First problem.
For each of the following six questions, four possible answers are provided, but only one of them is correct: write the corresponding letter in the box!

1. Let \( f: X \to Y \) be a function. Let \( x \) and \( x' \) be elements of \( X \) such that \( f(x) = f(x') \).
   What do we need to know about \( f \) to conclude that \( x = x' \)?
   A] Nothing: this is true for all functions \( f \).
   B] We need \( f \) to be injective.
   C] We need \( f \) to be surjective.
   D] We need \( f \) to be bijective.

2. Let \( f: X \to Y \) be a function. Let \( x \) and \( x' \) be elements of \( X \) such that \( x = x' \).
   What do we need to know about \( f \) to conclude that \( f(x) = f(x') \)?
   A] Nothing: this is true for all functions \( f \).
   B] We need \( f \) to be injective.
   C] We need \( f \) to be surjective.
   D] We need \( f \) to be bijective.

3. Let \( f: X \to Y \) be a function. Let \( y \) be an element of \( Y \).
   What do we need to know about \( f \) to conclude that \( y = f(x) \) for some \( x \in X \)?
   A] Nothing: this is true for all functions \( f \).
   B] We need \( f \) to be injective.
   C] We need \( f \) to be surjective.
   D] We need \( f \) to be bijective.

4. Let \( f: X \to Y \) be a function. Let \( y \) be an element of \( Y \).
   What do we need to know about \( f \) to conclude that \( y = f(x) \) for exactly one \( x \in X \)?
   A] Nothing: this is true for all functions \( f \).
   B] We need \( f \) to be injective.
   C] We need \( f \) to be surjective.
   D] We need \( f \) to be bijective.

5. Let \( f: X \to Y \) be a function. Let \( y \) be an element of \( Y \).
   What do we need to know about \( f \) to conclude that \( y = f(x) \) for at most one \( x \in X \)?
   A] Nothing: this is true for all functions \( f \).
   B] We need \( f \) to be injective.
   C] We need \( f \) to be surjective.
   D] We need \( f \) to be bijective.

6. Let \( f: X \to Y \) be a function. Let \( x \) be an element of \( X \).
   What do we need to know about \( f \) to conclude that \( f(x) = y \) for exactly one \( y \in Y \)?
   A] Nothing: this is true for all functions \( f \).
   B] We need \( f \) to be injective.
   C] We need \( f \) to be surjective.
   D] We need \( f \) to be bijective.
Second problem.
Let $X$ and $Y$ be sets, and $\varphi : X \to Y$ a function. Suppose that $W$ is a subset of $X$ and $Z$ is a subset of $Y$. Write the definitions of $\varphi(W)$ and of $\varphi^{-1}(Z)$.

Third problem.
Let $A$ and $B$ be sets, and let $f : A \to B$ be a function. Suppose that $A'$ and $A''$ are subsets of $A$, and that $B'$ is a subset of $B$. Are the following implications true or false? Prove or disprove them.

1. $B' \subset f(A' \cap A'') \implies B' \subset f(A')$ and $B' \subset f(A'')$  
   TRUE | FALSE

2. $B' \subset f(A')$ and $B' \subset f(A'') \implies B' \subset f(A' \cap A'')$  
   TRUE | FALSE