Abstract. I will survey the joint work with Gunnar Carlsson on the old conjecture of Armand Borel in topology. The conjecture states that if a closed aspherical manifold $M$ is homotopy equivalent to another manifold then the two manifolds have to be homeomorphic. The aspherical condition is equivalent to the universal cover of $M$ being contractible, which is common in geometry. Our approach studies the $K$-theoretic assembly map associated to $\pi_1(M)$ by factoring it through a controlled version of Grothendieck’s $G$-theory of the group ring $\mathbb{Z}\pi_1(M)$. The $G$-theory turns out to be easier to compute and is equivalent to $K$-theory in very general geometric situations, for example when $\pi_1(M)$ has finite decomposition complexity.