ABSTRACT. An interesting classical problem is the classification of integral lattices up to isomorphism. It turns out that self-dual (even, positive definite) integral lattices exist only for rank a multiple of 8, and that low rank examples are related to many beautiful, highly symmetrical objects—such as the E8 lattice in rank 8, and the 24 Niemeier lattices in rank 24.

The problem for lattices generalizes very naturally to the classification of simple vertex algebras. I will speak about recent progress (joint work with N. Scheithauer and S. Moeller) on the classification of these algebras in rank 24, which are conjecturally 71 in number. More precisely, I will explain our determination of the fusion rings of fixed-point vertex algebras, and the classification of possible affine structures on simple vertex algebras in rank 24.