kabaila in the 60s, and will examine questions of uniqueness that arise in this context. In particular, I will consider the problem of finding the maximum modulus of the derivative of a function at the origin in the unit ball of H^p, $0 < p < 1$, provided that the value at the origin is fixed, and will identify the corresponding extremal functions.

Quasi-similarity of contractions using their characteristic functions

Serigo Bermudo, Universitdad Pablo de Olavide

This is a joint work with Carmen H. Mancera, Pedro J. Paul, Vasily Vasyunin. Quasi-similarity is an equivalence relation between bounded operators which, being weaker than similarity, still preserves many interesting features as the eigenvalues, the spectral multiplicity or the non-triviality of the lattice of invariant subspaces. Two Hilbert space bounded operators $T_1 \in \mathcal{B}(\mathcal{H}_\infty)$ and $T_2 \in \mathcal{B}(\mathcal{H}_\in)$ are saide to be $quasi - similar$ if there exist two bounded operators $X : \mathcal{H}_\infty \to \mathcal{H}_\in$ and $W : \mathcal{H}_\in \to \mathcal{H}_\infty$ such that

$$XT_1 = T_2X, \mathrm{clos}[X\mathcal{H}_\infty] = \mathcal{H}_\in, \ker(\mathcal{X}) = \{\prime\}$$

$$T_1W = WT_2, \mathrm{clos}[W\mathcal{H}_\in] = \mathcal{H}_\infty, \ker(\mathcal{W}) = \{\prime\}$$

In that case, such operators, X and W, are called $deformations$ or $quasi - affinities$. We study, with the help of the coordinate-free function model deveoloped by Nikolski and Vasyunin, the quasi-similarity of contractions having 2×2 (non-trivial) characteristic function. This case seems to be already somewhat difficult to manage, but we hope that it will provide hints to tackle the general case of an $n \times n$ singular characteristic matrix.

Composition operators on the minimal space invariant under Möbius transformations

Oscar Blasco, University of Valencia

It is shown that if $\Phi : \mathbb{D} \to \mathbb{D}$ is an analytic function such that $M_p(\Phi'', r) \in L^{p'}(dr)$ for some $1 < p < \infty$ and $1/p + 1/p' = 1$ then $C_\Phi(f) = f \circ \Phi$ defines a bounded composition operator on the space B_1, the minimal space invariant under Möbius transformations. This was conjectured by J. Arazy, S. Fisher and J. Peetre.

Interpolating sequences on Besov type spaces

Daniel Blasi Babot, Universitat Autnoma de Barcelona

We characterize the interpolating sequences for the weighted analytic Besov spaces $B_p(\alpha)$, defined by the norm

$$\|f\|^p_{B_p(\alpha)} = |f(0)|^p + \int_{\mathbb{D}} |(1-|z|^2)f'(z)|^p (1-|z|^2)^{1-2\alpha} \frac{dA(z)}{(1-|z|^2)^2},$$

$1 < p < \infty$ and $0 < \alpha < 1/2$, and for the corresponding multiplier spaces $\mathcal{M}(B_p(\alpha))$.

Extremal problems for oscillating polynomials

Borislav Bojanov, Sofia University

We shall study a technique for solving extremal problems in the class of polynomials of given degree that have maximal number of real zeros in [-1,1]. The technique is based on an inheritance theorem, that is, on a theorem which states that if a certain relation holds for two polynomials P and Q,it holds also for their derivatives. The Markov observation that if the zeros of P and Q interlace, then the zeros of P' and Q' interlace too, is a remarkable classical example of such an inheritance theorem. In the first lecture we demonstrate that the values of an oscillating polynomial at its critical points define it completely and that certain important functionals depend on these values in a monotonic way. In the second part we illustrate the technique on Markov and Turan type inequalities for polynomials. Several open problems will be mentioned in the both lectures. No special knowledge beyond the standard course on mathematical analysis is required.

A uniqueness theorem for (sub-)harmonic functions in the disc

Alexander Borichev, University of Bordeaux I

We discuss several known boundary uniqueness theorems for harmonic and subharmonic functions in the unit disc, and give a new result with operator-theoretic applications.

On asymptotically optimal formulas of approximate integration along the ball which use inegrals along hyperspheres

Sergiy Borodachov, Georgia Institute of Technology

For a certain class of functions defined on a d-dimensional ball and r times differentiable along the radius we consider the problem about the best cubature formula for integration with a radial weight which uses mean values of the function and its radial derivatives along n concentric spheres inside the ball. Using known results on spline approximation by Ligun, Storchai and Shumeiko, we find the node spheres of an asymptotically optimal formula as n gets large and the asymptotic behavior of the minimal worst case error on this class of functions.

Remez type inequalities and Morrey-Campanato spaces on Ahlfors regular sets

Alex Brudnyi, University of Calgary

In the talk we present several new results on Remez type inequalities for real and complex polynomials in n variables on Ahlfors regular subsets of Lebesgue n-measure zero. As an application we prove an extension theorem for Morrey-Campanato spaces defined on such sets. This is a joint work with Yu.Brudnyi, Technion, Haifa, Israel.

On the index of invariant subspaces in spaces of vector valued analytic functions

Marcus Carlsson, Lund University

Let $\mathcal{H}$ be a Hilbert space of analytic functions on the unit disc $\mathbb{D}$ with $||M_z|| \leq 1$, where M_z denotes the operator of multiplication by the identity function on $\mathbb{D}$. Under certain conditions on $\mathcal{H}$ it has been shown by Aleman, Richter and Sundberg that there are subspaces of index greater than 1 if and only if $\lim_{k\to\infty} ||M_z^k f|| \neq 0$ for some $f \in \mathcal{H}$. We show that the corresponding statement in Hilbert spaces of $\mathbb{C}^n$-valued analytic functions is false and prove a correct generalization of the theorem. In doing so we also improve earlier results on the boundary behavior of functions in such spaces.

Inequalities for operators and applications

Gilles Cassier, Universite Lyon I 43

In the first part, we establish a Julia's type inequality for all operators whose spectrum are contained in the closed unit disc $\overline{D}$. When the operator is a strict contraction, we retrieve a nice result of K. Fan which was obtained by a different method. It is important to notice that a new parameter appears in the general setting. A particular case is very interresting, when the numerical range of the operator is contained in $\overline{D}$. We explore some applications in two areas. The first one focuses on the operatorial hyperbolic metric, the second one relies the behavior of the scalar spectral densities of an absolutely continuous ρ-contraction T and the existence of a non trivial invariant subspace for $f(T)$, where f is an holomorphic self map of D. The last part comes from a joint paper with N. Suciu, the purpose is to give some sharpened forms of the von Neumann inequality and of the Schwarz inequality for strict ρ-contractions.

Carleson measures for the area Nevanlinna spaces

Boo Rim Choe, Korea University

Let $1 \leq p < \infty$ and let μ be a positive finite Borel measure on the unit disk D. The area Nevanlinna-Lebesgue space $N^p(\mu)$ consists of all measurable functions h on D such that $\log^+ |h| \in L^p(\mu)$, and the area Nevanlinna space N^p_α is the subspace of $N^p((1-|z|^2)^\alpha d\nu(z))$, where $\alpha > -1$ and ν is area measure on D, consisting of all holomorphic functions. We characterize Carleson measures for N^p_α, defined to be those measures μ for which $N^p_\alpha \subset N^p(\mu)$. Among appications are:

1. Closedness of area Nevanlinna spaces under both differentiation and integration. This is in contrast to the classical Nevanlinna space which is closed under neither.
2. composition operators.
3. Volterra operators and their companion operators.
4. A remark on spherical derivatives.

This is a joint work with H. Koo and W. Smith.

Unitary equivalence for cyclic subnormal operators

John B. Conway, George Washington University

In this talk we give the background and present a problem in function theory that is equivalent to the problem of when two cyclic subnormal operators are unitarily equivalent. This relies heavily on Thomsons Theorem on bounded point evaluations. Partial progress will be discussed.

The space formed by the weighted special atom and their analytic characterizations

Geraldo De Souza, Auburn University

We define the special atom spaces B^1 as

$$B^1 = \left\{ f : [-\pi, \pi] \to \mathbb{R}, \text{periodic} \ , f(t) = \sum_{n=0}^{\infty} c_n b_n(t) \ ; \ \sum_{n=0}^{\infty} |c_n| < \infty \right\}$$

where $b_0(t) = \frac{1}{2\pi}$ or $b_n(t) = \frac{1}{|I_n|} [\chi_{L_n}(t) - \chi_{R_n}(t)]$, $I_n \subseteq [-\pi, \pi]$, $I_n = L_n \cup R_n$, L_n, R_n are the halves of the interval I_n.

B^1 is endowed with the norm $\|f\|_{B^1} = \inf \sum_{n=0}^{\infty} |c_n|$, where the infimum is taken over all possible representations of f.

We will have several different generalizations of this space, for example, where $|I_n|$ is replaced by $w(|I_n|)$, where $w(t)$ is a certain weight function.

We show that these generalized weighted special atom spaces are the boundary values of the spaces of weighted analytic functions F defined on the complex disc $|z| < 1$ so that

$$\int_0^1 \int_{-\pi}^{\pi} |F'(re^{i\theta})| w(r) \ d(\theta) \ d(r) \ < \ \infty$$

for several different weights $w(t)$.

Also we compute the duals of these spaces by showing that they are weighted generalized Lipschitz spaces.

Some important applications are given.

Nonnegativity criterion for a degenerate quadratic form and a two-dimensional generalization of Hardy's inequality

Andrei Dmitruk, CEMI RAN

We study an integral quadratic functional of the classical calculus of variations with two-dimensional control, having a Legendre coefficient degenerate at a point. We prove a criterion for the nonnegativity of this functional in the form of sharp estimates on its coefficients, that can be considered as a generalization of a Hardy's inequality.

Interpolation splines and Shannon-Kotelnikov theorem

Vladimir L. Dolnikov, Yaroslavl State University
Nikolay A. Strelkov, Yaroslavl State University

The following problem is considered. Let $S_n(x) = S_n(x, f, h)$ be interpolation spline of order n (for f) such that the following conditions are satisfied:

1) for all $i \in \mathbb{Z}$ the restriction of S_n to segment $[ih + (n-1)h/2, ih + (n+1)h/2]$ is the algebraic polynomial of degree n;

2) $S_n(ih) = f(ih)$ for all $i \in \mathbb{Z}$;

3) $S_n \in C^{n-1}(\mathbb{R})$.

Let h and $\{f(ih)\}_{i\in\mathbb{Z}}$ be fixed and $n \to \infty$. It is shown that limit interpolation formula coincides with Shannon–Kotelnikov theorem.

Extremal problems in Hardy and Bergman spaces

Peter Duren, University of Michigan

During the first half of the 20th Century, special extremal problems for Hardy spaces were studied by many authors. Then around 1950, in works of Macintyre and Rogosinski, S. Ya. Khavinson, and Rogosinski and Shapiro, an elegant theory of dual extremal problems emerged, based on principles of functional analysis. That theory does not extend readily to Bergman spaces, but there have been some advances in recent years. Special problems have been studied in Bergman spaces, and attempts to unify the theory have met with partial success. These lectures will review the classical theory of linear extremal problems in Hardy spaces and will attempt to describe the state of the art in Bergman spaces.

On the stability of some stochastic integral equations

Khairia El-Said El-Nadi, Faculty of Science-Alexandria University Egypt

Stochastic Volterra equations of the form:

$$dx(t) = f(x(t))dt + \int_0^t K(t-s)x(s)ds\ dt + g(x(t))dB(t),$$

are considered, where $\{B(t) : t \geq 0\}$ is standard one - dimensional Brownian motion and the kernel K decreases to zero non-exponentially. We study the convergence rate to zero of the stochastic solutions of the considered equation. It is proved under suitable conditions that :

$$\lim_{t\to\infty} \frac{|x(t)|}{K(t)} = \infty, \quad \text{almost surely.}$$

The considered stochastic integral equations arise if we consider the Black-Scholes market consists of a Bank account or a bond and a stock. These stochastic models can also be applied to population dynamics in biology.

Rigidity of holomorphic generators and one-parameter semigroups

Mark Elin, ORT Braude College

The talk is based on a joint work with M. Levenshtein, D. Shoikhet and R. Tauraso.
We establish a rigidity property of holomorphic generators by using their local behavior at a boundary point τ of the open unit disk Δ. Namely, if $f \in \mathrm{Hol}(\Delta, \mathbb{C})$ is the generator of a one-parameter continuous semigroup $\{F_t\}_{t\geq 0}$, we show that the equality $f(z) = o\left(|z-\tau|^3\right)$ when $z \to \tau$ in each non-tangential approach region at τ implies that f vanishes identically on Δ. Note, that if F is a self-mapping of Δ then $f = I - F$ is a generator, so our result extends the boundary version of the Schwarz Lemma obtained by D. Burns and S. Krantz.
We also prove that two semigroups $\{F_t\}_{t\geq 0}$ and $\{G_t\}_{t\geq 0}$, with generators f and g respectively, commute if and only if the equality $f = \alpha g$ holds for some complex constant α. This fact gives simple conditions on the generators of two commuting semigroups at their common null point τ under which the semigroups coincide identically on Δ.

The generalized Faber expansions for linear n-widths

Yu.A.Farkov, Russian State Geological Prospecting University,

Let Ω be an open subset of the plane $\mathbb{C}$ and let K be a compact subset of Ω. An analytic function $f : \Omega \to \mathbb{C}$ belongs to $BH^\infty(\Omega)$ if $|f(z)| \le 1$ for all $z \in \Omega$. Denote by $\mathrm{cap}\,(K, \Omega)$ the capacity of K related to Ω. For some K and Ω one can show that

$$\lim_{n\to\infty} [\lambda_n(BH^\infty(\Omega); C(K))]^{1/n} = \exp(-1/\mathrm{cap}\,(K, \Omega)),$$

where λ_n are the linear n-widths. In the case where K has a simply connected complement and Ω is a canonical neighbourhood of K, the classical tools for approximation of $f \in BH^\infty(\Omega)$ in $C(K)$ give the Faber series (cf. [1]).

The object of my talk is to consider the pairs (K, Ω) for which there exists a sequence functions $\{f_n\}$ with the following properties:

1. If f is analytic on Ω, then

$$f(z) = \sum_{k=0}^{\infty} a_k f_k(z), \quad z \in \Omega,$$

where the convergence is absolute and uniform over any compact subset of Ω. Moreover, this expansion is unique.

2. If $f \in BH^\infty(\Omega)$, then

$$\limsup_{n\to\infty} ||f - \sum_{k=0}^{n-1} a_k\, f_k||_{C(K)}^{1/n} \le \exp(-1/\mathrm{cap}\,(K, \Omega)).$$

In special cases such systems of analytic functions was studied by J.L. Walsh, V.D. Erokhin, V.P. Zakharyuta, and others (see [2, § 3.3]). If we assume that K is an union of several continua and $\partial K, \partial\Omega$ are smooth enough curves, then the relations

$$\lambda_n(BH^\infty(\Omega); C(K)) \asymp \exp(-n/\mathrm{cap}\,(K, \Omega)),$$

$$\lambda_n(BH^\infty(\Omega), L^q(K)) \asymp n^{-1/q} \exp(-n/\mathrm{cap}\,(K, \Omega)), \quad 1 \le q < \infty,$$

are obtained by the consided generalized Faber expansions.

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A version of Wiman-Valiron theory

Peter Fenton, University of Otago

Wiman-Valiron theory concerns the analysis of functions $f(z) = \sum_0^\infty a_n z^n$ by means of two associated functions, the *maximum term*, $\mu(r, f) = \max_{n\geq 0} |a_n| r^n$, and the *central index*, $N(r, f)$, which is the degree of the maximum term. One approach is to compare $f(z)$ with a smooth function $F(z) = \sum_0^\infty A_n z^n$ whose growth approximates that of $f(z)$. An account of this approach, which is very successful in the case in which $f(z)$ is entire, was given by Hayman in a long survey paper (1974). Hayman concluded the paper with a comment in which he expressed reservations about the possibility of extending this comparison idea to functions in the disc. Subsequently Strelitz (1993) established good results for functions in the disc using a quite different method. The intention here is to indicate how the comparison version of Wiman-Valiron theory can be extended to functions in the disc. This is work with my PhD student, Max Strumia.

Approximation of and by the Riemann zeta function

Paul Gauthier, University of Montreal

The Riemann zeta function can be approximated by functions for which the Riemann hypothesis fails. Any holomorphic function can be approximated by linear combinations of translates of the Riemann zeta function. These results are respectively joint work with Zeron and Tarkhanov.

On optimal embedding of generalized Bessel potentials

M.L. Goldman, People's Friendship University of Russia

For the space of generalzied Bessel potentials

$$F_E^a(R^n) = \{u = G_a * f, f \in E(R^n)\}, a > 0;$$

$$G_a * f(x) = \int_{R^n} G_a(x-y)f(y)dy; x \in R^n,$$

where G_a is Bessel-Macdonald kernel, $E(R^n)$ is rearrangement invariant space (shortly RIS), the criterion of embedding into RIS $X = X(R^n)$ is established:

$$F_E^a(R^n) \subset X(R^n) \qquad (1)$$

(without any a priory restrictions on RIS E, X). Order shar estimates are obtained for local growth envelope

$$A(t) = \sup\{u^*(t) : u \in F_E^a, ||u||_{F_E^a} \leq 1\}, t \in (0, T]$$

where u^* is decreasing rearrangement of function $u, T >$ is fixed. These results are concretized for the Bessel potentials based on generalized weighted Lorentz spaces $E = \Lambda_p(v)$(in particular for the case of Lorentz-Karamato spaces). The rearrangement invariant envelope is described (for given RIS E and given $a > 0$), i.e. such optimal RIS $X_0 = X_0(R^n)$ is found, that (1) is true for $X = X_0$, and if (1) holds for some RIS X then $X_0 \subset X$. The detailed discussion of corresponding problems and many references is contained in our survey: M.L. Goldman "Rearrangement Invariant Envelopes of Generalized Besov, Sobolev and Calderon Spaces", Contemporary Mathematics, v. 424 (2007) "The Interaction of Analysis and Geometry", pp 53 - 81.

Quadrature formulas on the unit circle with prescribed nodes and maximal domain of validity

P. González-Vera, La Laguna University

As it is known, Gaussian formulas are quadrature rules with the highest algebraic precision degree when considering the approximate calculation of weighted integrals on intervals of the real line. Their analogue on the unit circle are the so-called "Szego fomulas" which are quadrature rules with the highest trigonometric precision degree. These quadratures and other related topics as Szego polynomials and the trigonometric moment problem have received much recent attention as a result of their applications in the field of the digital signal processing so that they have become an active area of reserach. This has motivated, when dealing with the computation of integrals exhibiting singularities near the unit circle, the introduction of the so-called "rational Szego quadrature formulas" exactly integrating rational functions with prescribed poles not on the unit circle and depending on a free parameter. In this talk, an appropriate election of this parameter is made in order to enlarge at maximum the domain of validity of the quadratures. The case when some nodes are previosuly fixed on the unit circle is separately analyzed.

On the continuity of $\mathbf{L_p}$ balls and an application

Khalik G. Guseinov, Anadolu University
Ali S. Nazlipinar, Anadolu University

The need to evaluate the distance between the sets arise in various problems of theory and applications. To define the distance between the subsets of the given metric space, the Hausdorff distance notion is used. The distance between the subsets of the different metric spaces is defined by Gromov-Hausdorff distance. Other distance notions between the sets are also introduced in investigation of various problems.

Theorem: Let $\parallel \cdot \parallel$ be Euclidean norm in R^m, $\parallel u(\cdot) \parallel_p$ $(1 \leq p < +\infty)$ be a norm in $L_p([t_0,\theta], R^m)$ where

$$\parallel u(\cdot) \parallel_p = \left(\int_{t_0}^{\theta} \parallel u(t) \parallel^p \ dt \right)^{\frac{1}{p}}.$$

For $p \geq 1$ and $\mu_0 > 0$ we set

$$B_p(\mu_0) = \left\{ u(\cdot) \in L_p\big([t_0,\theta], \mathbb{R}^m\big) : \|u(\cdot)\|_p \leq \mu_0 \right\}.$$

The Hausdorff distance between the sets $U \subset L_{p_1}([t_0,\theta], R^m)$ and $V \subset L_{p_2}([t_0,\theta], R^m)$ is denoted by $h_1(U,V)$ and is defined as

$$h_1(U,V) = \max \left\{ \sup_{x(\cdot)\in V} d_1(x(\cdot), U), \sup_{y(\cdot)\in U} d_1(y(\cdot), V) \right\}$$

where $d_1(x(\cdot), U) = \inf \{ \parallel x(\cdot) - y(\cdot) \parallel_1 : y(\cdot) \in U \}$, $p_1 \in [1,\infty)$, $p_2 \in [1,\infty)$.

We prove the validity of following theorem, which characterizes continuity of the set valued map $p \to B_p(\mu_0)$ with respect to p where $p \in (1 + \infty)$.

Let $p_* > 1$ and $\varepsilon \in (0, \alpha_0)$. Then there exists $\delta = \delta(\varepsilon) > 0$ such that the inequality

$$h_1(B_p(\mu_0), B_{p_*}(\mu_0)) \leq \varepsilon$$

holds for all $p \in (p_* - \delta, p_* + \delta)$ where $\alpha_0 = \min\left\{\frac{\mu_0}{2}, 1\right\}$

As an application, we consider attainable sets of the nonlinear control system with integral constraints on control. $B_p(\mu_0)$ is chosen as the set of admissible control functions. Using Theorem 1, it is proved that the attainable set of the control system is continuous with respect to p.

Improved inverse theorems in weighted Lebesgue and weighted Smirnov spaces

Ali Guven, Balikesir University
Daniyal M. Israfilov, Balikesir University

Let $\mathbb{T}$:=$[-\pi,\pi]$ and let $\omega:\mathbb{T}\to[0,\infty]$ be a weight function, i. e. a measurable function on $\mathbb{T}$ such that $\omega^{-1}(\{0,\infty\})$ has measure zero. With any given weight ω, we associate the $\omega-$*weighted Lebesgue space* $L_p(\mathbb{T},\omega)$, $1\le p<\infty$. Let also

$$\sigma_h(g)(x):=\frac{1}{2h}\int_{-h}^{h}g(x+t)\,dt,\quad 0<h<\pi,\quad x\in\mathbb{T}$$

for $g\in L_p(\mathbb{T},\omega)$ and let

$$\Omega_k(g,\delta)_{p,\omega}:=\sup_{0<h\le\delta}\left\|T_h^k g\right\|_{L_p(\mathbb{T},\omega)},\quad \delta>0,$$

where

$$T_h g=T_h^1 g:=g-\sigma_h(g),\quad T_h^k g:=T_h\left(T_h^{k-1}g\right),\quad k=1,2,...$$

is the kth *modulus of smoothness* of $g\in L_p(\mathbb{T},\omega)$.

Let G be a domain in the complex plane, bounded by a Carleson curve Γ and let $E_p(G,\omega):=\{f\in E_1(G):f\in L_p(\Gamma,\omega)\}$, $1\le p<\infty$,be the $\omega-$weighted Smirnov space of analytic functions in G. We put

$$f^+(z):=\frac{1}{2\pi i}\int_\Gamma\frac{f(\varsigma)}{\varsigma-z}d\varsigma,\quad z\in G.$$

With every weight function ω on the rectifiable Jordan curve Γ, we associate another weight ω_0 on $\mathbb{T}$, defined by $\omega_0:=\omega\circ\psi$, where ψ is the conformal mapping of $G^-:=C\backslash\overline{G}$ onto $|w|>1$ with the conditions

$$\psi(\infty)=\infty,\quad \lim_{z\to\infty}\frac{\psi(z)}{z}>0.$$

Let $\omega\in A_p(\Gamma)$ and $\omega_0\in A_p(\mathbb{T})$, where $1<p<\infty$. If $f\in L_p(\Gamma,\omega)$, then $f_0:=(f\circ\psi)\left(\psi'\right)^{1/p}\in L_p(\mathbb{T},\omega_0)$.We define the kth *modulus of smoothness* of the function $f\in L_p(\Gamma,\omega)$ by

$$\Omega_k(f,\delta)_{\Gamma,p,\omega}:=\Omega_k\left(f_0^+,\delta\right)_{p,\omega_0},\quad \delta>0.$$

In this work we discuss the inverse and improved inverse theorems in the weighded spaces $L_p(\mathbb{T},\omega)$ and $E_p(G,\omega)$, using the modulus $\Omega_k(g,\delta)_{p,\omega}$ and $\Omega_k(f,\delta)_{\Gamma,p,\omega}$, respectively.

Multipliers of Faber series in weighted Smirnov-Orlicz Classes

Ali Guven, Balikesir University
Daniyal M. Israfilov, Balikesir University

Let G be a finite domain in the complex plane, bounded by a Dini-Smooth curve Γ. In this talk, we discuss a multiplier theorem for Faber series of analytic functions of the weighted Smirnov-Orlicz class $E_M(G, \omega)$, where ω is a weight function belonging to the Muckenhoupt class on Γ.

On the computation of contiguous relations for ${}_2F_1$ hypergeormetric Series

Adel K. Ibrahim, Suez Canal University
Medhat A. Rakha, Sultan Qaboos University

Contiguous relations are a fundamental concept within the theory of hypergeometric series and orthogonal polynomials. Their study goes back to Gauss who gave a list of 15 fundamental relations for the ${}_2F_1$ hypergeometric series. Applications of contiguous relations range from the evaluation of hypergeometric series to the derivation of summation and transformation formulas for such series. In this paper, we will present a new formula joining three Gauss functions of the form ${}_2F_1[a_1, a_2, a_3, z]$ with arbitrary integer shifts. Our analysis relies on the use of shifted operators attached to the three parameters $a_1, a_2 and a_3$. We also, discuss the sufficient conditions for such formula.

Approximation properties of the generalized Bieberbach polynomials in the closed Dini-Smooth domains

Daniyal M. Israfilov, Balikesir University
Burcin Oktay, Balikesir University

Let G be a finite Dini-smooth domain, $z_0 \in G$, and let

$$\varphi_p(z) := \int_{z_0}^{z} \left[\varphi_0'(\zeta)\right]^{2/p} d\zeta, \quad z \in G, \quad p > 0,$$

where φ_0 is the conformal mapping of G onto $D(0, r_0) = \{w : \mid w \mid < r_0\}$ with the normalization $\varphi_0(z_0) = 0$, $\varphi_0'(z_0) = 1$. In this talk, the approximation properties of the generalized Bieberbach polynomials $\pi_{n,p}(z)$, $1 < p < \infty$, $n = 1, 2, ...$ of the pair $(G,\ z_0)$ are studied and the error

$$\|\varphi_p - \pi_{n,p}\|_{\overline{G}} := \max_{z \in \overline{G}} |\varphi_p(z) - \pi_{n,p}(z)|$$

with respect to the geometric properties of $\overline{G}$ is estimated.

On the imbedding of Calderon-Orlicz space in L_∞

Davaadulam Jamsranjav, National University of Mongolia

Let $E = E(R^n)$ be a rearrangement invariant space, $\varphi_E = \|\chi_{(0,t]}\|_{\widetilde{E}}, (t \geq 0)$ -its fundamental function and $\mu_E(t) = [\varphi_E(t^{-1})]^{-1}$.

Let Φ and Ψ be complementary Young functions and ν be non-negative measurable function on $(0,\infty)$. Recall that the weighted Orlicz space $L_{\Phi,\nu}(R_+)$ is generated by the norm

$$\|f\|_{\Phi,\nu} = \inf\left\{\lambda > 0 : \int_0^\infty \Phi\left(\lambda^{-1}|f(x)|\right)\nu(x)dx \leq 1\right\}.$$

For a rearrangement invariant space (RIS) $E = E(R^n)$, a Calderon-Orlicz space $\Lambda_{E,\Phi}(R^n;\nu)$ is defined as a subspace that is consists of functions $f \in E$, such that their best approximations $e_t(f)_E$ in the norm E by entire functions of exponential type of degree $t > 0$ belong to an ideal space $F = L_{\Phi,\nu}(R_+) \bigcap L_\infty(R_+)$. In this case

$$\|f\|_{\Lambda_{E,\Phi}} = \|f\|_E + \|e_t(f)_E\|_{L_\infty} + \|e_t(f)_E\|_{\Phi,\nu}.$$

Theorem. *Under the notation above let $\Phi \in \Delta_2$ and $V(\infty) = \int_0^\infty \nu(t)dt = \infty$. Then the imbedding*

$$\Lambda_{E,\Phi}(R^n;\nu) \hookrightarrow L_\infty$$

is valid under the condition

$$\Phi\left(\frac{1}{w(t)}\right)\mu_E(t) \in L_{\Psi,\nu},$$

where $w(t) = \|\chi_{(0,t]}\|_F = 1 + [\Phi^{-1}\left(V(t)^{-1}\right)]^{-1}$.

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Extremal problems with linear constraints

Vladimir Janković, University of Belgrade

The Kuhn-Tucker theorem for the convex problem with linear constraints states that the Lagrange multiplier λ_0 is different from zero, without the additional assumption that Slater condition holds. In this talk we show that analogous statements hold for various extremal problems in the calculus of variations and optimal control.

Toeplitz C^*-algebras on Dirichlet spaces of the ball

H. Turgay Kaptanoğlu, Bilkent University

Dirichlet spaces $\mathcal{D}_q$ ($q \in \mathbb{R}$) are Hilbert spaces of holomorphic functions on the unit ball of $\mathbb{C}^N$ defined by a reproducing kernel which is $K_q(z,w) = (1-\langle z,w\rangle)^{-(N+1+q)}$ for $q > -(N+1)$ and given by a hypergeometric function for $q \leq -(N+1)$.

We consider the C^*-algebra $\mathcal{T}_q$ generated by the N-tuple of operators of multiplication by the coordinate functions (the so-called N-shift) on $\mathcal{D}_q$. We show that $\mathcal{T}_q$ contains all compact operators and Toeplitz operators with continuous symbols, a quotient map sends it onto the continuous functions on the boundary of the unit ball, and thus obtain the related short exact sequence of C^*-algebras. This result generalizes what is known for the Hardy space ($q = -1$), Bergman spaces ($q > -1$), and the Drury-Arveson space ($q = -N$).

We also show that the symmetric Fock space over $\mathbb{C}^N$ can be realized as each of the Dirichlet spaces under a suitable norm as noticed earlier for the Arveson space ($q = -N$).

Nonlinear extremal problems in spaces of analytic functions

Dmitry Khavinson, University of South Florida

In this talk I plan to give a survey of a number of still open basic extremal problems for non-vanishing functions in Hardy and Bergman spaces of analytic functions. I will discuss recent developments and show why the methods developed by S. Ya. Khavinson and V. M.Terpigoreva in the early 1960s that allowed at least qualitatively to solve a large variety of extremal problems in Hardy spaces run aground in the Bergman spaces context. Some examples and open questions will be discussed as well.

On algebras of bounded holomorphic functions in the unit polydisk generated by almost-periodic functions

Damir Kinzebulatov, University of Calgary

We introduce the algebra of bounded holomorphic functions in the unit polydisk which possess the weakest possible discontinuities on the boundary torus. The boundary values of the elements of this algebra are generated by semi almost-periodic functions. In our talk we discuss the properties of this algebra, in particular, the properties of its maximal ideal space.

The inequalities for derivatives of functions in the spaces L_p

V. A. Kofanov, Dnepropetrovsk National University

Let G denotes the interval $[a,b]$, real line $\mathbf{R}$, or the unit circle $\mathbf{T}$ which is realised as interval $[0,2\pi]$ with identified endpoints. We shall consider the spaces $L_p(G)$, $0<p\le\infty$, of all measurable functions $x: G\to\mathbf{R}$ such that $\|x\|_p=\|x\|_{L_p(G)}<\infty$, where functional $\|x\|_p$ is defined as usual. Let $L^r_p(G)$ be the spaces of functions x such that $x^{(r-1)}$ be the locally absolutely continuous and $\|x^{(r)}\|_p<\infty$. Set $L^r_{p,s}(G):=L_p(G)\cap L^r_s(G)$.

Our main result is the following

Theorem Let $r\in\mathbf{N}$, $r\ge 2$, $x\in L^r_{\infty,\infty}(\mathbf{R})$, and numbers $a,b\in\mathbf{R}$ satisfying the conditions $x'(a)=x'(b)=0$, $|x'(t)|>0$ for $t\in(a,b)$. Then for any $q\ge 1$, and if $r=2$ or $r=3$, then for any $q>0$

$$\frac{1}{b-a}\int\limits_a^b |x'(t)|^q dt \le \frac{1}{\pi}\int\limits_0^\pi |\varphi_{r-1}(t)|^q dt \left(\frac{\|x\|_{L_\infty(\mathbf{R})}}{\|\varphi_r\|_\infty}\right)^{\frac{r-1}{r}q} \left\|x^{(r)}\right\|_\infty^{\frac{q}{r}},$$

where φ_r is the perfect Euler's spline of order r.

The following inequalities of Kolmogorov type for periodic functions of small smoothness are proved with the help of Theorem 1.

Theorem. The following inequalities hold:

$$\left\|x^{(k)}\right\|_q \le \frac{\|\varphi_{r-k}\|_q}{\|\varphi_r\|_\infty^{1-\frac{k}{r}}}\|x\|_\infty^{1-\frac{k}{r}}\left\|x^{(r)}\right\|_\infty^{\frac{k}{r}}, \quad q>0,$$

for $x\in L^r_{\infty,\infty}(\mathbf{T})$, $r=2$, $k=1$ or $r=3$, $k=1,2$,
and

$$\left\|x^{(k)}\right\|_1 \le \frac{\|g_{r-k}\|_1}{\|g_r\|_\infty^{1-\frac{k}{r}}}\|x\|_\infty^{1-\frac{k}{r}}\left\|x^{(r)}\right\|_1^{\frac{k}{r}}$$

for $x\in L^r_{\infty,1}(\mathbf{T})$, $r=3$, $k=1,2$, where $g_r:=4^{-1}\varphi_{r-1}$.

Some other applications of Theorem 1 will be presented.

Finite rank Toeplitz operators with harmonic symbols

Hyungwoon Koo, Korea University and University at Albany

On the Bergman space of the unit ball of $\mathbb{C}^n$, we study the finite rank problem for Toeplitz products with harmonic symbols. We first solve the problem with two factors in case symbols have local continuous extension property up to the boundary. Also, in case symbols have additional Lipschitz continuity up to (some part of) the boundary, we solve the problem for multiple products with number of factors depending on the dimension n. An analogous theorem on the polydisk is also obtained.

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Upper bounds on classical orthogonal polynomials

Ilia Krasikov, Brunel University

T. Erdélyi, A.P. Magnus and P. Nevai conjectured that for $\alpha, \beta \geq -\frac{1}{2}$, the orthonormal Jacobi polynomials $\mathbf{P}_k^{(\alpha,\beta)}(x)$ satisfy the inequality

$$\max_{x\in[-1,1]}(1-x)^{\alpha+\frac{1}{2}}(1+x)^{\beta+\frac{1}{2}}\left(\mathbf{P}_k^{(\alpha,\beta)}(x)\right)^2 = O\left(\max\left\{1, (\alpha^2+\beta^2)^{1/4}\right\}\right).$$

We will confirm this conjecture for $k \geq 6, \quad \alpha = \beta \geq \frac{1+\sqrt{2}}{4}$, even in a stronger form by giving very explicit upper bounds.

Moreover, for the ultraspherical case $\beta = \alpha$, and k even, we establish the sharper inequality

$$\sqrt{\delta^2 - x^2}\,(1-x^2)^{\alpha}\left(\mathbf{P}_{2k}^{(\alpha,\alpha)}(x)\right)^2 < \frac{2}{\pi}\left(1 + \frac{1}{8(2k+\alpha)^2}\right)$$

for a certain choice of δ, such that the interval $(-\delta, \delta)$ contains all the zeros of $\mathbf{P}_{2k}^{(\alpha,\alpha)}(x)$.

Similar inequalities of the form

$$C_1 < \sqrt{(x-\delta_1)(\delta_2 - x)}\,w(x)p_k^2(x) < C_2$$

with the explicit absolute constants C_1, C_2 are obtained for p_k being the Hermite or general Laguerre polynomial of degree k. Here w is the corresponding weight function, and $[\delta_1, \delta_2]$ is an explicitly given interval containing all the zeros of p_k.

Some algebraic properties of Toeplitz operators on the Dirichlet space

Young Joo Lee, Chonnam National University

In this talk, we consider Toeplitz operators on the Dirichlet space of the unit disk and investigate some algebraic properties. We first characterize (semi-)commuting Toeplitz operators with harmonic symbols. Next we study the product problem of when the product of two Toeplitz operators is another Toeplitz operator. As an application, we show that the zero product of two Toeplitz operators with harmonic symbol has only the trivial solution. Also, the corresponding compact product problem will be discussed.

Universality limits for exponential weights

Evi Levin, Open University of Israel

We establish universality in the bulk for fixed exponential weights on the whole real line. Our methods involve first order asymptotics for orthogonal polynomials and localization techniques. In particular we allow exponential weights such as $|x|^{2\beta} g^2(x) \exp(-2Q(x))$, where $\beta > -1/2$, Q is convex and Q'' satisfies some regularity conditions, while g is positive, and has uniformly continuous and slowly growing or decaying logarithm. We apply universality limits to asymptotics of spacing of zeros of orthogonal polynomials.
This is joint work with Doron S. Lubinsky.

Optimal recovery of linear functionals in normed spaces

L.S. Maergoiz, Siberian Federal University
N. Tarkhanov, Potsdam University

The talk is devoted to development of an approach to the problem of optimal recovery of continuous linear functionals in normed spaces through information on a finite number of given functionals. The results obtained are applied to the problem of the best analytic continuation from a finite set in the complex space $\mathbb{C}^n$, $n \geq 1$, for the Wiener class $W_\sigma^p = \{f\}$ of entire functions of exponential type such that

$$|f(z)| \leqslant A \exp\{\sigma_1 |\Im z_1| + \cdots + \sigma_n |\Im z_n|\} \quad z \in \mathbb{C}^n$$

for some $A > 0,\ \sigma_j > 0,\ j = 1, ..., n$. Moreover, every element f belongs to the space L^p, $1 < p < \infty$, on the real subspace of $\mathbb{C}^n$. These results are generalization of investigations in [1]-[2]. They are published in [3].

Let $T : V \to B$ be an algebraic isomorphism of a vector space V onto a normed space B, both V and B being over the same field $\mathbb{K}$ where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. We give a norm to V by setting $\|f\|_V := \|Tf\|_B$ for $f \in V$, thus making V a normed space, too. Suppose that U is a closed ball of radius $R > 0$ about the origin in V. Given a finite number of independent functionals $L_1, \ldots, L_N$ on U, we consider the problem of recovering any fixed functional L on U through $L_1, \ldots, L_N$.

Our result is used for a solution of the problem of the best analytic continuation from a finite set in the complex space $\mathbb{C}^n$, $n \geq 1$, for the Wiener class $V = W_\sigma^p$. This solution based on the classical Plancherel-Pólya theorem [4].

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Approximation by ridge and radial functions in L_p-spaces

V.E. Maiorov, Technion

We investigate the approximation of classes of multivariate functions by the manifold R_n formed by all possible linear combinations of n ridge functions of the form $r(a \cdot x)$. We calculate an exact asymptotic for the deviation of the Sobolev class W_p^r from the set R_n of ridge functions in the space L_q, when $1 \leq q \leq p \leq \infty$. Moreover, we obtain analogous results for the approximation by radial functions.

Hyperbolic Besov functions

Shamil Makhmutov, Institute of Mathematics Ufa and Sultan Qaboos University

Let B be the family of all φ analytic and bounded, $|\varphi| < 1$, in the unit disk $D = \{|z| < 1\}$. The hyperbolic Besov classes B_p^h, $1 < p < \infty$, ([1], [2], [3]) are defined as the sets of holomorphic self-maps φ of D such that

$$\int_D (1 - |z|^2)^{p-2} (\varphi^\star(z))^p dA(z) < \infty$$

where $\varphi^\star(z) = |\varphi'(z)|(1 - |\varphi(z)|^2)^{-1}$ is the hyperbolic derivative of φ. Hyperbolic Besov functions induce compact composition operator C_φ that takes the Bloch space $\mathcal{B}$ into the corresponding analytic Besov space B_p. We will discuss various properties of hyperbolic Besov functions in terms of oscillation, value distribution, boundary behavior and etc.

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Cusp algebras

John E. McCarthy, Washington University of St. Louis

A cusp algebra is a subalgebra of the disk algebra given by a finite number of linear conditions on a function's derivatives at the origin. We shall discuss descriptions of these algebras, and realizations of them in C^n. This is joint work with Jim Agler of UCSD.

Universal integral means spectrum for Bloch functions

Jose Ignacio Monreal Galn, Universitat Autnoma de Barcelona

In 1994 N. Makarov introduced the Universal Integral Means Spectrum $B(p)$ defined as

$$B(p) := \sup \limsup \frac{\log \int_0^{2\pi} |f'(re^{i\theta})|^p d\theta}{\log(1-r)^{-1}}$$

where the supremum is taken over all univalent functions f in the disc. The exact value of $B(p)$ is only known for some ranges of p, and a complete identifcation of $B(p)$ is open. For instance, Brennan's conjecture can be restated as $B(-2) = 1$. Using the well-known relation between Bloch functions and univalent functions, an analogue of the function $B(p)$ will be introduced as

$$B_d^*(p) := \sup \limsup \frac{\log \int_0^{2\pi} e^{pReg(re^{i\theta})} d\theta}{\log(1-r)^{-1}}$$

where the supremum is taken over all Bloch functions g in the unit disc with $||g||_{B_d} \leq 1$. Here $|| \cdot ||_{B_d}$ is a convenient dyadic version of the Bloch norm. The main result of the talk is that

$$B_d^*(p) = \log_2(\cosh p).$$

Generalized Segal-Bargmann-Fock spaces and spectral theory of Landau Hamiltonians on C^n with application to some evolution equations

Zouhair Mouayn, Cadi Ayyad University

We introduce a class of generalized Segal-Bargmann-Fock spaces onthe Euclidean complex n-spaces. Functions of these spaces are Gaussian square integrable eingenfunctions of the Landau Hamiltonian, which are associated with the discret eigenvalues of the spectrum. The expression of the reproducing kernels of these spaces enabls us to obtain an L_2-spectral theory for the Hamiltonian operator as well as the corresponding Berezin transforms. We dicuss also some associated evolution equations (the obtained results have been realized in joint works with N. Askour and A.Intissar)

Analytical and numerical methods in the problem of extiguishing of the string vibrations by the point damper

Leonid A. Muravey, Russian State Technological University

The problem of damping of string vibrations with the help of moving dotted damper alone the part of string is considered. The analitical decisions problem is corresponding moment problems solviability conserned with existance the special Riesz basis system, for example, the product of trigonometrical functions in L_2 (0, T) for some $T > 0$. The numerical method is offered for decision of the common problem. Examples of calculations are given.

Compact differences of composition operators on Bloch and Lipschitz spaces

Pekka Nieminen, University of Helsinki

We consider the difference $T = C_\phi - C_\psi$ of two analytic composition operators in the unit disc. We characterize the compactness and weak compactness of T on the standard Bloch space, improving an earlier result of Hosokawa and Ohno. We also characterize the compactness and weak compactness of T on analytic Lipschitz spaces. These results are derived from a general one dealing with differences of weighted composition operators on weighted Banach spaces of analytic functions.

On a certain problem for algebraic polynomials in the Sobolev spaces

B.P. Osilenker, Moscow State Civil Engineering University

Let $\{\mu_k\}(k=0,1,...,m;m\in\mathbb{Z}_+)$ be a set of $(m+1)$ finite positive Borel measures with the support δ_k of μ_k . Assume that at least on of the measures μ_k has infinity many points of increase . If $f^{(k)}$ denotes kth derivative of the function f ,then expression

$$\langle f,g\rangle_S=\sum_{k=0}^{m}\int_{\delta_k}f^{(k)}(x)g^{(k)}(x)d\mu_k(x) \qquad (1)$$

defines an inner product in the linear space $\mathbb{P}$ of polynomials with real coefficients. Completion of $\mathbb{P}$ with respect to the norm $\|f\|_S=\langle f,f\rangle_S{}^{\frac{1}{2}}$ leads to the suitable Sobolev space of functions.
The Gram — Schmidt process with respect to the (1) applied to the canonical basis of $\mathbb{P}$ generates sequence of the Sobolev orthonormal polynomials
$\{\hat{q}_n\}(n\in Z_+;\deg\hat{q}_n=n)$.
We consider polynomial

$$\Pi_N^{(r)}(x)=\sum_{s=N-r+1}^{N}a_s^0x^s+\sum_{s=0}^{N-r}a_sx^s,$$

where $a_s^0(s=N-r+1,...,N-1,N;a_N^0>0)$ are fixed numbers and investigate the following extremal problem:
To find

$$\inf_{a_0,a_1,...,a_{N-r}}\langle\Pi_N^{(r)},\Pi_N^{(r)}\rangle_S$$

and to construct the extremal polynomial(the Zolotarev problem in a quadratic metric).
Using the properties of the corresponding polynomial systems $\{\hat{q}_n\}$ we solve this problem for the different discrete and continuous Sobolev spaces (in particular, for the Gegenbauer — Sobolev discrete space, Jacobi-type loading spaces, and Legendre — Sobolev continuous space).

The work of the author was partially supported by RFFI (Grant 05-01-00192).

On the exact values of the best relative approximations of certain functional classes by splines

Natalia Parfinovich, Dnepropetrovsk National University

Let L_p $(1 \le p \le \infty)$ be the spaces of 2π–periodic functions $f : \mathbb{R} \to \mathbb{R}$ with corresponding norms $\|\cdot\|_p$, M and M' be the certain subsets of L_p.

The value

$$E(M, H \cap M')_p = \sup_{f \in M} \inf_{h \in H \cap M'} \|f - h\|_p$$

is called the best relative approximation of the set M by the set $H \subset L_p$ in the space L_p.

We denote by W_p^r $(r \in \mathbb{N})$ the class of functions $f \in L_p$ such that $f^{(r-1)}$ $(f^{(0)} := f)$ is locally absolutely continuous and $\|f^{(r)}\|_p \le 1$.

In addition, let $S_{2n,r}$ $(n, r \in \mathbb{N})$ be the spaces of polynomial splines of order r defect 1 with knots in the points $\dfrac{k\pi}{n}$ $(k \in \mathbb{Z})$ and $\varphi_{\lambda,r}(\cdot)$ $(\lambda > 0,\ r \in \mathbb{N})$ be the r–th $\dfrac{2\pi}{\lambda}$–periodic integral of $\varphi_{\lambda,0}(x) = \operatorname{sign} \sin \lambda x$ with zero mean value on the period.

The values of the numbers $M_n > 0$ under which the best relative approximations of classes W_1^r by sets $S_{2n,r+k} \cap M_n W_1^{r+k}$ in the metric L_1 coincide with the L_1–approximations of these classes by sets $S_{2n,r+k}$ without restrictions are found.

Theorem. *For all* $r = 2, 3, ...,$ $k = 1, 2, ...$ *the following correlations are valid.*
If $M_n \ge r n^k \dfrac{\|\varphi_{1,r}\|_\infty}{\|\varphi_{1,r+k}\|_\infty}$, *then*

$$E(W_1^r, S_{2n,r+k} \cap M_n W_1^{r+k})_1 = \frac{\|\varphi_{1,r}\|_\infty}{n^r}$$

for all $n = 1, 2, ...$
If $M_n < (1-\varepsilon) r n^k \dfrac{\|\varphi_{1,r}\|_\infty}{\|\varphi_{1,r+k}\|_\infty}$ $(0 < \varepsilon < 1)$, *then*

$$E(W_1^r, S_{2n,r+k} \cap M_n W_1^{r+k})_1 > \frac{\|\varphi_{1,r}\|_\infty}{n^r}$$

for all $n > \max\left\{\dfrac{2^r - r - 1}{\varepsilon^2 r}, \dfrac{2^k(2^k - 1)}{\varepsilon}\right\}$.

Let us note that the same problem in the case $r = 3, 4, ...$ and $k = 0$ was solved by author [1], a similar questions as to relative widths of classes W_1^r and W_∞^r were studied by Yu.N. Subbotin and S.A. Telyakovskii [2].

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Inner functions in the Besov-type space

Fernando Pérez-González, Universidad de La Laguna

The talk will based on a joint work with Jouni Rättyä. Let $F(p,q,s)$ be the space of all functions f, analytic in the unit disc $\mathbb{D}$, such that $\sup_{a\in\mathbb{D}}\int_{\mathbb{D}}|f'(z)|^p(1-|z|^2)^q g^s(z,a)\,dA<\infty$, where $g(z,a)$ is the Green's function of $\mathbb{D}$. It is shown that a singular inner function of the form $\exp\left(\gamma\frac{z+w}{z-w}\right)$, where $0<\gamma<\infty$ and $|w|=1$, belongs to $F(p,q,s)$, $0<s\le 1$, if and only if $p\le q+\frac{s+3}{2}$. Morever, it is proved that, if $0<s<1$, then an inner function S belongs to the Möbius invariant Besov-type space $B^p_s=F(p,p-2,s)$ for some $p>\max\{s,1-s\}$ (equivalently for all $p>\max\{s,1-s\}$) if and only if S is a Blaschke product whose zero sequence $\{z_n\}$ satisfies

$$\sup_{a\in\mathbb{D}}\sum_{n=1}^{\infty}(1-|\varphi_a(z_n)|^2)^s<\infty,$$

where $\varphi_a(z)=\frac{a-z}{1-\bar{a}z}$.

Hardy and Bergman spaces on hyperconvex domains and their composition operators

Evgeny Poletsky, Syracuse University
Michael Stessin, University at Albany

We will discuss:

1. Definitions of Hardy and Bergman spaces on hyperconvex domains and basic properties of such spaces.

2. The notion of Nevanlinna counting function for holomorphic functions on such domains.

3. The basic technical tool: change of variables formula.

4. Applications of these results to composition operators generated by holomorphic mappings between hyperconvex domains. We will give sufficient and necessary conditions for such operators to be bounded or compact.

Invariant subspaces of parabolic non-automorphisms in the Hardy space

Manuel Ponce Escudero, University of Sevilla

It will be shown that the lattice of invariant subspaces of the operator of multiplication by a cyclic element of a Banach algebra consists of the closed ideals of this algebra. This result is used to find the lattice of invariant subspaces of composition operators acting on the Hardy space and whose inducing symbol is a parabolic non-automorphism self-map of the unit disk. Thus,

$$\operatorname{Lat} C_\varphi = \bigl\{\overline{\operatorname{span}}\,\{e_t\colon t \in F\}\colon F \in \mathbb{F}[0,\infty)\bigr\},$$

where $\mathbb{F}[0,\infty)$ is the set of closed subset of $[0,\infty)$. In particular, each invariant subspace always consists of the closed span of a set of eigenfunctions of the composition operator C_φ.

Joint work with Alfonso Montes-Rodriguez and Stanislav A. Shkarin.

On the Computations of Contiguous Relations for ${}_2F_1$ Hypergeometric Series

Medhat A. Rakha, Sultan Qaboos University

The hypergeometric function ${}_2F_1[a_1, a_2; a_3; z]$ plays an importance role in mathematical analysis and its application. Gauss defined two hypergeometric functions to be contiguous if they have the same power-series variable, if two of the parameters are pairwise equal, and if the third pair differe by ± 1. He showed that a hypergeometric function and any two other contiguous to it are linearly related.

In this paper, we will present a new formula joining three Gauss functions of the form ${}_2F_1[a_1, a_2; a_3; z]$ with arbitrary integer shifts. Our analysis relies on the use of shifted operators attached to the three parameters a_1, a_2 and a_3. We also, discuss the sufficient conditions for such formula.

Extremal properties of some modified Bessel function approximations

Juri M. Rappoport, Russian Academy of Sciences

The extremal problems of the Lanczos Tau method's application for the numerical solution of the second order differential equations with polynomial coefficients are elaborated. The computational scheme of Tau method is extended for the systems of hypergeometric type differential equations [1]. The extremal properties of various vector perturbations are discussed. Our choice of the perturbation term is a shifted Chebyshev polynomial with a special form of selected transition and normalization. The extremal conditions for the perturbation term are found for one equation. They are sufficiently simple for the verification in a number of important cases. Several approaches for the computation of kernels of Kontorovich–Lebedev integral transforms–modified Bessel functions of the second kind with pure imaginary order $K_{i\beta}(x)$ and with complex order $K_{1/2+i\beta}(x)$ are elaborated. The codes of the evaluation are constructed and tables of the modified Bessel functions $K_{1/2+i\beta}(x)$ are published. The advantages of discussed algorithms and codes in accuracy and timing are shown [2].

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Cyclicity of Volterra type operators

A. Rodríguez-Martínez, University of Seville
S. Shkarin, Queen's University

Essentially, the only known examples of quasinilpotent supercyclic operators are the weighted shifts studied by Hilden, Wallen and Salas. Here, we present new examples of quasinilpotent supercyclic operators. Their spectrum will be also exhibited.

The Hardy space of a slit domain

William T. Ross, University of Richmond

In this joint work with Alexandru Aleman and Nathan Feldman, we completely characterize the invariant subspaces for the Hardy space of certain types of slit domains.

Discrete minimal energy problems

Edward B. Saff, Vanderbilt University

For a compact set A in Euclidean space we shall investigate the asymptotic behavior of optimal (and near optimal) N-point configurations that minimize the Riesz s-energy (corresponding to the potential $\frac{1}{r^s}$ for $s > 0$ and $\log(\frac{1}{r})$ for $s = 0$) over all N-point subsets of A, where r denotes Euclidean distance. If A has finite and positive d-dimensional Hausdorff measure and $s < d$, then the analysis of such points falls under the umbrella of classical potential theory and is a consequence of the continuous theory. But what if $s > d$ or $s = d$? In such cases, the classical theory does not apply since A has s-capacity zero and so new techniques are needed to analyze the behavior of minimal energy configurations. We shall describe these techniques, which also yield information about "best-packing points" on A; that is, N points of A for which the minimal pairwise distance is as large as possible.

On certain integral operators

Pravati Sahoo, Banaras Hindu University

Let $\mathcal{A}$ be the class of functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, which are analytic in the open unit disc E. We consider the following one-parameter families of integral operators and study them in a new class of univalent functions

$$P_\beta^\alpha f(z) = \frac{(\beta+1)^\alpha}{\Gamma(\alpha) z^\beta} \int_0^z \tau^{\beta-1} (log\frac{z}{t})^{\alpha-1} f(t) dt (\alpha \geq 0; \beta > -1; z \in E)$$

$$Q_\beta^\alpha f(z) = \binom{\alpha+\beta}{\beta} \frac{\alpha}{z^\beta} \int_0^z (1-\frac{t}{z})^{\alpha-1} t^{\beta-1} f(t) dt (\alpha \geq 0; \beta > -1; z \in E)$$

and

$$J_\beta f(z) = \frac{\beta+1}{z^\beta} \int_0^z t^{\beta-1} f(t) dt (\beta > -1; z \in E)$$

where Γ denotes the familiar Gamma function.

Old and new on the component problem for composition operators on H^2

Eero Saksman, University of Jyvaskyla

We will review what is known in the component problem of Shapiro and Sundberg for analytic composition operators on the Hardy space H^2. Some new results are also presented. The talk is based on a joint work with Maria Jose Fuentes (Cadiz), Eva Gallardo (Saragoza), and Pekka Nieminen (Helsinki).

Distances from Bloch-type functions to some spaces of BMOA-type

Romi Shamoyan, Bryansk State University

In [1],[2] a simple proof of a theorem of Peter W. Jones was provided. This proof and some generalizations to the case of fractional derivatives and general weights will be discussed.

[1] R.Zhao;Distances from Bloch functions to some Möbius invariant spaces,Preprint;2006
[2] R.Zhao; Distances from Bloch functions to some Möbius invariant spaces,Abstracts of the conference,Analytic function spaces,Finland;2006.

Parameterization of multivariate ideal projectors

Boris Shekhtman, University of South Florida

In the talk we present a convenient parameterization of ideal projectors onto the space of polynomials of fixed degree via a certain variety. We show how the properties of the resulting variety reflect the properties of ideal projectors. In particular we present solution to two questions asked by Carl de Boor. This demonstrates a significant difference between bivariate and trivariate case.

A flower structure of backward flow invariant domains for semigroups and rigidity of holomorphic mappings

David Shoikhet, Galilee Research Center

This talk is based on joint work with M. Elin and L. Zalcman. We study the asymptotic behavior of semigroups generated by holomorphic functions by using infinitesimal versions of the Schwarz-Wolff Lemma and the Julia-Caratheodory Theorem.We consider conditions which ensure the existence of backward flow invariant domains for semigroups of holomorphic self-mappings of a simply connected domain D. More precisely, the problem is the following. Given a one-parameter semigroup S on D, find a simply connected subset U in D such that each element of S is an automorphism of U, in other words, such that S forms a one-parameter group on U. On the way to solving this problem, we prove an angle distortion theorem for starlike and spirallike functions with respect to interior and boundary points. Further we study the eigen-value problem of composition operators defined by Schroeder's functional equation for a semigroup with a boundary Wolff's point and establish new rigidity results in the spirit of Burns and Krantz.

Some extremal problems in predictability of severe weather events: modeling and statistics

Mikhail Shvartsman, University of St. Thomas

The ensemble forecasts and probabilistic forecasts are compared to rate various notions of predictability. We show that the patterns near the maximal values of helicity correspond to the solutions that are sensitive to initial conditions (Lorentz instability) in models of mesoscale meteorology.

Semigroups of composition operators in $BMOA$ and the extension of a theorem of Sarason

Aristomenis G. Siskakis, Aristotle University of Thessaloniki

A one-parameter semigroup of analytic functions is any continuous homomorphism $\Phi : t \mapsto \Phi(t) = \varphi_t$ from the additive semigroup of nonnegative real numbers into the composition semigroup of all analytic functions which map the unit disc $\mathbb{D}$ into itself. Each such semigroup of functions induces a semigroup of composition operators $T_t(f) := f \circ \varphi_t$ on the space $\mathcal{H}(\mathbb{D})$ of all analytic functions on $\mathbb{D}$. If X is a linear subspace of $\mathcal{H}(\mathbb{D})$ which is a Banach space in its own norm and on which all composition operators T_t are bounded, there arises the question whether the operator semigroup $\{T_t\}$ is strongly continuous on X. When X is a Bergman, Hardy or Dirichlet space the answer is always yes, independently of the inducing semigroup $\{\varphi_t\}$. For H^∞ the answer is always no. For other choices of X, such as the space $BMOA$ or the disc algebra, the answer depends heavily on the particular inducing semigroup $\{\varphi_t\}$.

In this talk we concentrate on the space $BMOA$. Given a semigroup of functions $\Phi = \{\varphi_t\}$ we find that there is a maximal subspace $V_\Phi \subset BMOA$ on which the corresponding operator semigroup is strongly continuous. This subspace contains $VMOA$ in all cases, an we find conditions on $\{\varphi_t\}$ under which V_Φ is exactly equal to $VMOA$. Particular cases of this are related to a well-known theorem of Sarason about $VMOA$, and we provide a generalization.
Joint work with O. Blasco, M. D. Contreras, S. Díaz-Madrigal and J. Martínez.

Multidimensional analogs of the I. Schur and V. Markov inequalties

Valentin Skalyga, Ametist Design Buro Moscow

Multidimensional genearlizations of Schur and Markov inequalites are discussed, and presented as two theorems. This research was supported by the Russian Foundation for Basic Research under grant No. 05-01-0027

On solvability of some nonlinear equations and fixed-point theorem

Kamal N. Soltanov, Hacettepe University Beytepe

In this talk we will study continuous mappings acting in a reflexive Banach space. Here problems of the following type are considered: when does a (semi-) continuous mapping possesses a fixed-point; when can the image of a (semi-) continuous mapping of subsets from the domain be described, and a nonlinear equation with a continuous mapping is solvable. This article is generalization of [1] and moreover the results obtained here are such that they can also be used for the study of mixed problems for equations of parabolic and hyperbolic types.

Let X be a reflexive Banach space with strongly convex norm together his dual space X^*, $f : D(f) \subseteq X \to X^*$ be a nonlinear mapping and G be an convex body of $D(f)$. Thus we study the role of a local relation between geometrical disposition of each point $x \in G \subseteq D(f)$ with its image $f(x) \in f(G)$ through investigation of the image $f(G)$ of mapping f, consequently as a whole. Moreover the solvability of equation $f(x) = y$ (and inclusion $f(x) \ni y$) is also investigated here. In particular, here is proved a fixed-point theorem.

Let X be as above, and $f : D(f) \subseteq X \to X^*$. Let on $B^X_{r_0}(0) \subseteq D(f)$ $(r_0 > 0)$ the following conditions holds: i) f is continuous mapping (i.e. $f \in C^0$); ii) there are nondecreasing under $\tau \geq \tau_0$ functions $\mu, \nu \in C^0$ such that $\|f(x) - f(0)\|_{X^*} \leq \mu(\|x\|_X)$ for any $x \in B^X_{r_0}(0)$ and $\langle f(x) - f(0), x\rangle \geq \nu(\|x\|_X)\|x\|_X$ almost any $x \in B^X_{r_0}(0)$, moreover $\nu(r_0) \geq \delta_0 > 0$. Then $f\left(B^X_{r_0}(0)\right)$ is contained a subset such that is dense in the subset $M^*_0 \equiv \left\{x^* \in X^* \mid \langle f(x) - x^*, x\rangle \geq 0, \ \forall x \in S^X_{r_0}(0)\right\}$

(**Fixed point Theorem**) Let X be as above and $B^X_r(x_0) \subset X$ is a closed ball. Let the mapping f acting in X satisfy on ball $B^X_r(x_0)$ the following conditions: $f \in C^0$, $f\left(B^X_r(x_0)\right) \subseteq B^X_r(x_0)$ and $\|f(x) - x_0\|_X \leq \mu(\|x - x_0\|_X)$, here $\mu \in C^0$ such that function $\nu_0(\tau) \equiv \tau - \mu(\tau)$ satisfy the condition of Theorem 1 with the duality mapping $\Im$. If the image $f_1\left(B^X_r(x_0)\right)$ is closed for the mapping $f_1 : f_1(x) \equiv x - f(x)$, $\forall x \in B^X_r(x_0)$, then f possess of the fixed point in $B^X_r(x_0)$. (Here $\Im$ such that $X \overset{\Im}{\Longleftrightarrow} X^*$: $\forall x \in X$, $\Im(x) = x^* \in X^*$, $\langle x, x^*\rangle = \|x\|_X \cdot \|x^*\|_{X^*}$.)

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On the superconvergence of splines

Nikolay A. Strelkov, Yaroslavl State University

Let $S_n(x) = S_n(x, f, h)$ be interpolation spline of order n (for f) such that the following conditions are satisfied:

1) for all $i \in \mathbb{Z}$ the restriction of S_n to segment $[ih+(n-1)h/2, ih+(n+1)h/2]$ is the algebraic polynomial of degree n;

2) $S_n(ih) = f(ih)$ for all $i \in \mathbb{Z}$;

3) $S_n \in C^{n-1}(\mathbb{R})$.

Note that for every f there exists infinite set of its interpolation splines. Nevertheless if $\|f\|_{n,h} < \infty$, then there exists a unique interpolation spline S_n such that $\|S_n\|_n < \infty$.

Let $\Xi_n^{(k)}$ be the sets of the following type (here $n \in \mathbb{Z}^+, k = 0, \dots, n$):

$$\Xi_{2r}^{(2s)} = \begin{cases} \frac{1}{2}\mathbb{Z}, & r \geq 1,\ s = 0, \dots, r-1, \\ \mathbb{Z}, & s = r \geq 0; \end{cases}$$

$$\Xi_{2r}^{(2s+1)} = \pm\mu_{r-s} + \mathbb{Z},\ r \geq 1,\ s = 0, \dots, r-1;$$

$$\Xi_{2r+1}^{(2s)} = \begin{cases} \mathbb{Z}, & r \geq 0,\ s = 0, \\ \frac{1}{2} \pm \mu_{r-s+1} + \mathbb{Z}, & r \geq 1,\ s = 1, \dots, r; \end{cases}$$

$$\Xi_{2r+1}^{(2s+1)} = \begin{cases} \frac{1}{2}\mathbb{Z}, & r \geq 1,\ s = 0, \dots, r-1, \\ \frac{1}{2} + \mathbb{Z}, & s = r \geq 0, \end{cases}$$

where $\mu_k \in (0, 1/2)$ such that $1/2 \pm \mu_k$ is the pair of zeros of Bernoulli polynomial $B_{2k}(x)$.

THEOREM. *If $n \in \mathbb{Z}^+$ and $f \in L_\infty^{n+2}$, then*

$$\sup_{x \in \Xi_n^{(k)}} |(f - S_n)^{(k)}(xh)| \leq C_{k,n} h^{n+2-k} \|f^{(n+2)}\|_{L_\infty(\mathbb{R})}$$

for all $k = 0, \dots, n$.

Moreover, function $F_n : \mathbb{R}^2 \to \mathbb{R}$ (depending on some Appel–type polynomials) is constructed such that optimal constants $C_{k,n}$ are described in terms of F_n–extremal properties.

Extremal problems and zero sets

Carl Sundberg, University of Tennessee

We discuss various facts, old and new, known and unknown, true and untrue, concerning zero sets of spaces of analytic functions and extremal problems in those spaces.

Set separation and nonsmooth necessary conditions for optimality of curves via primal methods

Hector J. Sussmann, Rutgers University

There are two approaches for proving versions of the finite-dimensional Pontryagin Maximum Principle (PMP). One of them is "primal," using needle variations and a topological argument about set separation based on some version of the Brouwer fixed point theorem. The other one is "dual," and uses arguments based on limiting normal covectors. Until recently, it was hoped that these two approaches could be combined into one, yielding a unified version of the necessary conditions for optimality of curves. A recent couterexample due to A. Bressan has shown that this is almost certainly impossible, because there are "hybrid" cases, combining the technical conditions of both types of results, where the PMP is simply not true. The talk will explain how this happens, and will present some recent work in which the primal approach, involving a new theory of approximating multicones based on J. Warga's derivate containers, has been used to derive results usually proved by dual methods.

Non-smooth analysis: A historical prespective

Milosh Tadeusz, Warsaw University

An outline of the history of non-smooth analysis will include a survey of results of Clark, Ioffe, Mordukhovich, Varga and others.

Extremal problems, convex analysis and optimal methods of recovery

V.M. Tikhomirov, Moscow State University

1. Lagrange principle in the theory of extremal problems.
2. Duality and calculus in convex analysis.
3. Optimal methods of recovery and
   a) approximation of individual elements;
   b) inequalities of polynomial derivatives;
   c) inequalities of smooth and analytical functions;
   d) approximation of classes;
   e) n-widthes.

Sampling on sparse grids

Tino Ullrich, University of Jena

We consider a d-variate complex valued, periodic and continuous function $f : \mathbb{T}^d \to \mathbb{C}$, where d may be large. Our aim is to approximate f by sampling operators and investigate the error in the L_p-metric. Such a sampling process $\{A_m\}_m$ uses only discrete information about the function f in the following way

$$A_m f(x) = \sum_{k=1}^{Nm} f(x_k)\psi_k(x), x \in \mathbb{T}^d;$$

where the sampling points $x_k \in \mathbb{T}^d$ and the functions $\psi_k : \mathbb{T}^d \to \mathbb{C}, k = 1; ...; N_m$, are fixed. We will focus on the Smolyak algorithm. Starting with a sequence of sampling operators for the univariate case we obtain, via a special tensor product construction, a sequence $\{A_m\}_m$ of sampling operators for the d-variate case acting on a so called sparse grid. This construction provides useful properties. Having more information about f, for instance, f belonging to some periodic Sobolev space with dominating mixed derivative, we are able to prove $L_p(\mathbb{T}^d)$-error estimates depending on the size of the used sampling grid and present new upper bounds for the problem of optimal recovery of functions.

Reconstruction of holomorphic functions in the disc from their values on two disjoint arcs of the unit circle

Alekos Vidras, University of Cyprus

In the talk we describe the class of holomorphic functions in the disc representable by Carleman formula supported on two disjoint arcs of the unit circle. Some preliminary extension results are discussed.

Optimal recovery of solutions of the wave equation from inaccurate initial conditions

Natalia Vysk, Russian State Technological University

Optimal recovery problems of solutions of the wave equation are considered. We assume that the first N Fourier coefficients of the function defined the initial form of the string are known with some errors in L_2 and L_∞ metrics. The solution of this problem follows from a more general optimal recovery problem of linear operators from inaccurate initial data. It will be shown that an optimal recovery method uses only a part of the given information.

Optimal recovery of linear ordinary differential equations system solutions with self-adjoined matrix of constant coefficients and simple eigenvalues

Elena Wedenskaya, Russian State Technological University

We study the problem of optimal recovery of the linear differential equations system solutions from inaccurate data. We have solutions of the system at times $t = 0$ and $t = T > 0$ given with the errors δ_0 and δ_T in the Euclidean norm. An optimal method of recovery of the solution at time moment $\tau : 0 < \tau < T$ is obtained, and the error of optimal recovery is calculated.

Smoothness and tractability of multivariate approximation

Henryk Woźniakowski, Columbia University, University of Warsaw, University of Jena

We consider multivariate approximation for smooth classes of d variate functions. Let $n(\varepsilon, d)$ denote the minimal number of linear functionals that is necessary for solving the d variate approximation problem to within ε. Typically, $n(\varepsilon, d)$ goes to infinity as ε goes to zero but the speed of convergence of $n(\varepsilon, d)$ decreases with the increased smoothness of functions. For example, for infinitely differentiable functions, for any positive r we have $n(\varepsilon, d) = o(\varepsilon^{-r})$.

Tractability means that $n(\varepsilon, d)$ does *not* depend exponentially on d. Does large or infinite smoothness imply tractability? This is the question we address in our talk. It turns out that the answer depends on the norm of the target space and in many cases is negative. That is, we have the *curse of dimensionality even for infinite smoothness* since $n(\varepsilon, d)$ depends exponentially on d. The talk is based on joint work with Erich Novak.

Invariant subspaces in the Hardy space over the bidisk

Rongwei Yang, University at Albany

A closed susbpace in the classical Hardy space $H^2(D)$ is said to be invariant if it is invariant under multiplication by coordinate function z. The well-known Beurling's Theorem characterizes the invariant subspace in this classical setting, and the characterization turned out to have far reaching impacts on function theory and operator theory. In the Hardy space over the bidisk $H^2(D^2)$, a closed susbpace M is said to be invariant if it is invariant under multiplication by both coordinate functions z_1 and z_2. Invariant subspaces in this setting are known to be much more complex, and a complete characterization seems far from reach. Nevertheless, much progresses have been made in recent years. This talk surveys some of the developments.

A new class of uniform functions

V.K. Zakharov, Moscow State University

New class of uniform functions and its connection with integral representation of functionals and with week compactness of Radon measures.

Harmonic polynomial approximation and interpolation

Vyacheslav Zakharyuta, Sabanci University

Methods of Lh-potential theory, developed in [1] [2], proved to be an adequate tool for investigating some classical problems on harmonic functions (isomorhic classification of spaces of harmonic functions, extendible bases, separate harmonicity, orthogonal polynomials, etc.) that are unmanagable by traditional potential theory methods. Lh-functions and maximal Lh-functions play the same basic role as plurisubharmonic functions and maximal plurisubharmonic functions do in analogous applications of pluripotential theory to analytic functions of several complex variables.

We discuss here how those methods help in solving some problems on harmonic polynomial approximation and interpolation in $\mathbb{R}^n$ [3,4]. In particular, we demonstrate an irreducible gap between necessary and sufficient conditions in results that describe optimal domains of harmonicity corresponding to a given rate of convergence of the best harmonic polynomial approximations or extremal harmonic interpolation polynomials.

References

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[3] Skiba N. and Zahariuta V., *Bernstein-Walsh Theorems for harmonic functions in* $\mathbb{R}^n$, Israel Mathematical Conferences Proceedings 15 (2001), 357-382.

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Various kinds of resovent conditions

Jaroslav Zemanek, Institute of Mathematics of the Polish Academy of Sciences

We investigate various kinds of resolvent conditions and their mutual relations. The recent joint work with A. M. Gomilko will reported.

Application of interpolated methods to the free boundary problems.

A.A. Zhensykbaev, Institute of Mathematics

We consider the interpolation of multivariate functions by the following set of interpolants

$$S_{nr} = \left\{ s(x,t) = \frac{2}{r!\sqrt{\pi}} \int_{-\infty}^{\infty} \sum_{i=1}^{n} c_i (2a\sqrt{t-t_i}\,\xi - x)_+^r \, e^{-\xi^2} \, d\xi \right\}$$

It is based on the Hartry functions

$$e_n(x) = \frac{2}{n!\sqrt{\pi}} \int_x^{\infty} (\xi - x)^n e^{-\xi^2} d\xi.$$

In view of $u_n(c \pm x, t) := t^{\frac{n}{2}} e_n\left(\frac{c \pm x}{2a\sqrt{t}}\right)$ are solutions of the heat conductivity equation we apply these interpolants to the approximate solving of free boundary problems for the parabolic equations:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 \le x \le \alpha(t), \quad t \in [0,T],$$

$$u(x,0) = f(x),$$

$$u(0,t) = f(t),$$

$$u(\alpha(t),t) = g(t).$$

Pointwise estimates of extremal functions for invariant subspaces of weighted Bergman spaces

Kehe Zhu, University at Albany

Let $\omega(z)$ be a weight function on the unit disk and $K^\omega(z,w)$ be the reproducing kernel of the weighted Bergman space A^2_ω. If ω is logarithmically convex or if ω is a standard radial weight, we show that the extremal function φ for any invariant subspace of the weighted Bergman space A^p_ω must satisfy the pointwise estimate $|\varphi(z)|^p \leq (1-|z|^2)K^\omega(z,z)$. Certain properties of the kernel of the little hankel operator h_φ, which is an invariant subspace of A^2_ω, will also be discussed.

Necessary and sufficient conditions of the solvability of the Gauss variational problem

Natalia Zorii, National Academy of Sciences of Ukraine

We shall be concerned with the well-known Gauss variational problem on the minimum of the energy in the presence of an external field, the infimum being taken over fairly general classes of signed Radon measures in a locally compact space. We shall show that in the noncompact case the problem is in general unsolvable, and this occurs even under extremely natural assumptions (in particular, for the Newtonian, Green, or Riesz kernels in an Euclidean space). Necessary and sufficient conditions for the problem to be solvable will be given. Some related extremal problems are also supposed to be discussed.